Annexure 1. (a) Manchester news release of 13 Aug 2007 and (b) some derived news reports.

MANCHESTER 1824

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Indians predated Newton 'discovery' by 250 years

13 Aug 2007

A little known school of scholars in southwest India discovered one of the founding principles of modern mathematics hundreds of years before Newton according to new research.

Dr George Gheverghese Joseph from The University of Manchester says the 'Kerala School' identified the 'infinite series'- one of the basic components of calculus - in about 1350.

The discovery is currently - and wrongly - attributed in books to Sir Isaac Newton and Gottfried Leibnitz at the end of the seventeenth centuries.

The team from the Universities of Manchester and Exeter reveal the Kerala School also discovered what amounted to the Pi series and used it to calculate Pi correct to 9, 10 and later 17 decimal places.

And there is strong circumstantial evidence that the Indians passed on their discoveries to mathematically knowledgeable Jesuit missionaries who visited India during the fifteenth century.

That knowledge, they argue, may have eventually been passed on to Newton himself.

Dr Joseph made the revelations while trawling through obscure Indian papers for a yet to be published third edition of his best selling book 'The Crest of the Peacock: the Non-European Roots of Mathematics' by Princeton University Press.

He said: "The beginnings of modern maths is usually seen as a European achievement but the discoveries in medieval India between the fourteenth and sixteenth centuries have been ignored or forgotten.

"The brilliance of Newton's work at the end of the seventeenth century stands undiminished - especially when it came to the algorithms of calculus.

"But other names from the Kerala School, notably Madhava and Nilakantha, should stand shoulder to shoulder with him as they discovered the other great component of calculus-infinite series.

"There were many reasons why the contribution of the Kerala school has not been acknowledged - a prime reason is neglect of scientific ideas emanating from the Non-European world - a legacy of European colonialism and beyond.

"But there is also little knowledge of the medieval form of the local language of Kerala, Malayalam, in which some of most seminal texts, such as the Yuktibhasa, from much of the documentation of this remarkable mathematics is written."

He added: "For some unfathomable reasons, the standard of evidence required to claim transmission of knowledge from East to West is greater than the standard of evidence required to knowledge from West to East.

"Certainly it's hard to imagine that the West would abandon a 500-year-old tradition of importing knowledge and books from India and the Islamic world.

"But we've found evidence which goes far beyond that: for example, there was plenty of opportunity to collect the information as European Jesuits were present in the area at that time.

"They were learned with a strong background in maths and were well versed in the local languages.

"And there was strong motivation: Pope Gregory XIII set up a committee to look into modernising the Julian calendar.

"On the committee was the German Jesuit astronomer/mathematician Clavius who repeatedly requested information on how people constructed calendars in other parts of the world. The Kerala School was undoubtedly a leading light in this area.

"Similarly there was a rising need for better navigational methods including keeping accurate time on voyages of exploration and large prizes were offered to mathematicians who specialised in astronomy.

"Again, there were many such requests for information across the world from leading Jesuit researchers in Europe. Kerala mathematicians were hugely skilled in this area."

NOTES FOR EDITORS

The research was carried out by Dr George Gheverghese Joseph, Honorary Reader, School of Education at The University of Manchester and Dennis Almeida, Teaching Fellow at the School of Education, The University of Exeter.

Dr Joseph and Mr Almeida are available for comment.

Images are available

Copies of a related research paper are available.

For more details, contact:
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Media Relations Officer
Faculty of Humanities
The University of Manchester
0161 275 0790
07717 881567

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Royal Charter Number: RC000797

HINDUSTAN TIMES 14 AUG p. 1

ANALYSE THIS

Kerala scholars cracked math code before Newton

JAYANTO

Vijay Dutt
London, August 13

A LITTLE-KNOWN school of scholars in Kerala discovered one of the founding principles of modern mathematics much before Sir Isaac Newton, to whom the finding is currently attributed, a new research here says.

Dr George Gheverghese Joseph, an Honorary Reader of the University of Manchester, says the 'Kerala School of Mathematics and Astronomy,' identified the 'Infinite Series' — one of the basic components of calculus — in about 1350.

"The 'Infinite Series' was identified by these little-known scholars in Kerala all of whom were from within 500 km of Cochin," Dr Joseph, hailing from Kottayam, told HT. The scholars of the school also discovered what amounted to the Pi series and used it to calculate Pi correct to 17 decimals.

The research, carried out by teams led by Dr Joseph and Dennis Almeida of the University of Exeter, found evidence that the Indians passed on their discoveries to Jesuit missionaries who visited India during the 15th century.

vdutt@aol.com

THE HINDU 14 AUG 2007, p. 20

Calculus from Kerala School

LONDON: A little-known school of scholars in southern India discovered one of the founding principles of modern mathematics hundreds of years before Sir Isaac Newton, to whom the finding is currently attributed, according to new research findings announced here.

George Gheverghese Joseph, an academic and author, says the 'Kerala School' identified the 'infinite series', one of the basic components of calculus, circa 1350.

The discovery is attributed in books to Sir Isaac Newton and Gottfried Leibnitz at the end of the 17th century, the University of Manchester reported on in its website.

The Manchester-Exeter universities team said the Kerala School had also discovered what amounted to the Pi series and used it to calculate Pi correct to 9, 10 and later 17 decimal places.

And there is strong circumstantial evidence that Indians passed on their discoveries to mathematically savvy Jesuit missionaries who visited India during the 15th century. That knowledge, the researchers argue, may have been passed on to Newton.

The research was carried out by Dr. Joseph, Honorary Reader, School of Education at The University of Manchester and Dennis Almeida, Teaching Fellow at the School of Education, The University of Exeter.

Dr. Joseph, who hails from Kerala, made the finding while trawling through obscure Indian papers for a third edition of his book *The Crest of the Peacock: the Non-European Roots of Mathematics*, the report said.

Forgotten

Dr. Joseph said: "The beginnings of modern maths is usually seen as a European achievement but the discoveries in medieval India between the 14th and 16th centuries have been ignored or forgotten... The brilliance of Newton's work at the end of the 17th century stands undiminished — especially when it came to the algorithms of calculus. But other names from the Kerala School, notably Madhava and Nilakantha, should stand shoulder to shoulder with him as they discovered the other great component of calculus — infinite series."

Dr. Joseph attributed the non-acknowledgment of the contribution of the Kerala school to the neglect of scientific ideas emanating from the Non-European world, "a legacy of European colonialism and beyond." — PTI

TIMES OF INDIA 15 AUG 2007 p.36

Indian scholars predated Newton find by 250 yrs

London: A little-known school of scholars in south India discovered one of the founding principles of modern mathematics hundreds of years before Sir Isaac Newton, to whom the finding is currently attributed, according to new research here.

Dr George Gheverghese Joseph from The University of Manchester says the 'Kerala School' identified the 'infinite series' — one of the basic components of calculus — in about 1350.

The discovery is currently attributed in books to Sir Isaac Newton and Gottfried Leibnitz at the end of the 17th centuries, the University of Manchester reported in its website on Monday.

The team from the Universities of Manchester and Exeter reveal the Kerala School also discovered what amounted to the Pi series and used it to calculate Pi correct to 9, 10 and later 17 decimal places.

And there is strong circumstantial evidence that the Indians passed on their discoveries to mathematically knowledgeable Jesuit missionaries who visited India during the 15th century.

That knowledge, the researchers argue, may have eventually been passed on to Newton himself.

ANCIENT TRUTH

> 'The beginnings of modern maths is usually seen as a European achievement but the discoveries in medieval India between the 14th and 16th centuries have been ignored or forgotten'
>
> **Dr George G Joseph**
> UNIVERSITY OF MANCHESTER

The research was carried out by Dr George Gheverghese Joseph, Honourary Reader, School of Education at The University of Manchester and Dennis Almeida, Teaching Fellow at the School of Education, The University of Exeter.

Joseph made the revelations while trawling through obscure Indian papers for a yet to be published third edition of his best selling book *The Crest of the Peacock: the Non-European Roots of Mathematics*, the report said.

"The beginnings of modern maths is usually seen as a European achievement but the discoveries in medieval India between the 14th and 16th centuries have been ignored or forgotten," Joseph said. "The brilliance of Newton's work at the end of the 17th century stands undiminished — especially when it came to the algorithms of calculus. PTI

INDIAN EXPRESS 14 AUG 07 p.15

Indians got maths right 250 yrs before Newton: study

PRESS TRUST OF INDIA
LONDON, AUGUST 13

A LITTLE-known school of scholars in south India discovered one of the founding principles of modern mathematics hundreds of years before Sir Isaac Newton, to whom the finding is currently attributed, according to a new research here.

Dr George Gheverghese Joseph from The University of Manchester says the 'Kerala School' identified the 'infinite series' — one of the basic components of calculus — in about 1350.

The discovery is currently attributed in books to Sir Isaac Newton and Gottfried Leibnitz at the end of the 17th centuries, the University of Manchester reported in its website today.

The team from the Universities of Manchester and Exeter reveal the Kerala School also discovered what amounted to the Pi series and used it to calculate Pi correct to 9, 10 and later 17 decimal places.

And there is strong circumstantial evidence that the Indians passed on their discoveries to mathematically knowledgeable Jesuit missionaries who visited India during the 15th century.

That knowledge, the researchers argue, may have eventually been passed on to Newton himself. The research was carried out by Dr George Gheverghese Joseph, Hony Reader, School of Education at The University of Manchester and Dennis Almeida, Teaching Fellow at the School of Education at Exeter University. Joseph made the revelations while trawling through obscure Indian papers for a yet to be published third edition of his best selling book *The Crest of the Peacock: the Non-European Roots of Mathematics*, the report said.

"The beginnings of modern maths is usually seen as a European achievement but the discoveries in medieval India between the 14th and 16th centuries have been ignored," Joseph said.

Annexure 2. Email exchange with M. Addelman of 23 Aug 2007.

Your news item of 13 Aug 2007

From: **C. K. Raju** (c_k_raju@hotmail.com)
Sent: 23 August 2007 09:21AM
To: michael.addelman@manchester.ac.uk
Cc: president@manchester.ac.uk
Attachments:
Exeter Investigation Report.pdf (323.9 KB)

Mr Michael Addelman
Media Relations Officer
Manchester University

Dear Mr Addelman,

This refers to the news item "Indians predated Newton 'discovery' by 250 years" put on the Manchester University website on 13 August 2007 at

http://www.manchester.ac.uk/aboutus/news/display/index.htm?id=121685

This is to inform you that I intend to complain regarding this news item and a related research report to both Manchester and Exeter Universities.

Therefore, I must request you to preserve as evidence the news item and the related research reports (two of which you earlier supplied to my Asst. Dean). For confirmation, please supply a fresh soft copy of these reports to this email address.

As prima facie indication of the problem at hand, I attach a copy of an investigation report from the University of Exeter, dated August 2004, on my earlier complaint against Mr Dennis Almeida. Please refer to page 2, para 8 (b) "Under no circumstances should attempts be made to publish with the Aryabhata Group identified as the author."

Note that contrary to the explicit warning, one of the research reports is in the name of the "Aryabhata group". Evidence of the exact report supplied by you needs to be perserved for any future enquiry.

In case a change of any sort becomes completely unavoidable on the website, details of the changes should be logged, and incremental archives should be kept, including hard-copy archives of the news item and the research reports.

Depending upon the circumstances, I might also at some stage want to take the matter to the British National Audit Organization in which case, it would be absolutely essential that there is no material alteration of any sort.

Please acknowledge receipt of this email, and please confirm that you would be maintaining detailed records for evidence as requested.

With good wishes,

Yours sincerely,

C. K. Raju

Attachment: Exeter Investigation report.pdf (2 pages)

Copy to:

(1) The President, Manchester University
president@manchester.ac.uk

(2) Related press correspondents of newspapers which have carried the news and correction, and may wish to cross-check the facts for further corrections.

==========
C. K. Raju, Ph.D.
Distinguished Professor & Director (Academic) Inmantec
&
Editorial Fellow, Centre for Studies in Civilizations

Res: B-56, Tarang Apartments
19, I. P. Extension
Delhi 110 092
Tel: 2272-6015
Fax: 2272-4533

Re: "Your news item of 13 Aug 2007"

From: **Michael Addelman** (Michael.Addelman@manchester.ac.uk)
Sent: 23 August 2007 11:46AM
To: c_k_raju@hotmail.com

Hi. I am on leave until Tuesday 28 August. You might be
able to get hold of me for urgent enquiries by calling
my mobile on 07717 881 567. For all other enquiries
call the press office 0161 2758258.

Annexure 3. *Hindustan Times* correction of 25 Aug 2007.

HT CORRECTION

The claim made by two British researchers that they were the ones who unearthed the fact that Kerala mathematicians invented the calculus long before Sir Isaac Newton (Hindustan Times, August 14, 2007) was incorrect. The Kerala infinite series has been known to British scholars since 1832. Recent work on transmission of the calculus was first done by C.K. Raju, Editorial Fellow of the Project of History of Indian Science, Philosophy and Culture and is published in his book, Cultural Foundations of Mathematics: the Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th century. One of the British researchers, Dennis Almeida, was even warned in 2004 by Exeter University against plagiarising Raju's work." The error is regretted.

HT: 25 Aug 2007

Annexure 4.
(a) Exeter investigation report
(b) related news item of 8 Nov 2004
(c) Almeida's two apologies.

Investigation Report

1 This report arises from an investigation that we were requested by the Vice-Chancellor to undertake following the receipt of a complaint by Professor C K Raju, currently Head of the Centre for Computer Science at the Makhanlal Chaturvedi National University of Journalism, India against Mr Dennis Almeida of the School of Education and Lifelong Learning, claiming plagiarism of Professor Raju's ideas.

2 In considering this complaint, we took into account the following documentation:

(a) Professor Raju's complaint of 25 March 2004 and its attachments
(b) A response to the complaint dated 1 July 2004 by Mr Almeida
(c) Subsequent correspondence with Mr Almeida
(d) Correspondence between us and Dr George Joseph and Professor Chandra Sharma

3 We met with Mr Almeida on 12 July 2004. He was accompanied by his Head of School, Professor William Richardson.

4 At the heart of the complaint is the ownership of an idea – the transmission of some mathematical knowledge, relating to elements of calculus, from Kerala in India to Europe in the late Middle Ages and early modern period. Ownership of a general idea is difficult to prove (or disprove). We are satisfied that sufficient evidence exists of the idea being in the public domain before Professor Raju started presenting conference papers and publishing on the subject (from 1997-98). For instance, earlier publications by Bag[1], Joseph[2], and Katz[3] all refer to the possibility of such a transfer of knowledge.

5 What is undisputed is that Mr Almeida and others of a University of Exeter-based team did collaborate with Professor Raju from late 1998 to early 2000 and that during that time refinements and development of the idea would have occurred. It is wrong for Professor Raju's contribution to this collaboration not to have been given appropriate credit in publications both then and subsequently. It is acknowledgement that is required, not necessarily joint authorship, though given the team research involved it would have been sensible to include his name as a contributing author.

6 The issue of acknowledgement is compounded by the continued use by Mr Almeida of the Aryabhata Group as the author of publications which has raised the following issues:

(a) Membership of the Group is not defined.
(b) Membership is not stable; Professor Raju was included for a period of time.
(c) Disassociation is not formally recorded.

7 Mr Almeida has stated that he had the support of more experienced academics in the School of Education and Lifelong Learning for his strategy of citing authorship of work as by "The Aryabhata Group". That may have been the case, but the problems of this approach were questioned explicitly by Professor Chandra Sharma, Editor of the

---

[1] A K Bag (1979), Mathematics in Medieval and Ancient India
[2] G G Joseph (1991), The Crest of the Peacock: Non-European Roots of Mathematics
[3] V.J.Katz (1993), Ideas of Calculus in Islam and India

Journal of Natural Geometry, in which the article most prominently at the centre of this complaint was published in 2001. Ahead of publication, Professor Sharma emailed to Mr Almeida in the following terms:

"We cannot have "Aryabhata Group" as author: authors must be named individuals. In an Acknowledgement at the end you can mention that authors belong to "Aryabhata Group" and explain what it is and what support or recognition the group has received and whether it has other members besides the authors."

8 In a written submission to the investigation, Mr Almeida raises the issue of the future, asking if Professor Raju would not continue to press his allegations. That could well be the case, which underscores the need for Professor Raju's past contributions to the team research to be properly acknowledged in current and future publications that contain the fruits of the collaboration of which he was originally a part. The ethics of this seem clear:

(a) If the joint work with Professor Raju is part of the explicit content of a future paper, his contribution should be acknowledged by name.
(b) Under no circumstances should attempts be made to publish with the Aryabhata Group identified as the author. Publication must state the names of the authors; reference can be made in the publication that the authors are part of a collective team known as the "Aryabhata Group".

9 If proper, scientifically-accepted conventions for paper authorship had been followed from the outset, this case would not have arisen – Professor Raju's contribution would have been unequivocally acknowledged.

10 In conclusion:

(a) We do not consider that the world of science would accept that use of the term "Aryabhata Group" as author provides proper acknowledgement of Professor Raju's contribution. This conclusion was previously and independently communicated to Mr Almeida by the editor of a journal to which he had submitted a paper. (See paragraph 7 above.)

(b) We do not accept that the umbrella author name "Aryabhata Group" is sufficient acknowledgement of Professor Raju's contribution to the collaborative research of Mr Almeida's team 1998-2000.

(c) However, we do not consider that Mr Almeida's actions constituted a pre-meditated and deliberate attempt to plagiarise the ideas of Professor Raju with intent to pass them off as his own. Rather Mr Almeida was naïve in both the way in which he handled the collaboration and the subsequent and continuing reporting of its results.

PROFESSOR ROGER KAIN
PROFESSOR STUART TOWNLEY

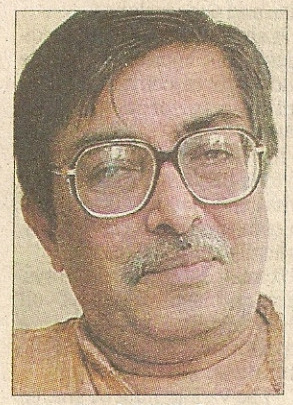
August 2004

Hindustan Times, Bhopal Live, 8 Nov 2004

Prof Raju's charge of plagiarism found correct

UK varsity warns lecturer

Shams Ur Rehman Alavi
Bhopal, November 7



Prof C K Raju

THE SCIENTIFIC fraternity in the United Kingdom (UK) has been taken by a storm after the charges of plagiarism levelled against a faculty member of the prestigious University of Exeter by the renowned City-based mathematician Professor C K Raju were found to be correct.

The university has issued a warning to the defaulter faculty member Dennis Almeida who allegedly plagiarised the path-breaking works of Prof Raju in the field of mathematics and published them in his name.

A mathematician and physicist, Dr C K Raju is well known for his research in which he has claimed that European mathematicians did not invent calculus, which was perhaps transferred from India to the West. He was staggered to find his unpublished work plagiarised by the UK lecturer early this year.

Soon after the 'discovery', Dr Raju, who is Head of Centre for Computer Science with the Makhanlal Chaturvedi National University of Journalism, and the university's the then DG Sharad Chandra Behar took up the issue with the Vice-Chancellor of the University of Exeter in March this year.

Recently, the VC of the University of Exeter, Professor Steve Smith, wrote a letter to Behar that a investigation was undertaken after the complaint of Dr Raju and after the enquiry was conducted, a written warning has been issued to Almeida that he must adhere to conventionally accepted tenets of authorship/acknowledgement in publications and any failure in future would be referred to a tribunal constituted under the university's statutes which could lead to his dismissal from the university.

Continued on page 5

UK varsity warns lecturer

Continued from page I

Dr Raju had charged Almeida, Lecturer in Mathematics Education at the School of Education in Exeter, Dr Zadorzhnyy, Department of Classics, University of Liverpool and another person for plagiarism of his innovative research.

According to Dr Raju, he had placed an advertisement for Research Associate on his project, Madhava and the origin of calculus on the internet in 1998, following which, Almeida had contacted him showing his interest in history of mathematics.

Almeida came to India to meet Raju and after his return to England raised some funding from the university for the project. In process, he reportedly gained access to Raju's unpublished works by saying that this was needed to raise more funds.

The association between the two continued until mid-2000.

However, Almeida then allegedly co-opted Dr George Joseph, Reader in Economics Department of the University of Manchester, on the grounds that his British citizenship will help in raising more funding. The terms subsequently became unacceptable for Dr Raju because it appeared to him that Almeida wanted to take over Dr Raju's ideas but giving him only a side role.

The association thus ended but to Raju's astonishment, he found that his thesis was presented by Dr Joseph in a conference in December 2000 and then Almeida, John and Zadorzhnyy wrote a paper, "Keralese mathematics: its possible transmission to Europe and consequential educational implications" in a journal in 2001.

Dr C K Raju, who played a leading role in the development of India's first supercomputer, Param, is an author of several books. He has questioned scientists like Einstein and Isaac Newton and challenged the long-existing beliefs that calculus was invented in Europe.

He has written several books including the famous, 'The Eleven Pictures of Time'.

The eminent Bhopal mathematician was astounded when he found that his unpublished work was freely used without credit to him and it appeared on a website recently. The complaint to the VC, University of Exeter was made and now the reply from the university and the findings of enquiry have at last proved the Indian mathematician correct.

MESSAGE DISPLAY

Message number 6

6

neomail-trash Move

Simple headers

From da2697@tutor.open.ac.uk Sat Dec 03 21:51:41 2005
Return-path: <da2697@tutor.open.ac.uk>
Envelope-to: c_k_raju@mcu.ac.in
Delivery-date: Sat, 03 Dec 2005 21:51:41 +0000
Received: from mcuac by host.focusindia.com with local-bsmtp (Exim 4.52)
id 1EifIB-0003ad-PF
for c_k_raju@mcu.ac.in; Sat, 03 Dec 2005 21:51:40 +0000
X-Spam-Level:
X-Spam-Checker-Version: SpamAssassin 3.1.0 (2005-09-13) on host.focusindia.com
X-Spam-Status: No, score=-2.6 required=5.0 tests=BAYES_00,UNPARSEABLE_RELAY
autolearn=ham version=3.1.0
Received: from [137.108.246.32] (helo=venus.open.ac.uk)
by host.focusindia.com with esmtp (Exim 4.52)
id 1EifIB-0003aZ-JK
for c_k_raju@mcu.ac.in; Sat, 03 Dec 2005 21:51:39 +0000
Received: from oufcnt2.open.ac.uk (actually host halley.open.ac.uk)
by venus.open.ac.uk via SMTP Local (Mailer 3.1) with ESMTP;
Sat, 3 Dec 2005 21:51:24 +0000
Message-id: <fc.004c540103cc4067004c540103cc4067.3cc41ed@oufcnt2.open.ac.uk>
Date: Sat, 03 Dec 2005 21:51:22 GMT
Subject: An apology
X-FC-Form-ID: 141
X-FC-SERVER-TZ: 28837376
To: c_k_raju@mcu.ac.in
From: Dennis Almeida <da2697@tutor.open.ac.uk>
MIME-Version: 1.0
Content-Type: text/plain; charset=ISO-8859-1
Content-Transfer-Encoding: 8bit
Status: R

Dear Professor Raju,

I will be brief. I wish to apologise unconditionally for the hurt and distress I have caused you. The content of the complaints you issued against me certainly suggests this and, given the critical retrospection that I have undertaken since leaving Exeter University, some of this have genuine merit.

I have no idea what the Vice Chancellor of Exeter University communicated to you after the investigation committee concluded its findings. However nothing he may have said would have conveyed the ugliness that I feel having participated in the actions of that period. Indeed I could not have expressed such sentiments at the time for events dictated otherwise.

I do not expect you to accept this apology. It makes little difference to me whether you do or not. The important thing for me is that I have cleared my conscience.

For the record I relinquished my post at Exeter in July 2005 and now teach part time on distance learning courses at the OU and for Paul Ernest. However I do intend to continue my research on the Jesuit conduit as an amateur on my own initiative and funds.

Yours,

Dennis Almeida

School of Education
University of Exeter
Heavitree Road
Exeter EX1 2LU
24 April 2007

Dear Professor Raju,

As you know the ethics committee at Exeter University who invesitgated your complaints found that judgement was flawed. I sincerely apologise for any distress and hurt that may have been caused to you by the manner in which I pursued the investigation on the conjecture of the transmission of Kerala mathematics.

Yours sincerely,

Dennis Almeida

Annexure 5. Lacunae in Exeter investigation report.

Lacunae in the earlier investigation report of 2004

1. To be fair, the committee should have contacted me, and given me a chance to reply. Somehow this did not happen. (Apparently the mail addressed to me was returned for some reason, though it is very unusual for the Indian postal department to return overseas mail. Moreover, there were other means of contacting me, by email, telephone, or fax, even if just to determine my address —which had *not* changed in the interim.)

2. My substantive complaint was about the *ethics* of non-citation of my work in a single paper. This was a *specific* complaint about a *specific* paper, written by Almeida et al. in 2001.[1] That 2001 paper cited *some* of my works, but omitted two key references,[2, 3] around which the central theme of the impugned paper was built. However, the committee first converted this specific complaint into a generalised complaint about "ownership" of an idea, and then opined that at that level of generality things were difficult to establish! It would have been better if the committee had restricted itself to just the specific complaint I had made.

   2.1. The committee accepted that it was wrong for my work not to be acknowledged. Since it is granted that a wrong has been done, it ought to have been remedied, through a retraction or other public correction of the earlier publication. But this was not done.

   2.2. There were several conceptual errors in the impugned paper published by Almeida et al. in 2001. These errors demonstrate that Almeida et al. did *not* understand my thesis which they claimed to have originated through the impugned paper. They could hardly have authored something they did not understand. **This is my epistemic test of transmission: in the context of doubtful authorship, conceptual mistakes are proof of copying.** The committee did not address this compelling evidence of guilt that I had provided.

   2.3. One of the papers which was not cited in the impugned paper of 2001 was the paper I presented in Hawai'i in Jan 2000 (cited in footnote 2). This Hawai'i paper was solely in my own name. Specifically, t**his paper was no part of any collaborative work with Exeter,** and does not acknowledge any grant from Exeter university. It **does** acknowledge grants from other sources. The ideas in my Hawai'i paper were critical for the applications to education in Almeida et al.'s impugned 2001 paper. They had *privileged access* to that Hawai'i paper *prior to publication*, to justify the bid for funds from the Leverhulme foundation through Exeter university in 2000. Nevertheless, that Hawai'i paper was **not** cited by them in the impugned 2001 paper (and I pointed out the conceptual mistakes they made also with regard to those ideas they copied). This was a key aspect of my complaint. The committee report fails even to mention this key fact. Now, just because there is *some* collaboration, does that entitle Almeida to claim *all* my work done during that period to

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1 D. F. Almeida, J. K. John and A. Zadorozhnyy, "Keralese mathematics: its possible transmission to Europe and the consequential educational implications", *Journal of Natural Geometry* **20** (2001) pp. 77–104.

2 C. K. Raju, "Computers, Mathematics Education, and the "Computers, mathematics education, and the alternative epistemology of the calculus in the *YuktiBhasa*", invited plenary talk at the *8th East-West Conference*, University of Hawai'i, Jan 2000. In *Philosophy East and West*, **51**:3, July 2001, pp. 325–362. The issue of transmission of the calculus from India to Europe is taken up in the section on the "History of the Calculus", especially from paras 3, 4 et seq., p. 351, while the next section deals with mathematics education.

3 C. K. Raju and D. F. Almeida (Aryabhata Group), "Transmission of the calculus from India to Europe, Part I: motivation and opportunity". "Part II: circumstantial and documentary evidence", papers presented at the International *Aryabhata Conference*, Trivandrum, Jan 2000.

which he had access? The committee, however, neglected this ethical aspect, and its report reads as if the issue concerned solely the work done during formal collaboration. **Nowhere does the committee report even mention the appropriation of my non-collaborative work in the impugned 2001 paper.** This too would have directly negated the opinion offered by the committee.

3. The committee consulted G. Joseph who evidently had a vested interest, for he was a party to the bid for some UKP 100,000 from Leverhulme foundation through Exeter, for research on these ideas (as stated on p. 3 of my letter of 25 March 2004). The committee should have recorded this conflict of interests, instead of treating Joseph as a neutral party. [**Joseph's attempt to appropriate credit for my work at a talk he gave in Bangalore, in Dec 2000 was also explicitly mentioned in my letter** of 25 March 2004 (p. 4, second para). Clear proof that Joseph too used my work without acknowledgment, and with the intention of appropriating credit for it, is now available (Annexure 7).]

   3.1. Joseph misled the committee, which hence opined that the idea of transmission of calculus was in the public domain! Really? If it was in the public domain for decades before 2004, then why were Joseph and Almeida so keen to get in the news with it in 2007? The committee cited the works Bag, Katz, and Joseph. This is incorrect. I have already sent a copy of a 2003 letter in which **Dr Bag has explicitly denied this,** stating: "I personally feel that...**the question of the transmission [of calculus] from India to Europe is basically a hypothetical issue...**". Katz has made no such claim regarding the transmission of the calculus; he has only pointed out the Indian infinite series (*that* is in the public domain, and surely known to the English-speaking world since 1832). But asserting the existence of infinite series in India is quite different from claiming transmission of calculus (the two are separated by 175 years; see Annexure 8 for a quick account of the enormously complex arguments required to go from "existence of infinite series" to "transmission of calculus"). Likewise, **there is no claim of transmission of calculus** in the 1991 edition of Joseph's book. (There *is* a clear mention of transmission in the 2000 edition of his book, but that is taken directly from my 1999 paper on Yuktibhasa, as described in the attached complaint to Princeton university press, Annexure 7.) The committee has confounded all sorts of issues in a matter in which it had no expertise. It should have stuck to my specific complaint, and not gone into such issues. However, doing so evidently provided an easy escape route for Almeida, through the "expert" guidance provided by his collaborator Joseph. The committee should at least have put down the page numbers, the passages cited, and the interpretations accorded to them—that would have helped to make manifest how the facts were stretched. Thus, although the committee based its whole report on this claim, I had no opportunity to rebut it then, and there is inadequate opportunity to do so even now because the citations are so vague.

4. The committee also consulted C. S. Sharma, the person who brings out the journal in which the impugned paper of 2001 was published. However, **Sharma is a family friend of Almeida, and has written joint papers with him in the past.**[4] It was inappropriate to treat him as a neutral

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4 E.g., D. F. Almeida (with C. S. Sharma), "A Direct Proof of Wigner's Theorem on Maps Which Preserve Transition Probabilities Between Pure States of Quantum Systems", *Ann Phys*, 1990, **197**(2), 300-309. Almeida gave a seminar on this in NISTADS, but, in response to my question, Almeida was unable to explain what exactly he had done on Wigner's theorem, stating he did it long ago, and had forgotten it. It later emerged that this was one of several papers which C. S. Sharma contributed in an

party. Even if the committee so decided, it should at least have recorded his lengthy past association with Almeida and the conflict of interests involved.

5. The committee should have stuck to readily demonstrable things like actions and facts, instead of going off into another grey area: that of intentions. The committee opined that Almeida et al.'s false claims of authorship were not premeditated. (Incidentally, this was based on the premise that Sharma was a neutral party.) However, a falsehood does not cease to be one even if it is not premeditated. At best that can be an ameliorating factor. Clearly, Almeida et al. gained personally. If this gain was not intended, they should subsequently have publicly denied authorship. The committee too was willing to allow these false claims to pass into posterity, merely saying that the same thing should not be done in the future. It is not clear to me: **on what ethical principles did the committee approve of misplaced credit, even for one paper, through false claims of authorship?** The ideas involved are important enough, so that misplaced credit through just ONE paper distorts history, as is clear from current accounts on the Wikipedia and the Internet and the subsequent coverage given to those ideas by the newspapers. (The other leg of the argument that the ideas were already in the public domain has already been shown to be incorrect.) The subsequent 2007 events show that his decision of the committee emboldened Almeida: he evidently felt that if false claims of authorship were permissible in respect of one paper, then why not two?

   5.1. The 2007 Manchester press release is a nakedly premeditated attempt to grab credit. Did this intent develop *after* the committee's report in 2004? That is unlikely, for in 2005 Almeida apologised. In his apology (to clear his conscience) Almeida admitted his participation in the "ugly events" (of 2000). That was a clear admission of premeditation. **Certainly there was a strong motive**, since Almeida and Joseph then hoped to get funding of around UKP 100,000 from the Leverhulme foundation. The committee should at least have mentioned the facts of this funding attempt alongside its judgment about lack of premeditation.

   5.2. Indeed, in retrospect, it is clear to me that premeditation was present right from the time in 1998 when Almeida contacted me after my July 1998 advertisement had publicly stated my research plans. Since Almeida had absolutely zero earlier background in this area, the sole connecting link was Joseph who presumably planted Almeida on me in a premeditated way that Almeida must have known about. Premeditation is irrelevant to my *specific* complaint, but (since the committee's opinion about lack of premeditation did induce them to let off Almeida lightly) the committee ought to have probed this aspect a little deeper: what *really* motivated Almeida to contact me in 1998? Was it Joseph?

6. As for the formally collaborative work under the name of the "Aryabhata Group", when I withdrew from the Leverhulme funding bid in 2000, I explicitly told Almeida that it would be unethical to persist in the bid without my participation (p. 3, para 3 of my letter of 25 March 2004). Apparently he did persist. Even supposing that the formally collaborative work was really collaborative (which it is no longer really possible or necessary to suppose) is it ethical to use collaborative work without the permission of one author? Is it ethical to do so when that permission has been expressly denied? This was another point which the committee did not address.

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attempt to help Almeida get a Ph.D., but Almeida was not confident of facing the *viva voce*.

Annexure 6. Plagiarism details
(including half a dozen cases of verbatim copying of my earlier published work by the Manchester 2007 paper claimed by Almeida and Joseph).

Table 1: Summary of my work used without acknowledgment/permission by G. Joseph and D. Almeida 2000-2007

S. No.	When/Where	What was copied without acknowledgment	Source/fact suppressed	Remarks
1	*Crest of the Peacock*, 2nd ed., 2000, p.356 : passage on Jesuits	Passage and idea that calculus was transmitted from India to Europe by Jesuits	Agra paper of Feb 1999. (Note its citation in the Hawai'i paper of Jan 2000, in note 58/59.)	Copied passage includes a mistake blindly copied by Joseph.
2.	*Crest of the Peacock*, 2nd ed., 2000, p. 415, reference to "Aryabhata's" "octagon method" compared to Greek "hexagon" method of determining pi.	Statement that Aryabhata used an "octagon method" compared to "hexagon" method used by Greeks and Arabs. Novel terms introduced be me were copied.	Indo-China paper of 1998, section 4.2, first so named the "octagon" method, explained it, and compared it with the "hexagon" methods. Note peculiarity of these terms used as short form for "octagon *doubling* method" etc. These long (and original) sections are paraphrased in a single line by Joseph, giving only the short form that I used.	Joseph again blindly copied a mistake I made. Actually, Aryabhata did *not* use the "octagon" method which I mistakenly attributed to him using this peculiar nomenclature. Joseph neither explains these methods, nor gives any source. No other source existed/exists for the "octagon" method prior to my paper.
3.	*Crest of the Peacock*, 2nd ed., 2000, p. 416-17	Observation that Euclidean geometry existed in India in parallel with traditional geometry to which it was *not* transmitted for centuries..	1. My Indo-China paper of 1998, section 2.5 on "non-transmission of information" due to epistemological barriers, which mentions Euclidean geometry. This is elaborated in sec 4.1 on "non-transmission of Euclidean geometry" due to epistemological differences, 2. Those differences were further explained in my "Euclid" paper of Feb 1999, at a Mumbai conference which Joseph attended.	No one earlier talked of epistemological differences in mathematics or geometry, which were treated as universal in Western thought. I did in 1998 to demonstrate epistemological barriers to transmission, and clearly explained these epistemological differences, further detailed in my "Euclid" paper.
4.	Joseph's talk at seminar in Bangalore, Dec 2000	Proposal that proof of transmission must use "opportunity, motivation, circumstantial and documentary evidence."	1. Hawai'i paper of Jan 2000 (published 2001) mentions "current legal standard of evidence" (p. 352), and details motivation, opportunity, circumstantial and documentary evidence. 2. Trivandrum paper of Jan 2000 based around this, .	Challenged Joseph publicly. (He had no answer from the floor.) Later complained to organizers of conference, but did not pursue matters. [Copy of email available/sent to you earlier.]
5.	Paper by Almeida et al in *J. Natural Geometry*, 2001.	1. Idea of calculus transmission from India to Europe. 2. Idea that Indian mathematics has a unique epistemology, and used floating point numbers which would be useful for mathematics education.	1. Trivandrum paper of Jan 2000. 2. Hawaii paper of Jan 2000 (published 2001).	Publicity stunt for this paper brought it to my notice and led to Exeter complaint of 2004. 1.Copy of Exeter investigation report 2004 sent. Note the injunction to not use "Aryabhata group" as author in future. 2. Section on "Transmission of the transmission thesis" in my 2007 book points out elementary mistakes made by these authors as clinching proof of copying.

S. No.	When/Where	What was copied without acknowledgment	Source/fact suppressed	Remarks
6.	Paper by "Aryabhata Group", Proceedings of Trivandrum conference, 2002, for which Joseph was the chief academic advisor.	Entire paper was published without seeking my approval and even without informing me. **This is a clear violation of copyright law by Joseph and Almeida acting together.**	(a) My authorship of the paper was suppressed, and my permission was not taken for anonymous publication; (b) my permission was not sought for copyright transfer, and (c) I was not even informed of the publication of this paper, and believed until recently that it was unpublished.	Several proofs that paper was written by me. (a) Several of my unpublished papers are cited, (b) I had planted mistakes in the paper as explained in my 2004 complaint and 2007 book (when I still thought the paper was unpublished). The relevant mistakes are reproduced verbatim in this paper.
6.	Paper by Almeida and Joseph in *Race and Class* 2004, 2nd paragraph of introduction, p. 46, repeated on p. 51 and in conclusions.	1. Key idea of an alternative Indian epistemology of mathematics, 2. the clarification that *pramana* is not the same as deductive proof, since it accepts empirical methods, 3. the use of numbers *like* floating point numbers.	1. Mathematics and Culture paper, published 1998 and 1999. 2. Indo-China paper of 1998, published 2002, sec. 4.1 3. "Euclid" paper, Mumbai conference Feb 1999, on alternative epistemology of Indian geometry, published 2001. 4. Hawai'i paper of Jan 2000 elaborates on difference between proof and pramana, and issues with floating point numbers, published 2001.	Mistakes made by Almeida et al. 2001 in asserting use of "floating point numbers" by "Kerala" mathematicians are repeated here. Indian numerals have a similar philosophical approach to the problem of representation: they are NOT floating point numbers as is wrongly asserted in this paper as well. An account of this mistake is published in my 2007 book. The idea that deduction is not infallible or universal is too big to be passed off without further discussion in one line.
7.	Paper by Almeida and Joseph in *Race and Class* 2004, remarks about Newton and Cavalieri, p. 47.	Key idea that European mathematicians (Newton, Cavalieri) had difficulty with the imported calculus because of the different way it handled infinites and infinitesimals. The authors neither explain nor provide a source.	1. The same points about Newton and Cavalieri are in *The Eleven Pictures of Time* (2003), e.g., p. 106, box on infinities. 2. My comments along these lines were publicly available on the web from the beginning of 2003.	The exact difficulties with infinity and infinitesimals that Newton and Cavalieri had are not explained, as they are in my books. This is highly non-trivial.
8.	Paper by Almeida and Joseph in *Race and Class* 2004, remarks on pp. 54-55 about no previous project to investigate the hypothesis of transmission of calculus.		1. My April 1998 INSA project on transmission of the calculus was suppressed. This was advertised in July 1998 (still archived on Historia Matematica list). 2. My 1999 plan for a PHISPC volume on this subject was suppressed.	1. Almeida coopted my former research associate from the project, wrote a joint paper with him, and Almeida and Joseph obtained my project materials from him. 2. Almeida has given a signed statement of 22 Dec 1999, agreeing to be co-author of a chapter in this volume. So he certainly knew of this project.
9.	2007 news release on Manchester website + paper by "Aryabhata group", Exeter and Manchester, supplied by Manchester Media relations officer + subsequent press statements by Joseph	Appendix gives 5 passages which agree near-verbatim with my previously published work. These are just examples and there is a far more substantial overlap with my earlier published work.	Various. But several pre-2000 unpublished papers of mine acknowledged (apparently by oversight).	This proves conclusively that Joseph had access to several of my unpublished papers (including Agra and Indo-China papers) from before 2000. Copied passages also conclusively prove plagiarism of cut-paste type by Joseph.

Appendix : Details

Quotation from C. K. Raju, "Approximation and proof in the Yuktibhasa derivation of Madhava's sine series" paper presented at the National Conference on Applied Sciences in Sanskrit Literature: Various Aspects of Utility, Univ. of Agra, 20-22 Feb 1999. See p. 12-13 of the attached soft copy (from the version of 21 March 1999, the one sent for publication) for the detailed context. Comments in square brackets have been added here.

"It is not so well known that the famous Jesuit astronomer and mathematician, Matteo Ricci, was, at exactly this exciting time, very active in Cochin having completed his theological studies in India, and having been ordained in Goa in 1580. He was in touch with Ludovico Maselli, the then Rector of the Collegium Romano.[1] I find it difficult to believe that he whiled away two years in Cochin, without attempting to gather information about astronomy from local sources—especially since he was 'requested to apply himself to the scientific study of this new and imperfectly known country.'[2] The key repository of astronomical knowledge was located not too far away, at a distance of barely 70 km at the Brahmin college in Trichur, from where Heyne and Whish later acknowledged having obtained manuscripts. (16th century CE Jesuits like Fr Fernicio, who sought astronomical knowledge, from India, were aware that the knowledgeable people were located not in Madurai but in Trichur.) Indeed, it was 'the reputation of "mathematician" that Ricci had thus acquired that made him wanted in China.' Matteo Ricci's record of interactions with China in mathematics and astronomy is well known; though this is the first time his interactions with India are being brought out, it is clear from the prizes offered by the Spanish government in 1576 [for navigation] that the same motivation applied, *a fortiori*, for acquisition of astronomical information in 1580.

"Material gathered by the Jesuits could easily have diffused all over Europe. The key centres in Europe at that time were Pisa (the oldest university) and Paris (the most prestigious). At Pisa, Galileo never got around to writing on the calculus, and his student Cavalieri, after waiting patiently for five years, himself started writing on it. Wallis' spent time at Pisa, before his work on infinite series. Newton, as is well known, followed Wallis.[3] Gregory spent time in Padua. At Paris, Mersenne, as is well known, acted as a clearinghouse for mathematical ideas. He corresponded with Galileo, Wallis, Fermat, and Pascal. Therefore, the case for transmission is still wide open...."

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1 *Documenta Indica*, Vol 13, p 146, Letter from Matthaeus Ricci, Cochin, dated 29 Nov 1580. Ricci who, trained for seven years in mathematics and astronomy, reached India on 13 Sep 1578 was ordained in Goa in 1580, and worked with the Cochin mission. Ricci left for Macao reaching there on 7 Aug 1582.

2 H. Bernard, *Matteo Ricci's Scientific Contribution to China*. I am indebted to Dennis Almeida for this and the subsequent quotation. [This footnote was intended to delineate Almeida's *exact* contribution of two quotes of marginal value.]

3 e.g. Carl B. Boyer, *A History of Mathematics*, Wiley, 1968, p 424. C. H. Edwards, *The Historical Development of the Calculus*, Springer, 1979, p 113.

Quotation from G. Joseph, The Crest of the Peacock*, 2000 edition from the Princeton University press, p. 356. This material is NOT found in the earlier edition of this book, and was added only after the above Agra paper on the Yuktibhasa.*

"However, a more important connection was the possible role of the Jesuits and the Portuguese: there is evidence that Matteo Ricci, the Jesuit astronomer and mathematician who is generally credited with bringing European sciences to the Chinese, spent almost two years in Cochin, South India, after being ordained in Goa in 1580. During that time he was in correspondence with the Rector of the Collegio Romano, the primary institution for the education of those who wished to become Jesuits. The Jesuits of that time were not merely priests, but also scholars who were very knowledgeable in science and mathematics. In fact, if you wanted to be trained as a mathematician in Italy at that time, you couldn't do better than go to a Jesuit seminary. For a number of Jesuits who followed Ricci, Cochin was a staging post on the way to China. Cochin was only 70 km from the largest repository of astronomical manuscripts in Trissur (Trichur) from where, two hundred years later, Whish...and Heyne...obtained their manuscripts. ...Material gathered by Jesuits was scattered all over Europe: at Pisa where Galileo Cavalieri and Wallis spent time; at Padua, where James Gregory was engaged in mathematical studies, and at Paris, where Mersenne, through his correspondence with Galileo, Wallis, Fermat, and Pascal, acted as an agent for the transmission of mathematical ideas."

The earlier edition of Joseph's *Crest of the Peacock* did not contain this (the following page numbers are from the Penguin edition of 1991):

1. p. 281 suggests that Indian trigonometry originated from Greek trigonometry from Ptolemy and then went from Arabs to Europe to Regiomontanus. This is the stock belief commonly found in all secondary sources. **Nothing new here.**

2. p. 287 states that the quality of Kerala mathematics is fundamentally different from mathematics in the classical period, and that there must be "'missing links' to bridge the gap between the two period". (This is not correct; there is no fundamental difference at all, "Kerala" mathematics is just a direct extension of Aryabhata's techniques. Joseph's wrong statement is just another indication that he is neither a mathematician, nor familiar with the primary sources in Sanskrit, and therefore not qualified to research this area.)

3. Joseph's book goes on to consider and reject the possibility of a "convenient external agency". (There have been suggestions in the literature that the "Kerala calculus" may have originated through contacts with Europeans.)

4. It then states that "Kerala mathematics" is comparable with "later discoveries in European mathematics". He does raise the question "whether the developments in Kerala mathematics had any influence on European mathematics", but immediately retreats from even this vague question to say that this requires careful investigation, and cites Lach 1965 as providing "some evidence of transfer of technology and products". Note the vagueness of the question: **it does not even use the word "calculus", for the developments in India.** This is a critically important point, as is clear from a report of my book (Annexure 11). Further the 1991 edition of Joseph's book makes no mention of Jesuits, as agents, or navigation, as the motivation, both of which are again critically important to understand the process of transmission. Also "any influence" is another vague term, and the reference cited actually pertains to "transfer of

technology and products" which is commonly accepted, and but natural. Many other instances of transfer of technology are long known, but they do not count as evidence for transmission of calculus. For example, in my 1998 paper on the *kamal*, I had mentioned the well known fact that the *kamal*, an Indo-Arabic navigational instrument, was taken back to Portugal by Vasco da Gama, but even that is a far cry from a definite claim of transmission of the calculus by Jesuits.

5. Joseph is well aware that such vague and ambiguous statements can be exploited, since they can be retrospectively disambiguated in a wide variety of ways as convenient to the occasion, and this is a common sharp practice also used by astrologers. Note that Joseph is a trained lawyer.

6. In contrast, my statements (in my 1999 Agra paper) are a very definite assertion that Jesuits (and particularly Matteo Ricci) in Cochin were looking for a solution to the European navigational problem which was provided by the Indian astronomical texts. This definite statement was repeated by Joseph in the 2000 edition of his book.

7. Instead of a straightforward acknowledgment to my Agra paper, the 2000 edition of Joseph's book vaguely says "there is some research on this in India". The real reasons for this vagueness have become quite clear in retrospect: Joseph desired credit for this research himself, and left the door open for a future false claim. He did not acknowledge my work just because he well knew that the statements in the earlier edition of his book were grossly insufficient for him to claim any sort of priority in this regard.

Details for 2nd row in Table

My paper for a 1998 meeting ("Interactions between India, Western and Central Asia, and China in Mathematics and Astronomy") first used the *terminology* "octagon method" as a short form for octagon-doubling method and gave a detailed account of it in a whole section, while comparing it with the "hexagon method" a short-form for "hexagon-doubling method" (usually called Archimedes' method), AND made the mistake of attributing the "octagon" method as due to Aryabhata.

The full subsection 4.2.1 "Aryabhata's method of calculting π" spans several pages, and has several figures. The key point of the comparison between the octagon and hexagon method is that it involved a novel way to compare the Indian method with "Greek", Arabic, and Chinese methods, which comparison was relevant to my paper on transmission between Greeks, India and China. It gives all actual sources I have used, so that the speculative part can be clearly identified.)

> "Unlike infinitesimal techniques, there is not even a shadow of a doubt that Aryabhata could have used this technique for it requires only the extraction of square roots and the definition of area, both of which have been explicitly described earlier in the *Aryabhatiya*.
>
> "We reproduce this method in full...to bring out the flavour of the techniques used, which have not before been explained. This process relied on octagons rather then the hexagons used by Archimedes and Liu....[Fig 5. The octagon method]... [Fig 6. Part of the 16-gon in a small square]....
>
> "Thus, Aryabhata's method could not have been the method used by Liu Hui, who clearly used a technique similar to that of Archimedes, since $3072 = 3 \times 1024 = 3 \times 2^{10}$ is not a power of 2 but is a number that would be obtained on the hexagon-doubling method. The same method of hexagon-doubling was also apparently used by al-Kashi since he used a polygon with 3×2^{28} sides.
>
> "The question remains: what was it that enabled Liu Hui to carry out this computation to a 3072-gon? Was it sheeer labour, or did Liu Hui have access to a more efficient technique of square-root extraction than was available to Archimedes? ....
>
> "[Fig. 7. Aryabhata's technique of computing arcs and sines. Dividing the square into a number of equal parts (which could be done) divided the circle into a number of unequal arcs..."

Although my paper was eventually published only in 2002 (in : A. Rahman (ed) *Interactions between India, Western and Central Asia, and China*, Oxford Univ. Press, New Delhi, 2002, pp. 227–254), the *unpublished* version of the paper was available to Almeida and Joseph from before 2000. (Proof: this is cited in note 15 of the Manchester 2007 report to which these two have now laid claim.)

This Indo-China paper is also cited in the Feb 1999 Agra paper, at note 13.

In *Crest of the Peacock*, 2000 edition, p. 415, Joseph writes:

> "Indeed, it is likely that Aryabhata preferred the octagon method rather than the hexagon method used by Greek and Arab mathematicians to compute his accurate estimate of the circumference of the circle."

In line with his long term practice of using my work but never citing it, Joseph gives no source for this statement; nor does he give any explanation for what this "ocatagon method" is. (My detailed account was the first and is still the only account of the "ocatagon" method in the literature.)

But, **Joseph's has given away his source by copying not only the terminology, but also a serious mistake.** Aryabhaata did *not* use the "ocatagon method" or any geometric method. To say so is to fudamentally misunderstand the contribution made by Aryabhata, and I corrected this subsequently.

On the epistemic test, those who copy tend to copy mistakes, because they copy without understanding and knowledge. The peculiar terminology of "octagon method" makes no sense on a stand-alone basis, and the copied mistake of attributing to Aryabhata unambiguously establishes that Joseph copied without acknowledgment from the unpublished version of my paper to which he had access. The mistaken attribution to Aryabhata is particularly ironical since it comes in a paragraph which begins by calling this method the "Kerala approach":

> An unusual aspect of the Kerala approach to the derivation of a number of infinite series...is an interesting geometric technique different from the method...used in Arab and European mathematics. In the Kerala case you are subdividing the arc into unequal parts. [cf. Fig. 7 of Indo-China paper]...Indeed it is likely that Aryabhata preferred the octagon method....[cf. Fig. 5 of Indo-China paper]. The [Greek and Arab] method was probably avoided because the calculation involved working out the square roots of numbers at each stage of the calculation a tedious and time-consuming task.

Details for 3rd row in Table

Section 2 of my paper for a 1998 meeting ("Interactions between India, Western and Central Asia, and China in Mathematics and Astronomy") was on "Models of Information Transmission". Subsection 2.5 was on "Non-transmission of information"

> "If clerical apologetics for surplus extraction is not the goal, then it is clear that cases where information was NOT shared, despite extensive contact, are equally interesting. But this situation seems never before to have been studied in detail. There are many such cases where the information was freely available and was simply not used....
>
> "Abul Fazl learnt Euclidean geometry, in India, presumably from Arabic and Persian sources, and mentions it in detail in the *Ain-i-Akbari*.[20] India had contacts with the Greeks certainly since the time of Alexander. There were extensive trading contacts with the Roman Empire. Nevertheless, the influence of Euclidean geometry is not traceable in the writings of non-Muslims in India, until Kamalakar, Jehangir's court astronomer, after the arrival of Jesuit priests in Akbar's court. Though mathematics, we are told, is one and universal, there were two streams of geometry simultaneously prevalent in India. Eventally, parts of the *Elements* were translated into Sanskrit only in 1718 well after the arrival of the Europeans."

The earlier section 2.4 on "Epistemological issues" had asserted

> "it is difficult to answer questions of information exchange without reworking the entire epistemological foundations of traditional mathematics"

[as I subseqeuntly did]. Section 4.1 on "Non-transmission of Euclidean geometry" explained this through a key example. This was further explained in subsection 4.1.2 "The epistemological barrier" as follows.

> As remarked earlier, Euclidean geometry is also a strong Arabic tradition....In India, this Arabic tradition did not mix with the indigenous tradition of geometry. There remained two streams of geometry in India, an Arabic one used by Muslim scholars such as Abul Fazl, and an indigenous tradition dating back to the *sulba sutra*....I will argue that the non-mixing was the result of important epistemological differences, which created an epistemological barrier to transmission.

The next few subsections explain this epistemological difference which is also explained at length in my "Euclid" paper of Feb 1999 at the Mumbai conference.

From Joseph, *Crest of the Peacock*, 2nd edition, 2000, p. 416.

> "In Central and Western Asia under Islamic rule, Euclidean geometry, which developed as a decutive science , became dominant. As a result, students of mathematics and astronomy taught in Arabic-Persian language schools in India required a thorough knowledge of Euclidean geometry...In 1732...Jagannath Samrata translated the Persian text into a Sanskrit version...
>
> "the major difference between the two streams of mathematical activity, namely the one within the Sanskritic tradition, and the other within the Arabic-Persian tradition, is well brought out...The details are of little importance in the context of this dicussion. What would have been very unlikely is that such a problem would have engaged the interest of an Indian mathematician from the Sanskritic tradition. The two parallel traditions met in a few cases involving astronomy, but hardly ever on matters relating to pure mathematics. The lack of contact was a missed opportunity which has had considerable repercussions for the development of Indian mathematics. But that is another story."

Seen as an isolated passage, and in the narrow legalistic sense that everything apart from cut-paste copying is permissible, this passage may pass muster.

However, the point of mentioning it is, first, that this passage ought not to be seen in isolation. This is another key (and connected) idea in the Indo-China paper which reproduced and juxtaposed with the "octagon method" in Joseph's book. Further, the idea of epistemological barriers to transmission is taken up again in Almeida and Joseph's *Race and Class* 2004 paper.

Secondly, copying of ideas without acknowledgement is unethical even if the expression is changed, from "Muslim" to "Arabic-Persian tradition", from "indigenous" to "Sanskritic tradition", and from "two [non-mixing] streams of geometry" to "two parallel traditions...relating to pure mathematics". [I will not argue here that the concept of "pure mathematics" in Indian tradition exists only in Joseph's imagination.]

Examples of cut-paste copying by Joseph and Almeida

Comparison of extracts from C. K. Raju's earlier published work and the Manchester paper 2007 in the name of the "Aryabhata Group".

I

From C. K. Raju, "Computers, Mathematics Education, ...Yuktibhasa", talk at Hawaii, Jan 2000, published in *Philosophy East and West* **51**(3) 2001 pp. 325-61. Endnote 60 of MSS. (In the published version this is endnote 59, pp. 360-61, and there are some minor changes in the style of citation.)

> 60. The key passage is quoted in the *YuktiBhâsâ* and attributed to the *TantraSangraha*. *YuktiBhâsâ,* Part I (ed) with notes by Ramavarma (Maru) Thampuran and A. R. Akhileshwara Aiyar, Mangalodayam Ltd, Trichur, 1123 Malayalam Era, 1948 CE, p 190. The passage is NOT to be found in the *TantraSangraha* of the Trivandrum Sanskrit Series. *Tantrasangrah*a, S. K. Pillai (ed), Trivandrum Sanskrit Series, 188, Trivandrum, 1958, the English translation of which has been recently serialised in the *Indian Journal of History of Science*. The authors of the modern *YuktiBhâsâ* commentary have however used a transcript of the MSS of the *TantraSangrahaVyâkhyâ* in the Desamangalattu Mana, a well-known Namboodri household. This version of the *TantraSangraha* is found in the *TantraSangrahaVyâkhya*, Palm Leaf MS No. 697 and its transcript No. T12541, both of the Kerala University MS Library, Trivandrum. The missing verses are after II.21a of the Trivandrum Sanskrit Series MS. The same verses are also found on pp 68–69 of the transcript No. T-275 of the *TantraSangrahaVyâkhya* at Trippunitra Sanskrit College Library which is copied from the manuscript of the Desamangalattu Mana....For detailed quotations and a more mathematical account of the passages, see C. K. Raju, "Approximation and Proof in the YuktiBhâsâ Derivation of Madhava's Sine Series", Paper presented at the National Seminar on Applied Science in Sanskrit Literature: Various Aspects of Utility, Agra, 20–22 Feb 1999.

This is copied in the Manchester paper Aug 2007 Note 2, except for the last sentence referring to my Agra paper. (But the slip shows, for note 53 mentions the Agra paper as "cited earlier".)

> The version of the *TantraSangraha* which has been recently serialised (K. V. Sarma ed) together with its English translation (V. S. Narasimhan Tr.) in the *Indian Journal of History of Science* (issue starting Vol. 33, No. 1 of March 1998) is incomplete and does not contain the relevant passages. We have used the verion of the *TantraSangraha* as found in the *TantraSangrahaVyakhya*, Palm Leaf MS No 697 and its transcript No. T 1251, both of the Kerala University MS Library, Trivandrum. The missing verses are after II.21a of the Trivandrum Sanskrit Series MS. The same verses are also found on pp 68—69 of of the transcript No., T-275 of the *TantraSangrahaVyakhya* at Trippunitra Sanskrit College Library, copied from a palm leaf manuscript of the Desa Mangalatta Mana. We found Pl-697/T-1251 more useful since the commentary in Malayalam is clearly separated from the original text in Sanskrit. Needless to say these verses are also found in the YuktiBhasa etc.

II

Hawai'i paper note 27/28

The victory of algorismus over abacus was depicted by a smiling Boethius using Indian numerals, and a glum Pythagoras to whom the abacus technique was attributed. This picture first appeared in the *Margarita Philosophica* of Gregor Reisch, 1503, and is reproduced e.g. in Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, (Tr.) Paul Broneer, MIT Press, Cambridge, Mass., 1970, p 350.

Manchester paper note 12

The victory of algorismus over abacus was depicted by a smiling Boethius using Indian numerals, and a glum Pythagoras to whom the abacus technique was attributed. This picture first appeared in the *Margarita Philosophica* of Gregor Reisch, 1503, and is reproduced e.g. in Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, (Tr) Paul Broneer, MIT Press, Cambridge, Mass., 1970, p 350.

III

Hawai'i paper note 30/31

'Com tudo não me parece que sera impossivel saberse, mas has de ser por via d'algum mouro honorado ou brahmane muito intelligente que saiba as cronicas dos tiempos, dos quais eu procurarei saber tudo.' Letter by Matteo Ricci to Petri Maffei on 1 Dec 1581. *Goa* **38** I, ff 129r–130v, corrected and reproduced in *Documenta Indica*, **XII**, 472–477 (p 474). Also reproduced in Tacchi Venturi, *Matteo Ricci S.I., Le Lettre Dalla Cina 1580–1610*, vol. 2, Macerata, 1613.

Manchester 2007 paper, note 71

"Com tudo não me parece que sera impossivel saberse, mas has de ser por via d`algum mouro honorado ou brahmane muito intelligente que saiba as cronicas dos tiempos, dos quais eu procurarei saber tudo." Letter by Matteo Ricci to Petri Maffei on 1 Dec 1581. *Goa* **38** I, ff 129r--30v, corrected and reproduced in *Documenta Indica*, **XII**, 472-477 (p 474). Also reproduced in Tacchi Venturi, *Matteo Ricci S.I.*, *Le Lettre Dalla Cina 1580-1610*, vol 2, Macerata, 1613.

IV

Hawai'i paper note 32/33. (Note the peculiar way in which the title of Clavius's book is shortened to emphasize the points that (a) he was dealing with Rsine values, and not sine values as currently understood, and (b) that the value of the radius is the same as that used in the *Karanapaddhati.*)

> Christophori Clavii Bambergensis, *Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000...*, Ioannis Albini, 1607.

Manchester paper 2007 note 28

> Christophori Clavii Bambergensis, *Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000 ...,* Ioannis Albini, 1607.

V

Hawai'i paper pp. 351-52 (print) pp. 28-29 (typescript)

> "Since authoritative Western histories of mathematics are replete with wild claims of transmission from Greece, an appropriate standard is needed for the evidence for transmission. I have suggested that we follow the current legal standard of evidence, by establishing (i) motivation, (ii) opportunity, (iii) documentary evidence, and (iv) circumstantial evidence.

Manchester 2007 paper

> "Hence we propose to adopt a legal standard of evidence good enough to hang a person for murder. Briefly, we propose to test the hypothesis on the grounds of (1) motivation, (2) opportunity, (3) circumstantial evidence, and (4) documentary evidence."

(In fact, both sets of sentences are mine.)

VI

Hawai'i paper pp. 351-53 considers in detail all the four types of evidence listed above: motivation, opportunity, circumstantial evidence, and documentary evidence, and it is clear that the Manchester 2007 paper puts forward ideas with a very substantial overlap. Other examples include Fermat and Pell's equation, Descartes statement about curved lines being beyond the human mind etc. etc.

VIII

C. K. Raju, *Cultural Foundations of Mathematics*, April 2007, p. 365, "Transmission of the Transmission Thesis"

> "The source of these errors can be found in the first part of the Trivandrum paper NOT cited by the trio, which makes the same mistake, on p. 6:
>
> The widely distributed *Laghu Bhaskariya* (abridged works of Bhaskara) and *MahaBhaskariya* (extensive works of Bhaskara) of the first Bhaskara (629 CE) explicitly detailed methods of determining the local latitude and longitude, using **observations** of solar **declination** or pole star, and simple instruments like the gnomon, and the clepsydra. Since local latitude could easily be determined from solar **declination** by day and e.g. pole star altitude at night (using an instrument like the kamal) an accurate sine table was just what was required. . . . [Emphasis added]"

Note that the above is a quotation from the Trivandrum paper in my 2007 book, and this quotation was intended to prove plagiarism through blind copying of mistakes injected into the Trivandrum paper which was not cited by Almeida et al. 2001.

Manchester paper August 2007, p. 4

> The widely distributed *Laghu Bhaskariya* (abridged work of Bhaskara) and *Maha Bhaskariya* (extensive work of Bhaskara) of the first Bhaskara (629 CE) explicitly detailed methods of determining the local latitude *and* longitude, using observations of solar declination, or pole star altitude, and simple instruments like the gnomon, and the clepsydra.[i] Since local latitude could easily be determined from solar declination by day and e.g. pole star altitude at night (using a common instrument like the *kamal*), an accurate sine table was just what was required.

(The error is in the use of the term "declination", which cannot be readily observed at sea. The use of this term, in place of "altitude", also makes nonsense of my thesis that the Greogrian calendar reform was carried out because a good calendar was required by Europeans for navigation, to fix latitude.)

Annexure 7. Excerpts from Hawai'i paper (plenary talk 8th East-West conference, Jan 2000, published *Philosophy East and West* 51(3) 2001 pp. 325-62.

DRAFT: Please see
Philosophy East & West **51**(3) 2001
325–62 for the final version

Computers, mathematics education, and the alternative epistemology of the calculus in the *Yuktibhâsâ*

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Abstract

Current formal mathematics, being divorced from the empirical, is entirely a social construct, so that mathematical theorems are no more secure than the cultural belief in 2-valued logic, incorrectly regarded as universal. Computer technology, by enhancing the ability to calculate, has put pressure on this social construct, since proof-oriented formal mathematics is awkward for computation, while computational mathematics is regarded as epistemologically insecure. Historically, a similar epistemological fissure between computational/practical Indian mathematics and formal/spiritual Western mathematics persisted for centuries, during a dialogue of civilizations, when texts on 'algorismus' and 'infinitesimal' calculus were imported into Europe, enhancing the ability to calculate. I argue that this epistemological tension should be resolved by accepting mathematics as empirically-based and fallible, and by revising accordingly the mathematics syllabus outlined by Plato.

Paper based on an invited plenary talk at the 8th East-West Conference,
East-West Centre and University of Hawai'i, Hawai'i, Jan 2000
revised version in *Philosophy East and West*, **51** (3), 2001, 325–62.

Computers, mathematics education, and the alternative epistemology of the calculus in the *Yuktibhâsâ*

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0 Introduction

0.0 The East-West civilizational clash in mathematics: *pramâna* vs proof

In Huntington's terminology of a clash of civilizations, one might analyse the basis of the East-West civilizational clash as follows: the Platonic tradition is central to the West, even if we do not go to the extreme of Whitehead's remark, characterising all Western philosophy as no more than a series of footnotes to Plato. But the same Platonic tradition is completely irrelevant to the East.

In the present context of mathematics, the key issue concerns Plato's dislike of the empirical, so the civilizational clash is captured by the following central question: *can a mathematical proof have an empirical component?*

0.1 The Platonic and Neoplatonic rejection of the empirical

According to university mathematics, as currently taught, the answer to the above question is no. Current-day university mathematics has been enormously influenced by (Hilbert's analysis of) "Euclid's" *Elements*, and Proclus,[2] a Neoplatonist and the first actual source of the *Elements*, argued that

> Mathematics…occupies the middle ground between the partless realities…and divisible things. The unchangeable, stable and incontrovertible character of [mathematical] propositions shows that it [mathematics] is superior to the kinds of things that move about in matter.…Plato assigned different types of knowing to…the…grades of reality. To indivisible realities he assigned intellect, which discerns what is intelligible with simplicity and immediacy, and…is superior to all other forms of knowledge. To divisible things, in the lowest level of nature, that is, to all objects of sense-perception, he assigned opinion, which lays hold of truth obscurely, whereas to inter-

Extract

$$\frac{x \cdot 0 + \frac{x \cdot 0}{2}}{0} = 63$$

This suggests that, when we go beyond the empirical, the 'universal' may lie, as in a physical theory, in what Poincaré called 'convenience'. This criterion of 'convenience' can have profound consequences as in the case of the theory of relativity: the constancy of the speed of light is not an empirical fact (though elementary physics texts usually misrepresent it as such), Poincaré defined the speed of light as a constant as a matter of 'convenience'. I see this criterion of 'convenience' as more modest than the criterion of beauty which seeks to globalize a local sense of aesthetics.

3 History of the calculus

If mathematics is a social construct, which changes with changing social circumstances, then the question is: how should one teach mathematics today? Admitting the role of technology in shaping mathematics, accepting that the computer is going to play an increasingly important role in the future, and admitting that formal mathematics is not quite suited to computers, the conclusion seems to be forced that a different type of mathematics should be taught. The calculus is at the core of many numerical computations, but can one at all do the calculus without real numbers? An alternative mathematical epistemology could be invented *ab initio*. Or one could fall back on the alternative epistemology of mathematics in India, as described in the *Yuktibhâsa*. This alternative epistemology provided the natural soil in which the calculus grew. Recognizing the existence of this alternative epistemology of mathematics requires, however, an alternative account of the history of mathematics. This is an illustration of the general maxim that the history of mathematics has profoundly influenced its philosophy, so that to change the philosophy of mathematics, one must also revise its history. A condensed account of the suggested revision follows.

According to the Western history of the calculus, the calculus was the invention of Leibniz and Newton, particularly Newton, who used it to formulate his 'laws' of physics. In a series of papers, I have pointed out that this narrative needs to be significantly changed for several reasons.

(a) The key result of the calculus, attributed variously to Gregory,[58] Newton, and to Newton's student Brook Taylor,[59] is the infinite-series expansion today commonly known as the Taylor's series expansion. This infinite series expansion is found in India a few centuries before Newton in the work of Madhava of Sangamagrama and in the later works like Nilkantha's *TantraSangraha* (1501 CE), Jyeshtadeva's *YuktiBhâsa* ("Discourse on Rationale" c. 1530 CE)[60] the *TantraSangrahaVyâkhya*, the *YuktiDîpikâ*, the *Kriyâkramakari*, the *KaranaPadhati* and other such widely distributed and still existent works of what has been called the Kerala school of mathematics and astronomy.

निहत्य चापवर्गेण चापं तत्तत् फलानि च ।
हरेत समूलयुग्वर्गैस्त्रिज्यावर्गाहतैः क्रमात् ॥
चापं फलानि चाधोऽधो न्यस्योपर्युपरि त्यजेत् ।
जीवाप्त्यै ...

This key passage may be translated as follows.

> Multiply the arc by the square of the arc, and repeat [any number of times]. Divide by the product of the square of the radius times the square of successive even numbers increased by that number [multiplication being repeated the same number of times]. Place the arc and the results so obtained one below the other and subtract each from the one above. These together give the *jîvâ*…

Jîvâ relates to the sine function. Etymologically, the term sine derives from *sinus* (= fold) a Latin translation of the Arabic *jaib* (opening for the collar in a gown), which is a misreading of the Arabic term *jîbâ* (both terms are written as *jb*, omitting the vowels). Mathematically, however, as is well-known, *jîvâ* and *sara*, like the sine and cosine of Clavius' sine tables (as their very title shows),[61] were not the modern sine and cosine but these quantities multiplied by the radius r of a standard circle. The *jîvâ* corresponds to $r \sin \theta$, while the *sara* corresponds to $r\,(1-\cos \theta)$.

In current mathematical terminology, this passage says the following. Let r denote the radius of the circle, let s denote the arc and let t_n denote the nth expression obtained by applying the rule cited above. The rule requires us to calculate as follows. (1) Numerator: multiply the arc s by its square s^2, this multiplication being repeated n times to obtain $s \cdot \prod_1^n s^2$. (2) Denominator: Multiply the square of the radius, r^2, by $[(2\,k)^2 + 2\,k]$ ("square of successive even numbers increased by that number") for successive values of k , repeating this product n times to obtain $\prod_{k=1}^{n} r^2\,[(2k)^2 + 2k]$. Thus, the nth iterate is obtained by

$$t_n = \frac{s^{2n} \cdot s}{(2^2+2)\cdot(4^2+4)\cdot \ldots \cdot [(2n)^2+2n]\cdot r^{2n}}$$

The rule further says:

$$jîvâ = (s - t_1) + (t_2 - t_3) + (t_4 - t_5) + \ldots$$

Substituting:

(1) *jîvâ* $\equiv r \sin\theta$,
(2) $s = r\,\theta$, so that $s^{2n+1}/r^{2n} = r\,\theta^{2n+1}$, and noticing that
(3) $[(2k)^2+2k] = 2k\cdot(2k+1)$, so that
(4) $(2^2+2)(4^2+4)\ldots[(2n)^2 + 2n] = (2n+1)!$,
and cancelling r from both sides, we see that this is entirely equivalent to the well-known expression

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots.$$

This verse is followed by a verse describing an efficient numerical procedure for evaluating the polynomial.[62] The existence of these verses has been known to Western specialists for nearly two hundred years, and is today acknowledged in some Western texts on the history of mathematics, like those of Jushkevich,[63] Katz[64] etc.

In current mathematical terminology, the key step in the *Yuktibhâsâ* rationale for the above series is that

$$(1)\quad \lim_{n\to\infty} \frac{1}{n^{k+1}} \sum_{1}^{n} i^k = \frac{1}{k+1},\ k = 1, 2, 3,\ldots,$$

in the sense that the remaining terms are numerically insignificant, for large enough n.

(b) A relevant epistemological question is this: did Newton at all understand the result he is alleged to have invented? Did Newton have the wherewithal, the necessary mathematical resources, to understand infinite series? As is well known, Cavalieri in 1635 stated the above formula as what was later termed a conjecture. Wallis, too, simply stated the above result, without any proof.[65] Fermat tried to derive the key result above from a result on figurate numbers, while Pascal used the famous "Pascal's" triangle[66] long known in India and China. Though Newton followed Wallis, he had no proof either,[67] and neither did Leibniz who followed Pascal. Neither Newton nor any other mathematician in Europe had the mathematical wherewithal to understand the calculus for another two centuries, until the development of the real number system by Dedekind.

(c) The next question naturally is this: if Newton and Leibniz did not quite understand the calculus, how did they invent it? In the amplified version of the usual narrative, how did Galileo, Cavalieri, Fermat, Pascal, and Roberval etc. all contribute to the invention of a mathematical procedure they couldn't quite have understood? The frontiers of a discipline are usually foggy, but here we are talking of a gap which is typically 250 years.

(d) Clearly a more natural hypothesis to adopt is that the calculus was not invented in Europe, but was imported, and that the calculus took nearly as long to assimilate as did zero. Since authoritative Western histories of mathematics are replete with wild claims of transmission from Greece, an appropriate standard is needed for the evidence

for transmission. I have suggested that we follow the current legal standard of evidence, by establishing (i) motivation, (ii) opportunity, (iii) documentary evidence, and (iv) circumstantial evidence.

Motivation (a) : Europe had strong motivation to import mathematical and astronomical knowledge in the 16th and 17th centuries CE, because mathematics and astronomy were widely regarded as holding the key to navigation which was the route to prosperity hence the critical technology of the times. As is now widely known, Europe did not have a reliable technique of navigation, and European governments kept offering huge prizes for this purpose from the 16th until the 18th century CE. Indeed, the French Royal Academy, the Royal Society of London etc. were started in this way in an attempt to develop the astronomical and mathematical procedures needed for a reliable navigational technique.

The first navigational problem concerned latitude: right from Vasco da Gama, Europeans attempted to learn the Indo-Arabic techniques of determining latitude through instruments like the Kamâl. The Indo-Arabic technique of determining latitude in daytime assumed a good calendar, and this led to the Gregorian calendar reform. As a student and correspondent of Pedro Nunes, Clavius presumably understood that reforming the calendar, and changing the date of Easter was critical to the navigational problem of determining *latitude* from the observation of solar altitude at noon, as described in widely distributed Indian mathematical-astronomical texts, and calendrical manuals.

Opportunity: On the other hand, right from the 16th century there was ample opportunity for Europeans to collect Indian mathematical-astronomical and calendrical texts. The Jesuits were in India, with their strongest centre being Cochin, from where a copy of the *Tantrasangraha* or *Yuktibhâsâ* could easily have been procured. Each Jesuit was expected to know the local language, and Alexander Valignano declared that it was more important for the Jesuits to know the local language than to learn philosophy. They could hardly have functioned without a knowledge of the local calendar and days of festivity. One of the earliest Jesuit colleges was at Cochin, and it typically had an average of about 70 Jesuits during the period 1580–1660. Prior to this period, printing presses had already been started in languages like Malayalam and Tamil, and Malayalam was being taught at the Cochin college at the latest by 1590.

Documentary evidence: Moreover, the Jesuits were systematically collecting and translating local texts and sending them back to Europe. In particular, Christoph Clavius, head of the Gregorian Calendar Reform Committee changed the mathematics syllabus of the Collegio Romano, to correct the Jesuit ignorance of mathematics, and from the first batch of mathematically trained Jesuits he sent Matteo Ricci to Cochin to understand the available texts in India on the calendar, and the length of the year.[68]

Motivation (b): Pedro Nunes was also concerned with loxodromic curves, the key aspect of Mercator's navigational charts, which involved a problem equivalent to the fundamental theorem of calculus. Pedro Nunes obtained his loxodromic curves using

sine tables, which tables were later corrected by Christoph Clavius and Simon Stevin. Thus, precise sine values were a key concern of European astronomers and navigational theorists of the time. The infinite series expansion as used by Madhava to calculate high-precision sine values, the coefficients used for efficient numerical calculation of these values, and the 24 values themselves were incorporated in a single sloka each, the last two found also in the widely distributed calendrical manuals like *Karanapadhati*.

Motivation (c): Europeans could not use Indo-Arabic techniques of longitude determination because of a goof-up about the size of the earth. Columbus, to promote the financing of his project, downgraded the earlier accurate Indo-Arabic estimates of the size of the earth by 40%. But this size entered as a key parameter in the Indo-Arabic techniques. Nevertheless, Europeans remained interested in the Indo-Arabic techniques of longitude determination, and when the French Royal Academy ultimately developed a method to determine longitude on land, it was a slight improvement of the technique of eclipses mentioned in the texts of Bhaskara-I, and the tome of al Biruni.

Circumstantial evidence: Once in Europe the imported mathematical techniques could easily have diffused, and there is circumstantial evidence that many contemporary mathematicians knew something of the material in Indian texts. For example, Clavius' competitor and critic Julian Scaliger introduced the Julian day-number system, essentially the *ahârgana* system of numbering days followed in Indian astronomy since Aryabhata. Galileo's access to Jesuit sources is well documented, as is that of Gregory and Wallis. Cavalieri was Galileo's student, and Gregory does not claim originality for his series. Marin Mersenne was a clearinghouse for mathematical information, and his correspondence records his interest in the knowledge of Brahmins and 'Indicos'. Fermat, Pascal, Roberval were all in touch with him, and part of his discussion circle. There is other circumstantial evidence to connect Fermat to Indian mathematical texts, for instance his famous challenge problem to European mathematicians, and particularly Wallis, involves a solved problem in Bhaskara's *Beejganita*.[69] 'Julian' day-number, "Fermat's" challenge problem, and "Pascal's" triangle cover only some of the circumstantial evidence of the inflow of mathematical and astronomical knowledge into Europe of that period, but I will not examine more details here, since I regard the above as adequate to make a strong case for the transmission of the calculus from India to Europe in the 16th and 17th c. CE.

4 Mathematics Education

To jump from the past to the future: what bearing do these concerns have on current mathematics education? In the light of the revised history of the calculus, in the light of the argument that mathematics is a social construction that is likely to change with changing technology, especially the widespread use of computers, how should mathematics and calculus be taught today?

technique, or can be empirically validated, like a physical theory, or in conjunction with a physical theory. A given technique of calculation may be fallible, and may not work in another case: for example, the standard technique of extracting a finite value from a divergent integral, as used in renormalization in quantum field theory, does not work with shock waves. While one need have no qualms about non-universality, naturally, the most convenient conventions will be those that are most widely applicable.

(c) On the other hand, I feel Proclus did have a point, that at least at an elementary level, mathematics-as-proof does afford a certain aesthetic satisfaction, even if mathematics as proof does not fulfill the original promise of providing secure knowledge. Thus, I feel that the teaching of mathematics-as-proof, like the teaching of music, or other art form, ought not to be discontinued altogether, but it should be an optional matter, which could be taken up, especially at higher levels, by those interested in it.

Acknowledgments: The author gratefully acknowledges a grant by the Indian National Science Academy, which partly supported the work reported here, and a grant by the East-West Centre and the University of Hawai'i, which supported the travel to Hawai'i, to present this paper at the 8th East-West Conference.

13. In Christian rational theology, the empirical world had to be contingent, since a necessary proposition was regarded as a proposition that had to be true for all time, or at least for all future time (Rescher, cited above). But, a world which existed for all time past, or all time future, would go against the doctrine of creation and apocalypse.

14. Proclus, cited earlier, p 29.

15. Proclus, cited earlier, p 37: 'as Plato also remarks: "If you take a person to a diagram," he says, "then you can show most clearly that learning is recollection." '

16. D. Hilbert, *The Foundations of Geometry*, Open Court, La Salle, 1902.

17. B. Russell, *The Foundations of Geometry*, London, 1908.

18. *School Mathematics Study Group: Geometry*, Yale University Press, 1961.

19. Godel's attack on Hilbert's program concerned Hilbert belief that theorems could be *mechanically* derived from axioms. Godel's theorems do not challenge the notion of proof, which remains mechanical. That is, though it may be impossible to *generate* mechanically or recursively the proof of all theorems pertaining to the natural numbers, given a fully written-out proof, it is, in principle possible to *check* its correctness *mechanically*.

20. Schopenhauer, *Die Welt als Wille*, 2nd ed, 1844, p 130, cited in Heath, cited earlier, p 227.

21. G. D. Birkhoff, 'A Set of Postulates for Plane Geometry (based on scale and protractor),' *Ann. Math.* **33** (1932). For an elementary elaboration of the difference between the various types of geometry, see E. Moise, *Elementary Geometry from an Advanced Standpoint*, Addison Wesley, Reading Mass, 1968.

22. Tr. Davies and Vaughan, cited earlier, p 248. Jowett's translation, reads as follows: '—geometry and the like—they only dream about being, but never can they behold the waking reality so long as they leave the hypotheses which they use unexamined, and are unable to give an account of them. For when a man knows not his own first principle, and when the conclusion and intermediate step are also constructed out of he knows not what, how can he imagine that such a fabric of convention can ever become science?' B. Jowett, cited earlier, p 397.

23. Proclus, cited earlier, p 26.

24. Proclus, cited above, p 33.

25. Proclus, cited earlier, p 24.

26. Tr. Davies and Vaughan, cited earlier, p 240. Jowett's translation reads, 'They have in view practice only, and are always speaking, in a narrow and ridiculous manner, of squaring and extending and applying and the like—they confuse the necessities of geometry with those of daily life; whereas knowledge is the real object of the whole science.' B. Jowett, cited earlier, p 394.

27. Suzan Rose Benedict, *A Comparative Study of the Early Treatises Introducing into Europe the Hindu Art of Reckoning*, Ph.D. Thesis, University of Michigan, April, 1914, Rumford Press.

28. The victory of algorismus over abacus was depicted by a smiling Boethius using Indian numerals, and a glum Pythagoras to whom the abacus technique was attributed. This picture first appeared in the *Margarita Philosophica* of Gregor Reisch, 1503, and is reproduced e.g. in Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, (Tr.) Paul Broneer, MIT Press, Cambridge, Mass., 1970, p 350.

29. C. K. Raju and Dennis Almeida (Aryabhata Group), "The Transmission of the Calculus from Kerala to Europe, Part I: Motivation and Opportunity", "Part II: Documentary and Circumstantial Evidence". Paper presented at the Aryabhata Conference, Trivandrum, Jan, 2000.

30. More details, and quotations etc. may be found in C. K. Raju, "Kamâl or Râpalagai," paper presented at the Indo-Portuguese conference on history, INSA, Dec 1998. To appear, in Proc.

31. 'Com tudo não me parece que sera impossivel saberse, mas has de ser por via d'algum mouro honorado ou brahmane muito intelligente que saiba as cronicas dos tiempos, dos quais eu procurarei saber tudo.'

Letter by Matteo Ricci to Petri Maffei on 1 Dec 1581. *Goa* **38** I, ff 129r–130v, corrected and reproduced in *Documenta Indica*, **XII**, 472–477 (p 474). Also reproduced in Tacchi Venturi, *Matteo Ricci S.I., Le Lettre Dalla Cina 1580–1610*, vol. 2, Macerata, 1613.

32. *The Principal Works of Simon Stevin, Vol. III, Astronomy and Navigation*, (eds) A Pannekoek and Ernst Crone, Amsterdam, Swets and Seitlinger, 1961.

33. Christophori Clavii Bambergensis, Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000..., Ioannis Albini, 1607.

34. The proposition is: to construct an equilateral triangle on a given finite line segment.

35. Ubiratan D'Ambrosio, *Socio-Cultural Bases for Mathematics Education*, Unicamp, 1985.

36. ANSI/IEEE Standard 754-1985

37. Incidentally, there is the practical observation that this notion of implication is counter-intuitive: students have a hard time grasping that $A \Rightarrow B$ is true provided only that A is false. This suggests that the notion of implication in 2-valued logic, far from being universal, is not even the same as the notion of implication in natural language.

38. C. K. Raju, "The Mathematical Epistemology of Sûnya", summary of interventions at the Seminar on the Concept of Sînya, Indian National Science Academy and Indira Gandhi Centre for the Arts, New Delhi, Feb 1997. C. K. Raju, "Mathematics and Culture", in: *History, Culture and Truth*, Essays Presented to D. P. Chattopadhyaya, (eds) Daya Krishna and K. Satchidananda Murty, Kalki Prakash, New Delhi, 179–193, 1999. Reprinted in *Philosophy of Mathematics Education*, **11**, available at http://www.ex.ac.uk/~PErnest/pome11/art18.html. C. K. Raju, " Some Remarks on Ontology and Logic in Buddhism, Jainism and Quantum Mechanics," Invited talk at the conference on Science et engagement ontologique, Barbizon, October, 1999.

39. Prior to the Buddha, a different logic may have been prevalent, as Barua argues. In Barua's view Sanjaya Belatthaputta used a five-fold negation: *evam pi me no, tath ti pi me no, annatha ti pi me no, iti ti pi me no, no ti ti pe me no*. B. M. Barua, *A History of Pre-Buddhistic Indian Philosophy*, Calcutta 1921. Reprinted, Motilal Banarsidass, 1970.

40. Maurice Walshe (Tr.), *The Long Discourses of the Buddha. A Translation of the Dîgha Nikâya*, Wisdom publications, Boston, 1995, p 541, footnote 62 to Sutta 1

41. Maurice Walshe, cited earlier, pp 78–79.

42. *Dîgha Nikâya*. Tr. Maurice Walshe, cited above, pp 80–81.

43. *Mûlamâdhyamakakârika* 18.8, Sanskrit text and Eng. (Tr.) David J. Kalupahana, *Nagarjuna. The Philosophy of the Middle Way*, SUNY, New York, 1986, p 269.

44. Tr. D. Chatterji, "*Hetucakranirnaya*", *Indian Historical Quarterly* **9** (1933) 511–514. Reproduced in full in, R. S. Y. Chi, *Buddhist Formal Logic*, The Royal Asiatic Society, London, 1969, reprint Motilal Banarsidass, Delhi 1984. Chi objects to the exposition of Vidyabhushan (cited below).

45. B. K. Matilal, *Logic, Language, and Reality*, Motilal Banarsidass, Delhi, 1985, p 146: "My own feeling is that to make sense of the use of negation in Buddhist philosophy in general, one needs to venture outside the perspective of the standard notion of negation." See also, H. Herzberger, Double Negation in Buddhist Logic", *Journal of Indian Philosophy* **3** (1975) 1–16.

46. S. C. Vidyabhushan, *A History of Indian Logic*, Calcutta, 1921.

47. J. B. S. Haldane, "The Syadvada system of Predication", *Sankhya, Indian Journal of Statistics*, **18** (1957) 195. Reproduced in D. P. Chattopadhyaya, *History of Science and Technology in Ancient India*, Firma KLM, Calcutta, 1991, as Vol. 2, *Formation of the Theoretical Fundamentals of Natural Science*, Appendix IV, pp 417–432.

48. S. C. Vidyabhushan, *A History of Indian Logic*, Calcutta, 1921. Neither Jaina nor Buddhist records tell us which Bhadrabahu was associated with *syâdvâda*. I am inclined to think it was Bhadrabahu the junior, a contemporary of Dinnaga, and not the senior.

49. C. K. Raju, "Time in Indian and Western Traditions, and Time in Physics", in *Mathematics, Astronomy and Biology in Indian Tradition*, (eds) D. P. Chattopadhyaya and Ravinder Kumar, PHISPC, New Delhi, 1995, 56–93. C. K. Raju, "Kala and Dik" (to appear) in S. K. Sen, ed., PHISPC, New Delhi.

50. C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994, Chapter 6b.

51. Isn't socially approved knowledge the best that one can aspire for? That depends upon the nature of the society in question. Does social authority refer to unanimity or even to a democratically evolved consensus? As I have argued elsewhere, in industrial capitalist societies, for economic reasons, the social authority for scientific knowledge necessarily rests in certain specialists, and the social conferment of authority on these specialists often fully reflects the evils of these societies. Further, these specialists being under pressure to confirm, the agreement of many specialists is hardly a guarantee of secure knowledge. Thus mathematical knowledge in capitalist societies is exactly as insecure as the technology that arises from the capitalist way of getting quick practical results with the least resources. C. K. Raju, "Mathematics and Culture", cited earlier.

52. B. B. Dutta and A. N. Singh, *History of Hindu Mathematics, A Source Book*, Parts I and II, Asia Publishing House, Bombay [1935] 1962, p 245, "Brahmagupta has made the incorrect statement that 0/0=0."

53. *Seminar on the Concept of Sûnya*, INSA and IGNCA, New Delhi, Dec., 1998.

54. W. Rudin, *Real and Complex Analysis*, Tata McGraw Hill, New Delhi, 1968, p 18–19.

55. Rudin, cited above.

56. C. K. Raju, "Mathematical Epistemology of sûnya", cited earlier.

57. Datta and Singh, cited earlier, p 245. There is a misprint there about the value of x.

58. In a letter of 15 Feb 1671 to John Collins, Gregory had supplied Collins with seven power series around 0, for arc tan θ, tan θ, sec θ, log sec θ, etc., H. W. Turnbull, *James Gregory Tercentenary Memorial Volume*, London, 1939. Gregory's series, however, contained some minor errors in the calculation of the coefficient of the fifth-order term in the expansion.

59. Taylor Theorem appeared as Proposition 7, Corollary 2, in Brook Taylor, *Methodus Incrementorum directa et inversa*, London, 1715, 1717. Translation in L. Feigenbaum, *Brook Taylor's "Methodus Incrementorum": A Translation with Mathematical and Historical Commentary*, Ph.D. Dissertation, Yale University, 1981. Apart from Newton, the series was anticipated by James Gregory. L. Feigenbaum, "Brook Taylor and the Method of Increments, " *Arch. Hist. Exact. Sci.* **34** (1) (1984) 1–140.

60. The key passage is quoted in the *YuktiBhâsâ* and attributed to the *TantraSangraha*. *YuktiBhâsâ*, Part I (ed) with notes by Ramavarma (Maru) Thampuran and A. R. Akhileshwara Aiyar, Mangalodayam Ltd, Trichur, 1123 Malayalam Era, 1948 CE, p 190. The passage is NOT to be found in the *TantraSangraha* of the Trivandrum Sanskrit Series. *Tantrasangraha*, S. K. Pillai (ed), Trivandrum Sanskrit Series, 188, Trivandrum, 1958, the English translation of which has been recently serialised in the *Indian Journal of History of Science*. The authors of the modern *YuktiBhâsâ* commentary have however used a transcript of the MSS of the *TantraSangrahaVyâkhyâ* in the Desamangalattu Mana, a well-known Namboodri household. This version of the *TantraSangraha* is found in the *TantraSangrahaVyâkhya*, Palm Leaf MS No. 697 and its transcript No. T12541, both of the Kerala University MS Library, Trivandrum. The missing verses are after II.21a of the Trivandrum Sanskrit Series MS. The same verses are also found on pp 68–69 of the transcript No. T-275 of the *TantraSangrahaVyâkhya* at Trippunitra Sanskrit College Library which is copied from the manuscript of the Desamangalattu Mana. See also K. V. Sarma, *A History of the Kerala School of Astronomy (in perspective)*, Hoshiarpur, 1972, p 17; A. K. Bag, "Madhava's sine and cosine series," *Indian Journal of History of Science*, **11** (1976) 54–57; T. A. Sarasvati Amma, *Geometry in Ancient and Medieval India*, Motilal Banarsidass, New Delhi, 1979,

2nd ed, 1999, 184–190. For detailed quotations and a more mathematical account of the passages, see C. K. Raju, "Approximation and Proof in the YuktiBhâsâ Derivation of Madhava's Sine Series", Paper presented at the National Seminar on Applied Science in Sanskrit Literature: Various Aspects of Utility, Agra, 20–22 Feb 1999.

61. Clavius, cited above. The secant tables in Stevin, cited above, are similar.
62. The procedure is described in C. K. Raju, "Kamâl…", cited earlier.
63. A. P. Jushkevich, *Geschichte der Mathematik in Mittelater*, German Tr., Leipzig, 1964, of the original, Moscow, 1961.
64. Victor J. Katz, *A History of Mathematics: An Introduction*, HarperCollins*CollgePublishers*, 1992.
65. Edwards, cited above, p 114; Boyer, cited above, p 417.
66. e.g., Edwards, cited above, pp 109–113.
67. e.g. V. I. Arnol'd, *Barrow and Huygens, Newton and Hooke*, (Tr.) E. J. F. Primrose, Birkhauser Verlag, Basel, 1990, pp 35–42, states that "Newton's basic discovery was that everything had to be expanded in infinite series…. Newton, although he did not strictly prove convergence, had no doubts about it….What did Newton do in analysis? What was his main mathematical discovery? Newton invented Taylor series, the main instrument of analysis. "
68. Matteo Ricci, cited earlier.
69. Fermat's challenge problem to European mathematicians, particularly Wallis, concerned "Pell's" equation. (The name is due to Euler, and Pell had nothing to do with this equation.) Fermat's correspondence with Frenicle explicitly mentions the case n= 61 of "Pell's" equation, which has the solution x = 226153910, and y = 1766319049, which is the case that appears in Bhaskara's *Beejganita*. A similar problem had earlier been suggested by Brahmagupta, and Bhaskara II provides the general solution with his *cakravâla* method. D. Struik, *A Source Book in Mathematics 1200–1800*, Harvard University Press, Cambridge, Mass, 1969, p 29–30. I. S. Bhanu Murthy, *A Modern Introduction to Ancient Indian Mathematics*, Wiley Eastern, New Delhi, 1992.

Annexure 8. Excerpts from the section on "Transmission of the Transmission Thesis" from C. K. Raju, *Cultural Foundations of Mathematics*, Pearson Longman, April 2007, demonstrating blind copying of mistakes by Almeida 2001 which persists in the Manchester 2007 paper of Aug 2007.

present the paper in Trivandrum. Almeida, being in the School of Education, felt that this invitation for a plenary talk in an international conference, that too in a session related to technology and education, would greatly add to the credibility of these ideas with his colleagues, and was very particular that I should send him a copy of the paper,[67] when I last met him in Goa in Dec 1999. (This Hawai'i paper is the other key paper not cited by the trio.) At that time, Almeida also formally agreed to be co-author of a chapter in this book, originally conceived as a series of essays by different authors. Later he asked for the revised copy of the paper in connection with a bid for funds from the Leverhulme trust, a bid in which G. G. Joseph, then a Reader in Economics at the University of Manchester, was invited to join, on the grounds that a British citizen was required to obtain this funding, and he also had some popular writings on the history of mathematics. At this stage the collaboration was terminated, due to disagreements, and I pointed out that it would be unethical for others to continue pursuing these ideas without my participation.

Finally, my Bangalore talk on the transmission of the calculus,[68] in Dec 2000, happened to be in a session chaired by G. G. Joseph, who naturally had a copy of the detailed abstract, and was at that time giving the School of Education of the University of Exeter as one of his affiliations, and was obviously associated with at least one member of the trio, though he, himself, was not a signatory to the trio's paper submitted subsequently on 22 February 2001—it is not necessary to go here into what transpired in Bangalore.

It would not be appropriate to discuss motivation etc. in the context of this book, although I have discussed it elsewhere, for instance in my formal complaint to the University of Exeter.

Finally, there is the principle of epistemological discontinuity which can be very well illustrated in the context. The principle is very simple. Those who copy without acknowledgement, also very often copy without adequate understanding. Therefore, lack of understanding is a good indication of lack of originality.

This lack of understanding is barely illustrated here using a couple of the more obvious howlers in the trio's paper.

The authors state[69] (p. 87)

> latitude was determined in the northern hemisphere by **measuring** the polar star **declination** (the angle of the pole star)—latitude was approximately equal to the altitude of the pole star. [Emphasis added]

As the deliciously vague phrase "angle of the pole star" suggests, there is a confusion here between the two angles: DECLINATION and ALTITUDE. The meaning of the sentence is quite unambiguous: the authors intend that the declination of the pole star is to be measured, and the altitude is presumably to be calculated!

This, of course, defeats a key aspect of the novel[70] thesis that was advanced above: namely that Jesuits searched for calendrical manuals in India because Europe then needed a good calendar for navigation. Why was a good calendar needed for navigation? According to

my novel thesis, a good calendar was needed just because there was no easy way to measure declination at sea, but the (solar) declination could be easily estimated using a calendar, provided the calendar correctly fixed the day of the equinox. So if declination could have been *measured* so easily and directly at sea in the 16th c. CE, there would hardly have been any European need for a good calendar!

That this is no typo, but involves a conceptual confusion, is clear in the next howler, when the trio subsequently speak of

> **measuring** the solar **declination** at noon and then looking up tables correlated with the calendar. [Emphasis added]

Since, according to the repeated claim made by the authors, the solar declination could be directly measured at sea, and since it is the case that altitude could easily be observed with a simple instrument like a cross-staff (or *kamāl*), latitude could be readily calculated, using the *Laghu Bhāskarīya* formula. So what on earth was a "table correlated with the calendar" needed for? To help the navigator determine the date, perhaps!

That this is no typo, but a conceptual confusion, is proved beyond all reasonable doubt, when the authors repeat the same thing a third time, on the next page:

> **observations** of solar **declination** or pole star. . . . [Emphasis added]

It was, I believe, an established principle in Europe since the 17th c. CE to "booby trap" a mathematical table by deliberately injecting errors in it, just as some computer programmers (like me) have been known to booby trap source code (when compelled to disclose it against their wishes to persons whose credentials are not established) by deliberately injecting bugs in it. The source of these errors can be found in the first part of the Trivandrum paper NOT cited by the trio, which makes the same mistake, on p. 6:

> The widely distributed *Laghu Bhaskariya* (abridged works of Bhāskara) and *Maha Bhaskariya* (extensive works of Bhaskara) of the first Bhaskara (629 CE) explicitly detailed methods of determining the local latitude and longitude, using **observations** of solar **declination** or pole star, and simple instruments like the gnomon, and the clepsydra. Since local latitude could easily be determined from solar **declination** by day and e.g. pole star altitude at night (using an instrument like the kamal) an accurate sine table was just what was required. . . . [Emphasis added]

Since the objective here is only to illustrate the principles of evidence used to establish transmission, we take up just one more example to demonstrate the consequences of conceptual confusion regarding key aspects of the transmission thesis closely related to my other key (Hawai'i) paper that is also not cited by the trio. This involves a somewhat subtler point.

In the abstract of that paper, "Computers, mathematics education, and the alternative epistemology of the calculus in the Yuktibhāṣa", presented before a large number of scholars at Hawai'i, I had argued as follows:

> Current (formal) mathematics, being socially constructed, may change with technology.... Computers also use a different notion of 'number': unlike Turing machines, computers necessarily use floating point numbers, fundamentally different from real numbers on which mathematical analysis is currently based. An alternative pedagogy and epistemology of the calculus, bypassing real numbers is thus needed. A suitable alternative epistemology is found in the c. 1530 CE YuktiBhasa of Jyeshthadeva... Given the practical uses of computer simulation and the consequent social pressure to teach a changed notion of 'number' can the incompatible epistemologies of mathematics be reconciled?

Or even more succinctly, as stated in the four-line abstract of the paper for the table of contents of *Philosophy East and West*:

> Current formal mathematics, being divorced from the empirical, is entirely a social construct... Computer technology, by enhancing the ability to calculate, has put pressure on this social construct....

The paper pointed out the representation of real numbers involves a supertask not necessary for practical purposes.

> For practical purposes, no supertask is necessary; the representation of numbers on a computer is satisfactory for mathematics-as-calculation, but it is unsatisfactory or "approximate" or "erroneous" from the standpoint of mathematics-as-proof. Indian mathematics, which dealt with "real numbers" from the very beginning ($\sqrt{2}$ finds a place in the *śulba sūtras*), does not represent numbers by assuming that such supertasks can be performed, any more than it represents a line as lacking any breadth, for the goals of mathematics in the Indian tradition were practical not spiritual. The **Indian** tradition of mathematics worked with a finite set of numbers, **similar** to the numbers available on a computer, and similarly adequate for practical purposes. Excessively large numbers, like an excessively large number of decimal places after the decimal point, were of little practical interest. Exactly what constitutes "excessively large" is naturally to be decided by the practical problem at hand so that no universal or uniform rule is appropriate for it. [p. 340, emphasis added][71]

The trio seize without acknowledgement this thesis that I had presented a year earlier in Hawai'i:

> we believe that mathematics is a social construct that alters with changing technology and that the current revolution in information technology will induce changes in mathematics.... (p. 96)

How is this to be linked to Indian mathematics? They take off:

> we re-iterate that floating point numbers were used by the Kerala mathematicians.... (p. 96)

Note that the thesis has been slightly changed: the term *similar* has been dropped, changing the thesis from analogy to identity, and the term *Indian mathematics* has been replaced by "Kerala mathematicians". Note also how these slight changes have oversimplified the thesis, laying it open to all sorts of doubts. (Where did Kerala mathematicians use the concepts of non-normal numbers and gradual underflow that one associates with floating point numbers?[72] Why only mathematicians confined to Kerala? Did they use numbers in a way different from other Indian mathematicians? What are the sources for this belief about use of numbers? etc.)

Through this oversimplification, the trio betray their lack of acquaintance with the philosophy of number underlying Indian mathematics. The problem with this is, as Nāgārjuna remarks, a half-understood concept of *śūnya* can be as fatal as a snake grasped wrongly—even slightly wrongly. This lack of understanding proves fatal to the trio's thesis as follows. Not quite understanding the Indian philosophy underlying the use of number, the trio of authors revert to a seemingly safe and conventional Western position (p. 96):

> We accept that mathematical analysis is based on the complete real number system needed for the existence of limits and that limiting processes can never be accomplished [sic] by a computer which uses a floating point number system.

However, this sudden reversion introduces a clash of epistemologies which stalls the original thesis in mid air, resulting in the inevitable crash. For, what after all is the use of Indian mathematics in the context? The trio continues

> we believe that a study of Keralese calculus will provide insights into computer-assisted teaching strategies for introducing concepts in *mathematical analysis*.... [p. 96, emphasis mine]

But how on earth can floating point numbers be used to motivate or teach formal real numbers? That amounts to putting the cart before the horse! And even supposing that floating point numbers (and concepts like non-normal numbers) can somehow be used to motivate formal real numbers, why not simply use computers for this purpose? Thus, it seems quite obvious to me that the task of computer-aided mathematics teaching can be

performed perfectly well by software like my CALCODE (Calculator for Ordinary Differential Equations, which was purchased by the University of Exeter), especially since the ultimate object is to teach *mathematical analysis*! So, why bring in "Kerala mathematics" at all? Of course, the easiest way to understand the origin of these insoluble problems is to suppose that these problems have arisen from the oversimplification of a complex thesis, used without acknowledgement.[73]

The more important point here is to observe how the attempt to bring a novel thesis into a conventional epistemic fold so quickly makes it meaningless. This is exactly what happened also in the case of the calculus when it came to Europe with an epistemology of mathematics and number, that was incompatible with the European perspective into which it was forced to fit.