Transmission of the Calculus from Kerala to Europe
Part 1: Motivation and Opportunity

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It is by now widely recognised[2] that the calculus had already developed in India in the works of the mathematicians and astronomers of the Aryabhata school: Madhava, Nilkantha (*Tantrasangraha*), Jyeshtadeva (*Yuktibhasa*) etc, between the 14th and 16th centuries CE. These developments included infinite "Gregory/Taylor" series for sine, cosine and arctan functions,[3] with accurate remainder terms, and a numerically efficient algorithm, leading to a 9 decimal-place precision table for sines and cosines stated in sexagesimal *katapayadi* notation in two verses found also in the widely distributed *KaranaPaddhati* of Putumuna Somayaji.[4] The development also included the calculation of complex derivatives like that of arcsin ($p \sin x$) (*Tantrasangraha*

[1] The Aryabhata Group acknowledges financial support from the School of Education, University of Exeter, in the work that led to this paper.

[2] A.P. Jushkevich, *Geschichte der Matematik im Mittelater* German translation, Leipzig, 1964, of the original, Moscow, 1961. Victor J. Katz, *A History of Mathematics: An Introduction*, HarperCollins*CollegePublishers*, 1992. Srinivasiengar, *The History of Ancient Indian Mathematics*, World Press, Calcutta, 1967, A. K. Bag. *Mathematics in Ancient and Medieval India*, Chaukhambha Orientalia, Delhi, 1979. A popular account may be found in G. G. Joseph, *The Crest of the Peacock: non-European Roots of Mathematics*, Penguin, 1992.

[3] The version of the *TantraSangraha* which has been recently serialised (K. V. Sarma ed) together with its English translation (V. S. Narasimhan Tr.) in the *Indian Journal of History of Science* (issue starting Vol. 33, No. 1 of March 1998) is incomplete and does not contain the relevant passages. We have used the verion of the *TantraSangraha* as found in the *TantraSangrahaVyakhya*, Palm Leaf MS No 697 and its transcript No. T 1251, both of the Kerala University MS Library, Trivandrum. The missing verses are after II.21a of the Trivandrum Sanskrit Series MS. The same verses are also found on pp 68—69 of of the transcript No., T-275 of the *TantraSangrahaVyakhya* at Trippunitra Sanskrit College Library, copied from a palm leaf manuscript of the Desa Mangalatta Mana. We found Pl-697/T-1251 more useful since the commentary in Malayalam is clearly separated from the original text in Sanskrit. Needless to say these verses are also found in the YuktiBhasa etc. For detailed quotations, translations, and a mathematical exposition, collected in one place, see C. K. Raju, "Approximation and Proof in the Yuktibhasa derivation of Madhava's Sine series". Paper presented at the National Seminar on Applied Sciences in Sanskrit Literature, Various Aspects of Utility, Agra 20--22 Feb 1999. To appear in Proc.

[4] P. K. Koru (ed) *Karana-paddhati of Putumuna Somayaji*, Astro Printing and Publishing Co., Cherp (Kerala), 1953, p 203; S. K. Nayar (ed) *Karana Paddhati of Putumuna Somayaji*, Govt. Oriental Manuscript Library, Madras, 1956, pp 189--193. For an exposition, see C. K. Raju, "Kamal or Rapalagai", Paper presented at the Xth Indo-Portuguese Conference on History, Indian National Science Academy, New Delhi, Dec 1998. To appear in Proc.

V.53-54), and *p* sin *x*/(1+*p* cos *x*) (*Sphutanirnaya* III.19-20), to calculate the instantaneous velocities of the sun and the moon, and infinite series expansions, and high-precision computations of the value of π correct to 9, 10 (and later 17) decimal places.[5] (As already noted by Benjamin Heyne[6] in 1805 these developments were probably not confined to Kerala but were available also in Tamil Nadu, Telangana, and Karnataka, though this possibility has not yet been properly investigated.)

A key point, that has not been noticed earlier, is this: these developments *cannot* be dismissed as "pre-calculus", the way the works of Fermat, Pascal etc. usually are. Thus, the use of traditional floating point numbers, enabled the Indian mathematicians to provide a rigorous rationale for the infinite series and the infinitesimal calculus. This was quite unlike the case of Newton etc. who, lacking also the notion of real number, used "fluxions" or "infinitesimals", the exact meaning of which remained a mystery until the development of mathematical analysis and the clarification of the notion of "proof" in the late 19th century and early 20th century CE. Since the Indian mathematicians had a rigorous rationale which Newton could not possibly have had, the *Yuktibhasa* exposition should, *a fortiori*, count as calculus.

It is true that the *Yuktibhasa* ideas of mathematics and proof differ from the Platonic and Hilbertian idea that mathematics must be divorced from the empirical; however, it is hard to see, from either a theoretical or a practical point of view, why acceptance of the Platonic point of view and Platonic authority *ought* to be a key ingredient of mathematics. In particular, the Platonic insistence on divorce from the empirical leaves hanging in the air the question of what *logic* ought to underlie a proof,[7] whereas acceptance of the empirical would mean a change in the notion of proof, since different criteria are used to validate a physical theory. Since this paper is

[5] K. V. Sarma (Ed and Tr) *GanitaYuktiBhasa of Jeyshtadeva (Analytical Exposition of the Rationales of Indian Mathematics and Astronomy)*, unpublished typescript , p 33-34.

[6] J. Warren, *Kala Sankalita*, Madras 1825, pp 93, 309--310. This observation of Heyne precedes the better known paper of Charles Whish, presented in 1832: "On the Hindu quadrature of the circle and the infinite series of the proportion of the circumference to the diameter exhibited in the four Shastras, the Tantrasamgraham, Yukti-Bhasa, Carana Padhati, and Sadratnamala." *Tr. Royal Asiatic Society of Gr. Britain and Ireland*, **3** (1835) p 509-523.

[7] This is not an empty question since the logic employed in other cultural traditions, like the empirical logic of quantum mechanics, need be neither 2-valued nor even truth functional; see, C. K. Raju, "Mathematics and Culture", in: *History, Time and Truth: Essays in Honour of D. P. Chattopadhyaya*, (eds) Daya Krishna and K. Satchidananda Murty, Kalki Prakash, New Delhi 1998. Reproduced *in Philosophy of Mathematics Education*, **11**, available at http://www.ex.ac.uk/~PErnest.

concerned with transmission, rather than epistemology, we do not pursue this any further.

Prior to Vasco da Gama there is ample evidence of the import of Indian mathematical knowledge into Europe.[8] The history of Indian arithmetical techniques imported into Europe via the Arabs as "algorismus" texts is now well known. (Algorismus is the Latinized version of Al Khwarizmi (9th century CE), who translated the arithmetical and astronomical texts of Brahmagupta (7th century CE.) These "algorismus" techniques were first introduced into Europe by Gerbert (Pope Sylvester III) in the 10th century CE, but it is only in the 16th century CE that their final triumph over abacus techniques started being depicted on the covers of arithmetical texts.[9] On the other hand, there is also, from *after* the late 17th century, ample evidence of the large scale import of Indian texts and manuscripts, tens of thousands of which are today housed in European libraries[10]. Our primary hypothesis for investigation is that this process of importing Indian texts continued also during the unstudied intervening period of the 16th and 17th century. *Our hypothesis is that the arrival of Vasco da Gama in Calicut not only short-circuited the traditional Arab route for spices, it also short-circuited the traditional Arab route for knowledge of Indian mathematics and astronomy.*

Further, it is our hypothesis that the epistemological difficulties encountered with infinitesimals in Europe from the 17th to the 19th centuries CE arose, exactly like the difficulties with *sunya*/zero, due to the import of techniques with a different epistemological base.[11] This has important pedagogical implications. However, in this

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[8] Suzan Rose Benedict, *A Comparative Study of the Early Treatises Introducing into Europe the Hundu Art of Reckoning*, Ph.D. Thesis, University of Michigan, April, 1914, Rumford Press.

[9] The victory of algorismus over abacus was depicted by a smiling Boethius using Indian numerals, and a glum Pythagoras to whom the abacus technique was attributed. This picture first appeared in the *Margarita Philosophica* of Gregor Reisch, 1503, and is reproduced e.g. in Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, (Tr) Paul Broneer, MIT Press, Cambridge, Mass., 1970, p 350. According to the periodisation suggested by Menso Folkerts, the abacus period commenced in the 12th century. Menso Folkerts, Lecture at the Second Meeting of the International Laboratory for the History of Science, Max Planck Institute for the History of Science, Berlin, 19-26 June 1999.

[10] An early descriptive catalogue of the Indian manuscripts in the Vatican is P. Paulino A. S. Bartholomaeo, *Historico-Criticum Codicum Indicorum*, Rome, 1792.

[11] For the different epistemology underlying the notion of *sunya*, see C. K. Raju, "Mathematical Epistemology of *Sunya*", summary of interventions at the Seminar on the Concept of *Sunya*, Indian National Science Academy and Indira Gandhi Centre for the Arts, Delhi, Feb, 1997. To appear in Proceedings.

paper we will set aside the epistemological and pedagogical issues and focus on the question of transmission.

In the past there have been far too many claims of transmission, where the evidence produced is farcical; for example, in support of the widespread claim of the transmission of Ptolemaic astronomy from Alexandria to India, one line of evidence, proposed by Thibaut, is that Varahamihira's use of "Paulisha" suggests that it could have been derived from "Paul" (rather than Pulisha or Pulastya, one of the seven sages forming the constellation known as the Great Bear). If this be the standard of evidence, there is nothing for us to prove. For the works of Paramesvara, Madhava, Nilkantha, and Jyeshtadeva, clearly precede those of Fermat, Pascal, Gregory, Wallis, Newton, and Leibniz, and India was clearly known (and actively linked) to Europe by the 16th century CE.

However, we are aware that, for some unfathomable reasons, the standard of evidence required for an acceptable claim of transmission of knowledge from East to West is different from the standard of evidence required for a similar claim of transmission of knowledge from West to East. Priority and the possibility of contact *always* establish a socially acceptable case for transmission from West to East, but priority and definite contact *never* establish an acceptable case for transmission from East to West, for there always is the possibility that similar things could have been discovered independently. Hence we propose to adopt a legal standard of evidence good enough to hang a person for murder. Briefly, we propose to test the hypothesis on the grounds of (1) motivation, (2) opportunity, (3) circumstantial evidence, and (4) documentary evidence.

(1) Motivation

The motivation for import of knowledge derived from the needs of greater accuracy in (a) navigation, (b) the calendar, and (c) practical mathematics (as in algorismus texts). Navigation was clearly the key motivation, being then a matter of the greatest strategic and economic importance for Europe. Early navigators like Columbus, and Vasco da Gama did *not* know stellar navigation,[12] and dead reckoning was of little use in "uncharted" seas unless, like Columbus, one was aimed at so massive a shore line, that one could hardly hope to miss it! Official acknowledgment of both the ignorance of navigational techniques, and the great need to learn more, came from various European governments which instituted huge prizes for anyone who could provide an accurate technique of navigation. These included the Spanish government (1567, 1598) the Dutch government (1632), the French government (1670), and the British government (1711), which last prize was finally claimed in 1762 by Harrison with his chronometer (which came into general use only by the 19th century). To get an idea of the true value of these prizes we observe that e.g. the British government's prize was more than 300 times Newton's *annual* fellowship.

Consequently, not only kings and parliaments, but also very many of the leading scientists of the times were involved in these efforts. Galileo, for example, unsuccessfully competed for the revised Spanish prize for 16 years, before shifting his attention to the Dutch prize. Colbert wrote personally to all the leading scientists of Europe, offering large rewards, and selected from the replies received to start the French Royal Academy "to improve maps, sailing charts, and advance the science of navigation". This was also a key motivating problem for starting the British Royal Society. Prior to the Royal Society, groups of scientists began meeting in London and Oxford from 1645, the longitude problem being their main object of concern. The importance of the longitude problem, and the related difficulty about the size of the earth, both, are captured in a 1661 poem describing the work going on at Gresham College:

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[12] For references and other details see C. K. Raju, "Kamal or Rapalagai" Paper presented at the Xth Indo-Portuguese meeting on History, Indian National Science Academy, Dec 1998, to appear in Proc. The rapalagai was the instrument used by the Indian pilot who brought Vasco da Gama to India, from Melinde.

The Colledge will the whole world measure,
Which most impossible conclude,
And Navigators make a pleasure
By finding out the longitude.
Every Tarpalling shall then with ease
Sayle any ships to th'Antipodes.

[Tarpalling here means a sailor] The group from Gresham College include John Wallis and Robert Hooke, which later joined other groups to became the Royal Society of London for the Promotion of Natural Knowledge. Christopher Wren, also a member of the Gresham College group, wrote the preamble to the Royal Society's charter. One of the stated aims of the newly founded Royal Society was: Finding the longitude. Newton testified before the British parliament in connection with this problem of navigation.

Stellar navigation naturally involved the study of astronomy and timekeeping, which was, then, inseparably linked to mathematics. Most Indian texts on mathematics were located in the context of astronomy and timekeeping (*jyotisa*). Moreover, the navigational knowledge that the Europeans sought *was*, in fact, available in Indian mathematical and astronomical texts. The widely distributed *Laghu Bhaskariya* (abridged work of Bhaskara) and *Maha Bhaskariya* (extensive work of Bhaskara) of the first Bhaskara (629 CE) explicitly detailed methods of determining the local latitude *and* longitude, using observations of solar declination, or pole star, and simple instruments like the gnomon, and the clepsydra.[13] Since local latitude could easily be determined from solar declination by day and e.g. pole star altitude at night (using an instrument like the *kamal*), an accurate sine table was just what was required to determine local longitude from a knowledge of latitudinal differences and course angle. The *Laghu Bhaskariya* already states the criticism that determination of longitude by a calculation involving plane triangles is not adequate because of the roundness of the earth. Later texts like the *Siddhanta* of Vatesvara (904 CE) pointed

[13] For detailed quotations, and an exposition, see C. K. Raju, "Kamal or Rapalagai", cited earlier. Specifically, the method of determining longitude using a clepsydra is detailed in Laghu Bhaskariya ,

out[14] that these techniques needed to be corrected by applying spherical trigonometry. Al Biruni, the 10th-11th century scholar, who visited India on behalf of Mahmud of Ghazni, and systematically studied and translated Indian mathematical and astronomical texts, explicitly used these techniques of spherical trigonometry to determine local latitudes and longitudes in his treatise on mathematical geography.[15]

Exactly this technique started being tried in Europe in the 16th century, when the centre of navigational excellence in Europe had shifted from Florence to Coimbra. In the first half of the 16th century, Pedro Nunes studied motion on the sphere along a given rhumb line, or a given course. Evaluating such a path or a loxodromic (Gr. *loxos* = oblique, *dromos* = course) curve is exactly equivalent to the fundamental theorem of calculus: [16] given the tangent at every point, to determine the curve passing through these points which has those tangents. It is very interesting that these loxodromic curves were, in fact, studied by Pedro Nunes and Simon Stevin[17] *using sine tables, and the above stated technique*,[18] with a solution in spherical triangles, though the triangles involved were not strictly spherical, as Stevin observed. (The exact technique by which Mercator obtained the loxodromic curves for his famous chart is not known, but was probably similar.) The conjectures of Pedro Nunes were tested in a voyage to Goa, in the 1540's during which they reportedly failed, presumably due to inaccurate techniques of calculation, inadequate sine table, and other factors listed below.

Though the Europeans, motivated by navigation, were actively seeking the knowledge of determining local latitude and longitude through stellar astronomy, and though this knowledge was available in Indian mathematical texts, there were three things that impeded their search. They lacked: (a) knowledge of practical and mental mathematics, (b) an accurate calendar, and (c) an accurate estimate of the size of the

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II.8 (Ed and Tr. K. S. Shukla), Department of Mathematics and Astronomy, Lucknow University, 1963, p 53.

[14] For detailed quotations, see C. K. Raju, "Kamal..." cited above.

[15] E. S. Kennedy, *A Commentary Upon Biruni's Kitab Tahdid al Amakin, An 11th Century Treatise on Mathematical Geography*, American University of Beirut, Beirut, 1973.

[16] D. J. Struik, *A Source Book in Mathematics, 1200--1800*, Harvard University Press, Cambridge, Mass., 1969, p 253.

[17] *The Principal Works of Simon Stevin, Vol. III, Astronomy and Navigation* (eds) A Pannekoek and Ernst Crone, Amsterdam, Swets and Seitlinger, 1961.

[18] For a figure etc., see C. K. Raju, "Kamal..." cited above. Stevins, *The Haven Finding Art*, cited above, p 481 et. seq. Nunes ascribed to the loxodrome a particular property, viz. That the sines of the

earth. The last was needed for the calculation of longitudes/departures, from a knowledge of only latitudes and course angle.[19] While Caliph al Mamun had confirmed through empirical observations in the 9th centruy CE, the estimates of the equatorial radius of the earth given in Indian astronomical texts, and al Biruni had implemented a cheaper and easier technique to confirm these,[20] Columbus undid this. To sell his idea of sailing West to reach the East, he underestimated the size of the earth by 40%. This error persisted, with e.g. Newton's initial estimates being off by 25%, until Picard's accurate re-determination of the size of the earth, in 1671, funded by the French Royal Academy as its first scientific effort. However, in the first part of the 16th century lacking even an accurate calendar, and lacking techniques of calculation, the difficulty was with determining *latitude* correctly!

Like al Biruni was to Mahmud of Ghazni, the Jesuits were to the Portuguese an intelligence gathering arm. While the Jesuits learned the local languages like Malayalam and Tamil easily enough, they were (a) deficient in knowledge of mathematics, and (b) constrained by an inaccurate ritual calendar. Christoph Clavius, who had studied under the famous Pedro Nunes at Coimbra, realised this handicap. He reformed the Jesuit mathematical curriculum at the Collegio Romano in the 1570's, and later went on to head the committee which reformed the Gregorian Calendar to which the Pope gave his assent in 1582.

Clavius also wrote a text on practical mathematics, and compiled and published a tables of sines[21] which could be looked up without the need for any mental calculation. These tables, presumably, were intended to replace the tables of Regiomontanus, taken from Arabic sources, and those of Rheticus, who perhaps also obtained his information from Arabic sources, like Copernicus.[22] Thus, Clavius recognized and strove to remove all the drawbacks listed above. The need for more

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polar distances of the points of intersection with meridians at equal differences of longitude form a continued proportion. Stevins,(p 491) refuted this by direct calculation.

[19] For more details, see C. K. Raju, "Kamal..." cited above.

[20] For details of Al Biruni's technique, and for the connection to the Indo-Arabic navigational measure of zam, see C. K. Raju, "Kamal or Rapalagai," cited earlier.

[21] Christophori Clavii Bambergensis, *Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000 ...*, Ioannis Albini, 1607. As the title suggests, this table concerns not sine values proper, as today understaood, but RSine values which are what are given in Indian manuscripts. Stevins follows the same practice for his secant tables, The Haven Finding Art, cited above, p 483.

accurate sine tables for navigational purposes was stressed also by Clavius' contemporary, Simon Stevins, in his criticism of the work of Pedro Nunes. Stevins explicitly states Aryabhata's value of π, observing that it is more accurate than that of Regiomontanus, who came nearly a thousand years after Aryabhata.[23] (As is well known, Stevins contemporary, Ludolph von Ceulen devoted a lifetime to getting increasingly accurate values of π.)

(2) Opportunity

The famous Matteo Ricci was in the first batch of Jesuits trained in the new mathematics curriculum introduced in the Collegio Romano by Clavius. He also went to Lisbon to study cosmography and nautical science. Ricci was then sent to India. While the Portuguese had shifted their headquarters to Goa, the Jesuits maintained a large presence in Cochin (until the Protestant Dutch closed down the Cochin College around 1670). So it was to Cochin that Ricci went after taking his orders. He remained in touch with the Dean of the Collegio Romano. Writing from Cochin to Maffei, he explicitly acknowledged that he was trying to find out about the calendar from Indian sources, both Brahmins and Moors. (For details, see Section 4 below.)

We emphasise that the calendar is necessary for synchronising social activities, so the Jesuits could not but have already noticed the discrepancy between their calendar and the local calendar. For example, the Jesuits were accustomed to the idea that festivals like Christmas, or Easter came on a fixed day of their calendar. They could not have failed to notice that the major Indian festivals like Dussehara, Diwali, Holi, Sankranti etc., did not fall on the same days of the Julian calendar. However, the typical Jesuit before Matteo Ricci probably did not know enough about astronomy to have known the difference even between the sidereal year (the basis of the Indian calendar) and the tropical year (the basis of the Julian/Gregorian calendar); so they could hardly have been expected to understand the complexities of the Indian calendar.

Now, in India, preparing the calendar (*pancanga*) was and remains to this day the task of the *jyotishi*. The typical *jyotishi* relied (and still relies), like a clerk, on a handbook

[22] George Saliba, *A History of Arabic Astronomy in the Golden Age of Islam,* New York University Press, 1994.

of rules, without bothering to go into too many details of how the rules were derived. The standard treatises that were consulted and are today still consulted for this purpose are the *Laghu Bhaskariya*, and, in Kerala, the *KaranaPaddhati*. These are in the nature of calendrical manuals, and so are widely distributed throughout the country, since they are used every year to determine the dates of a large number of festivals. Depending upon differences of religion, caste, and region, each group of people only accept as authoritative a *pancanga* (almanac) prepared by a particular family. Thousands of families of *pancanga* makers were hence involved in this process of calendar making, across the country.[24] So, if Matteo Ricci did try to find out about the calendar from a Brahmin source in Cochin, in the heart of Kerala, as he explicitly stated he was trying to do, it is difficult to conceive that he did not run into the *KaranaPaddhati*. It may help to reiterate that the *Kriyakramakari*, the *KaranaPaddhati* etc. incorporate Madhava's sine table, in a single verse, along with the cosine table in another verse.

We emphasise that the Jesuits had much more than a casual interest in India. For at just about the time that Matteo Ricci was in Cochin, in 1580, the Mughal emperor Akbar invited the Jesuits to his court. This was represented in Rome as a sign of his imminent conversion, an event of the greatest importance, which could bring along with it all the political and material benefits that the Roman church obtained from Constantine; three high-level missions were sent to Akbar's court. Matteo Ricci was, at the same time, writing back sending details of the Mughal army.

We also emphasise that the Jesuits had much more than a casual interest in the calendar. For at just about that time, Matteo Ricci's teacher, Christoph Clavius was busy heading the commission that ultimately reformed the Gregorian calendar in 1582, an event that had been preceded by centuries of controversy. The 1545 Council of Trent had already acknowledged the error in the Julian calendar, and had authorised the Pope to correct it. So Matteo Ricci's interest in the Indian calendar was not a casual one, but was an effort preceded by years of preparation and study, and came at a time when the Jesuit interest in both India and the calendar was at a peak.

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[23] *The Principal Works of Simon Stevins*, Vol III, cited above, p 603.

[24] Printing has somewhat simplified this process. For a listing of 60 different currently printed *pancanga*-s, see Report of the Calendar Reform Committee, Govt of India, CSIR, 1955.

Finally, we emphasise that Matteo Ricci's mathematical preparation was most suited to the task at hand. Christoph Clavius had written a commentary on the *Sphere* of Sacro Bosco, (after studying the *Sphere* of Pedro Nunes) and had published in 1580 a large 645 page book on *Gnomonices*. The sphere (*gola*) and the gnomon (*shanku*) were the two key topics needed to understand Indian astronomy and timekeeping: Aryabhata devotes a chapter to the sphere, while Vatesvara has a whole book on it. Ricci had studied nautical science along with the *Cosmographia* of Apian, so he could hardly have missed the significance and importance of precise sine values.

Transmission of the Calculus from Kerala to Europe
Part 2: Circumstantial and documentary evidence

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Part I proposed to take a legal view of the evidence for the transmission to Europe of the calculus that had developed in the works of the mathematicians and astronomers of the Aryabhata school between the 14th and 16th centuries. We established strong motivation for the transmission in the needs of navigation and the calendar reform, which were recognised as the most important scientific problems of that age in Europe. We also established that Europeans had ample opportunity to access the texts and calendrical almanacs in which this information was to be found, not only in the straightforward sense that they knew the local languages well, but also in the sense that trained mathematicians were sent for the express purpose of collecting the knowledge available in these texts.

Given this it would seem a foregone conclusion that the Indian mathematical manuscripts incorporating the calculus had already reached Europe by around 1580. Nevertheless, it is important to detail the circumstantial and documentary evidence, for it helps also to understand how this knowledge, after arriving in Europe, diffused in Europe. (Apart from historical curiosity, this is also a matter of contemporary pedagogical significance, since it enables us to assess, in a non-destructive way, the possible impact of introducing today the study of calculus with a different epistemological basis.)

We know too little even to conjecture anything about the accumulation of knowledge in Coimbra , nor exactly what happened to this accumulated knowledge after Jesuits

[1] The Aryabhata Group acknowledges financial support from the School of Education, University of Exeter, in the work that led to this paper.

took over control of the university after about 1560. After 1560, in line with the above reasoning, our working hypothesis, or scenario, is that over a 50 year period, say from 1560 to 1610, knowledge of Indian mathematical, astronomical and calendrical techniques accumulated in Rome, and diffused to nearby universities like Padova and Pisa, and to wider regions through Cavalieri and Galileo, and through visitors to Padova, like James Gregory[2]. Subsequently, it also reached Paris where, through the agency of Mersenne, and his study circle, it diffused throughout Europe.

Mersenne, though a minim monk, had received a Jesuit education, and was closely linked to Jesuits. Mersenne's correspondence reveals that Goa and Cochin were famous places in his time,[3] and Mersenne writes of the knowledge of Brahmins and "Indicos",[4] and mentions the orientalist Erpen and his "les livres manuscrits Arabics, Syriaques, Persiens, Turcs, Indiens en langue Malaye".[5] Mersenne's study circle included Fermat, Pascal, Roberval etc., and Mersenne's well-known correspondence with leading scientists and mathematicians of his time, could have helped this knowledge diffuse throughout Europe. (Newton, as is well known,[6] followed Wallis, and Leibniz himself states, he followed Pascal.) Of course, acquisition of knowledge of Indian mathematics could hardly have been a controlled process, so that many others, like the Dutch and French, for instance, could have simultaneously acquired this knowledge directly from India, without the intervention of Rome.

(1) Circumstantial evidence

With this scenario as the background, we can ask: what sort of circumstantial evidence can we *hope* to find? Certainly it would be absurd to expect citations in

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[2] See H W Turnbull, *James Gregory Tercentenary Memorial Volume*, London , 1939. "Gregory spent three or four years (1664-1668) in Italy....having stayed most of the time in Padua, where Galileo taught." (p 4). That Gregory acquired his knowledge through other sources is made plain by A. Prag, "On James Gregory's Geometriae Pars Universalis", pp 487-509, in H W Turnbull, *James Gregory Tercentenary Memorial Volume*, London , 1939: "James Gregory published *Geometriae Pars Universalis* at the end of his visit to Italy in 1668. This book is the firs attempt to write a systematic text-book on what we should call the calculus. Gregory does not suggest that he is the actual author of all the theorems in this work …We cannot judge exactly how much Gregory borrowed from other authors, because we do not know which books he may have read and to what extent he had knowledge of the unprinted work of his contemporaries."

[3] *Correspondance du P. Marin Mersenne*, 18 volumes, Presses Universitaires de France, Paris, 1945-. ; A letter from the astronomer Ismael Boulliaud to Mersenne in Rome, Vol XIII, p 267-73.

[4] *Correspondance*, Vol XIII, p 518-521.

[5] *Correspondance*, Vol II, p 103-115.

[6] E.g. Carl B. Boyer, *A History of Mathematics*, Wiley, 1968, p 424; C. H. Edwards, *The Historical Development of the Calculus*, Springer, 1979, p 113.

published work! The tradition in Europe of that time was that mathematicians did not reveal their sources. When they could get hold of others' sources, they copied them without compunction. The case of e.g. Cardan is well known, and there are well documented cases against, e.g. Copernicus,[7] Galileo[8] etc., of copying from others, whether or not such copying amounted to "plagiarism". Under these circumstances, mathematicians naturally kept their sources a closely guarded secret: they published only problems, not their solutions, and challenged other mathematicians to solve them.

(a) Fermat and Pell's equation.

One such challenge problem was proposed by Fermat, and has come to be known as Pell's equation (for no fault of Pell). "In a letter of February 1657 (*Oeuvres*, II, 333-335, III, 312-313) Fermat challenged all mathematicians (thinking in the first place of John Wallis in England) to find an infinity of integer solutions of the equation $x^2 - Ay^2 = 1$, where A is any nonsquare integer."[9] Mathematicians in Europe were unable to solve Fermat's challenge problem for over 75 years, until Euler published a general solution in 1738. In February 1657, Fermat also wrote a letter to Frenicle, where he elaborated upon this problem:[10] "What is for example the smallest square which, multiplied by 61 with unity added, makes a square?"

As Struik further notes, Indian mathematicians had a solution to this problem. In fact, strangely enough, *exactly* the case of A=61 is given as a solved example in the *BeejGanita* text of Bhaskara II. This coincidence is not trivial when we consider that the solution x = 176631904, y = 226153980 involves rather large numbers.[11] A similar problem had earlier been suggested by the 7th century Brahmagupta, and Bhaskara II provides the general solution with his *chakravala* method. Thus, Fermat's challenge problem, strongly suggests a connection of Fermat with Indian

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[7] George Saliba, cited earlier.

[8] William Wallace, *Galileo and his Sources – The heritage of the Collegio Romano in Galileo's Science*, Princeton University Press, 1984, has direct evidence of Galileo borrowing heavily from Jesuit sources in the Collegio Romano.

[9] D. Struik, *A Source Book in Mathematics*, cited earlier, p 29.

[10] D. Struik, *A Source Book in Mathematics*, cited above, p 30.

[11] For an modern introduction to details of Bhaskara's *chakravala* method, see I. S. Bhanu Murthy, A Modern Introduction to Ancient Indian Mathematics, Wiley Eastern, new Delhi, 1992. Also see C.Selenius, 'Rationale of the Chakravala Process of Jayadeva and Bhaskara II', *Historia Mathematica*, **2**, 1975, 167-184

mathematics: Fermat probably had access to some Indian mathematical texts like the *BeejaGanita*.

Euler certainly knew about Indian astronomy (hence mathematics), for Giovanni Cassini, then the most reputed astronomer of France, had already published an account of "Hindu astronomy" in 1691, and Euler wrote on the "Hindu year" [12] (sidereal year; the pope's bull was still not acceptable e.g. to Protestant Britain, until 1752). Since numerous Indian astronomical texts deal with "Pell's" equation, Euler had presumably learnt about this as well. However, we are not aware that he acknowledged this when he published his solution to what he called Pell's equation.

This suggestion that Fermat knew something about Indian mathematics is reinforced by Fermat's friendship not only with Mersenne, but with Jacques de Billy (1602-1669), a Jesuit teacher of mathematics in Dijon. Fermat also had the habit (then a general proclivity) of searching for knowledge in ancient books.[13]

(b) Fermat, Pascal and the calculus

"One of Fermat's most stunning achievements," continues Aczel, "was to develop the main ideas of calculus, which he did thirteen years before the birth of Sir Isaac Newton [in 1642]." Fermat's and Pascal's approach to the calculus reinforces the belief in a connection with Indian mathematics. It was at Mersenne's place that Pascal met Descartes who remarked in his *La Geometrie* of 1637 about the impossibility of measuring the circumference of a circle: "The ratios between straight and curved lines are not known, and I even believe cannot be discovered by men, and therefore no conclusion based upon such ratios can be accepted as rigorous and exact." Therefore, there was not, at this point of time, anything that could be called an indigenous or acceptable tradition of calculus.

However, in Indian mathematics, from the time of the *sulba sutra*-s, because of the different epistemological base, measuring the length of a curved line by laying a rope (*sulba*) along it, and straightening it (or measuring a general area by triangulation) has

[12] Euler's paper on the "Hindu year" was an appendix to *Historia Regni Graecorum Bactriani* by T. S. Bayer; see G. R. Kaye, *Hindu Astronomy*, 1924, reprinted, Cosmo Publications, 1981, p 1.

been quite an acceptable process, used to obtain a value of π. In fact, Aryabhata states that the area of a general plane figure should be obtained by triangulation,[14] before going on to give an improved measure of the (curved) circumference of a circle in terms of the diameter (a straight line), in the next verse.[15] In the very next verse he explains how his sine table is derived by approximating small arcs by line segments.[16] It is therefore quite natural to find this eventually developing into the calculus in the *Tantrasangraha*, whose author Nilkantha belonged to the Aryabhata school, and wrote a lengthy commentary on the *Aryabhatiya*, almost exactly a thousand years after it.

Nilkantha's younger contemporary, Jyeshtadeva, author of the Malayalam *Yuktibhasa* ("Discourse on Rationales"), has a chapter on the circle, explaining the method of deriving the improved sine table stated in the *Tantrasangraha*. The key step in this derivation[17] is the evaluation:

$$\lim_{n\to\infty} \frac{1}{n^{k+1}}\sum_{i=1}^{n} i^k = \frac{1}{k+1}, \quad k = 1,2,3,... \tag{1}$$

This is also exactly the approach to calculus adopted by Fermat, Pascal etc. to evaluate the area under the parabolas $y = x^k$, or, equivalently, calculate $\int x^k \, dx$. As Pascal remarked, about this formula, it serves to solve all sorts of problems of the calculus. "Any person at all familiar with the doctrine of indivisibles will perceive the results that one can draw from the above for the determination of curvilinear areas. Nothing is easier, in fact, than to obtain immediately the quadrature of all types of parabolas and the measures of numberless other magnitudes." This formula has been

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[13] Amir D. Aczel, *Fermat's Last Theorem*, Penguin, p 5.
[14] *Ganita*, 9 a-b. Aryabhatiya of Aryabhata,(ed and Tr.) K. S. Shukla and K. V. Sarma, Indian National Science Academy, Delhi, 1976. It may be recollected here, that area is not defined anywhere in any well-known version of the Elements, and that Hilbert's synthetic approach to the Elements, which dfines neither length nor area, hence fails beyond Proposition 1.35 of the Elements. See C. K. Raju, "How Should `Euclidean' Geometry be Taught in Schools?" Paper Presented at the International Workshop on History of Science, Implications for Science Education, Homi Bhabha Centre for Science Education, TIFR, Mumbai, 22--26 Feb 1999. To appear, Proc.
[15] *Ganita*, 10. *Aryabhatiya*, cited above.
[16] *Ganita*, 11. *Aryabhatiya*, cited above.
[17] C. K. Raju, "Approximation and proof in the Yuktibhasa derivation of Madhava's sine series", cited earlier.

attributed to Fermat in 1629; Roberval, another member of Mersenne's circle, also worked on it. Earlier Cavalieri, a student of Galileo, had stated this formula, without proof, in 1635, after waiting five years for Galileo to write on infinitesimals. John Wallis, who visited Pisa, verified the formula for a few values of k, and obtained his value of π using similar series expansions.

The curious thing is that though so many European mathematicians seem to have suddenly "discovered" this formula at about the same time, the formula had no natural epistemological basis in European mathematics, either of that time or for the next two centuries, for European mathematics was oriented towards "proof" rather than "calculation", and shared the Greek "horror of the infinite". Even today, despite the compelling changes of technology, due to widespread use of supercomputers, the situation has not entirely changed, and as in the time of Clavius, calculation continues to be regarded as "inferior" to "proof".

Though the European mathematicians were unable to prove the formula (1), or provide a rigorous rationale for it within their epistemology,[18] even the techniques by which they attempted to prove formula (1) suggests transmission. For example, Pascal tried to establish[19] the formula (1) using the so-called Pascal's triangle, for the binomial coefficients. The triangle appears as the *meru prastara*, in Pingala's Chandahsutra of (-3 century CE), and another 1200 years later in the work of his 10th century CE commentator Halayudha.[20] It was known to the Arabs and the Chinese.[21] Among Renaissance European mathematicians it is found in the arithmetic of Apian, and in the work of authors like Bombelli[22].

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[18] E..g. Cavalieri's statement is now termed a conjecture, while Wallis is said to have stated the formula without proof. Edwards, cited above, p 114; Boyer, cited above, p 417.

[19] E.g. Edwards, cited above, p 109-113.

[20] S. N. Sen, "Mathematics", in D. M. Bose, S. N. Sen, and B. V. Subbarayappa (eds), A Concise History of Science in India, Indian National Science Academy, Delhi, 1971, pp 156-157.

[21] Photographs of a 1303 Chinese depiction of Pascal's triangle have been provided by Joseph Needham, *The Shorter Science & Civlisation in China*, Vol 2, Cambridge University Press, 1981, p 55.

[22] Bombelli acknowledged the transmission of Indian mathematics to the West. From J. Fauvel and J. Gray, *The History of Mathematics*, Macmillan, 1987, we have (page 264) from the preface of Bombelli's *Algebra* " …a Greek work on this discipline has been discovered in the Library of our Lord in the Vatican, composed by a certain Diophantus of Alexandria, a Greek author, who lived at the time of Antoninus Pius. When it had been shown to me by Master Antonio Maria Pazzi, from Reggio, public lecturer in mathematics in Rome…..(we) set ourselves to translate it … in this work we have found that he cites Indian authors many times, and thus I have been made aware that this discipline belonged to the Indians before the Arabs."

(c) The Ahargana and the Julian day-number system

The use of Julian day numbers is another kind of evidence. These day numbers, used in scientific specification of dates, are named, somewhat ambiguously, after Julius Scaliger, the father of Joseph Scaliger. Joseph Scaliger was a well-known opponent of Christoph Clavius, and he, too, introduced his numbering system from 1582.

Now, from at least the time of Aryabhata, all dates in Indian astronomy are specified in this way, using day numbers. This eliminates any possible ambiguity due to calendrical differences; such ambiguities did exist because of the variety of calendars in use. These day numbers are specified as *Ahargana* or "heap of days". Understanding the first stanza of the *Aryabhatiya* requires us to know about this system; in fact, the day number system could have been transmitted by absolutely any Indian astronomical text.

The difference between *Ahargana* and Julian day numbers is only this: the *Ahargana* count starts from the beginning of the Kaliyuga, (17 Feb -3102 CE) whereas the Julian day-number count starts from 1 Jan -4713 CE (an astronomically convenient date, presumably related to the date of Biblical creation). Thus, the *Ahargana* differs from the Julian day number by exactly 588,465 which is the Julian day number for the start of the Kaliyuga. Of course, the system is simple enough and could have been invented by anyone at any time. The strange thing is that the system was allegedly invented in Europe at *exactly* the time, in 1582, when it could have been transmitted through a stated earlier desire to learn about Indian calendrical techniques. If our conjecture about transmission of the day-number system is true, it would seem that well before their conquest of Cochin, the Dutch had independent sources of information from India.

(d) Planetary models and elliptic orbits

There are many other key instances that should count as circumstantial evidence. For example, Nilkantha's planetary model, in the *Tantrasangraha*, is exactly the "Tychonic" model (Tycho was a contemporary of Clavius), except that it involves elliptical orbits. (It is now known that Tycho's student, Kepler, obtained his elliptical orbits by computing his "observations".[23]) We do not go into these for reasons of space, since we first need to give an exposition of all the relevant planetary theories.

We hope to find compelling evidence through a statistical analysis of various sine tables. This analysis is, at present, being carried out, so it is not included in this paper.

(2) Documentary evidence of the role of the Jesuits

In this section we document how this knowledge may have been transmitted from the Malabar coast by the Jesuit missionaries.

(a) The arrival of the Jesuits. The period after Vasco da Gama's arrival in Calicut in 1498 and the establishment, shortly thereafter, of a Portuguese colony with bases in Cochin, Cannanore and Goa, by Afonso de Albuquerque in 1510, laid the foundations for Catholic missionary work in the Malabar coast.

Among the various missions, the Jesuit one was the most important in respect of transmitting local knowledge to Europe. While there is a paucity of literature on this subject (mainly due to the, as yet, uncatalogued nature of a vast quantity of oriental manuscripts in Portugal), this is not the case with French missionary work.[24] That the French Jesuits were actively engaged in the acquisition of Indian astronomy is reported by Otto Spies.[25] Spies makes explicit reference to Calmette's study of local astronomy "A notable part of his activity turned Calmette to the astronomy and he speaks for a long time and in technical form in this and other letters."

The earlier Jesuits in the Malabar coast did the same and we offer substantial evidence to support this claim.

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[23] "Planet fakery exposed. Falsified data: Johannes Kepler" *The Times* (London) 25 January 1990, 31a, including large excerpts from the article by William J. Broad, "After 400 years, a challenge to Kepler: He fabricated his data, scholars say". *New York Times* 23 January 1990, C1, 6. The key article is William Donahue, "Kepler's fabricated figures: Covering up the mess in the *New Astronomy" Journal for the History of Astronomy*, **19** (1988) p 217-37.
[24] E.g. Gérard Colas, *Les manuscrits envoyé de l'Inde par les jésuites français entre 1729 et 1735*, Bibliothèque nationale de France, Paris.
[25] O. Spies, "Il P. Calmette e le sue Conoscenze Indologiche", *Studia Indologica*, Bonn, 1955, p 53-64.

(b) Jesuit mastery of vernacular languages and of Brahmin culture. It was Jesuit policy to aim to master vernacular languages such as Malaylam.[26] Prominent Jesuits who became fluent native speakers included De Nobili who spoke Sanskrit and Tamil (the language spoken in Trichur) and Diogo Gonsalves who spoke Malaylam. The attempt by Jesuits to learn the vernacular was so widespread that they frequently used Malaylam to sign their names in letters to the Society of Jesus headquarters in Rome.[27] The rationale for learning the vernacular languages was to aid their work in converting the local populace to Jesuit Catholicism by understanding their science, culture and customs and, of course, by facilitating communication. The former was important for the Jesuits and this included, at the very least, awareness of *jyotisa*--- de Nobili, for instance, in 1615, wrote[28] a strong polemic against the Vedanga Jyotisa, a work that had been discarded as obsolete by Varahamihira, a thousand years earlier. To formalise the policy of educating Jesuit workers in the local culture, 'local' subjects such as astrology or *jyotisa* were included in the curriculum of the Jesuit colleges in the Malabar coast[29]. The Jesuits' study of the vernacular languages was not merely intended to facilitate their work in conversions. It was also intended to enable work on transmitting this knowledge back to Europe. This is supported by the acquisition of translations of Malyalam and Sanskrit manuscripts:

"In Portuguese India, hardly seven years after the death of St. Francis Xavier the fathers obtained the translation of a great part of the 18 Puranas and sent it to Europe. A Brahmin spent eight years in translating the works of Veaso (Vyasa)……several Hindu books were got from Brahmin houses, and brought to the Library of the Jesuit college. These translations are now preserved in the Roman Archives of the Society of Jesus. (*Goa* **46**)"[30]

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[26] *Documenta Indica*, **XIV** p 425 and **XV** p 34*, and D. Ferroli, *The Jesuits in Malabar*, Bangalore, 1939.
[27] see, e.g., the letters contained in the manuscript collection *Goa* **13** at the Archivum Romanicum Societate Iesu in Rome.
[28] V. Cronin, *A Pearl to India – The Life of Roberto de Nobili*, Darton, Longman and Todd, 1966, p 178-180
[49] *Documenta Indica*, **III**, p 307.
[50] D. Ferroli, *The Jesuits in Malabar*, 1939, Vol 2, p 402

Further evidence of this knowledge acquisition is contained in the ARSI collections *Goa* **38**, **46** and **58**. The last being the work of father Diogo Gonsalves and contains detailed notes on the judicial system and on the sciences and mechanical arts of the Malabar region.

(c) The arrival of Matteo Ricci. Arithmetic, astronomy and timekeeping were particular areas of interest to the Jesuits as is shown by the many references in the *Documenta Indica*[31]. The arrival of Matteo Ricci in Goa in September 13, 1578 was significant in this respect. Ricci had previously studied at the Collegio Romano (as did all Jesuit fathers); in particular he studied mathematics under the renowned Christopher Clavius. Furthermore, he studied the *Cosmographia* and nautical science. This knowledge made Ricci a candidate for discovering the knowledge of the colonies and he had specific instructions to investigate the science of India. The Jesuit historian Henri Bernard states that Ricci

"…had resided in the cities of Goa and of Cochin for more than three years and a half (September 13, 1578-April 15, 1582): he had been requested to apply himself to the scientific study of this new and imperfectly know country, in order to document his illustrious contemporary, Father Maffei, the 'Titus Livius' of Portuguese explorations."[32]

Bernard reports that Ricci had begun his task prior to arriving in India by setting about a study of nautical science. It is also known that Ricci had enquiries about Indian calendrical science; in a letter to Maffei he states that he requires the assistance of an "intelligent Brahmin or an honest Moor" to help him understand the local ways of recording and measuring time (lit. *jyotisa*).

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[31] See, for example, *Documenta Indica*, **IV** p 293 and **VIII** p 458.
[32] Henri Bernard, *Matteo Ricci's Scientific Contribution to China*, Hyperion Press, Westport, Conn., 1973, p38. It is relevant to point out that it was Maffei (I. Petri Maffei, *History of the Indies*, in book 1, Venice 1589) who details the navigational help received by da Gama to cross the Arabian sea and arrive at Calicut.

"Com tudo não me parece que sera impossivel saberse, mas has de ser por via d`algum mouro honorado ou brahmane muito intelligente que saiba as cronicas dos tiempos, dos quais eu procurarei saber tudo."[33]

(d) Printing presses and translation work. The Jesuits established printing presses all over the Malabar region; in 1550 in Goa which used Roman types, in 1577 in Vaipicota using Tamil and Malyalam types, in 1602 Vaipicota using Syro-Chaldic, and in 1578 in Tuticorin with Tamil types. The aim of these presses was to publish the catechism so essential for missionary work; for example, St Francis Xavier's catechism was published in 1557 by the Goa press. The aim was also to translate the local science into Portuguese prior to transmission to Europe; for example, Garcia da Orta's *Colloquios dos simples e drogas he cousas mediçinas da India* published in Goa in 1563[34]. There were many other publications of this type but they remain obscure because, as Sarton points out

"A Portuguese book printed in Goa could not attract much attention outside the Portuguese world."[35]

(e) A possible source for Jesuit acquisition of Indian mathematics and astronomy. After the 1580 annexation of Portugal by Spain and subsequent loss of funding from Lisbon, the rationale for transmission acquired another dimension, that of profit[36] but how might the Jesuits have obtained key manuscripts of Indian astronomy such as the *Tantrasangraha* and the *Yuktibhasa*? Such manuscripts would require the Jesuits being in close contact with scholarly Brahmins; there is concrete evidence that they were in contact with such people. There is plenty of evidence of the Jesuits being in contacts with kings across the country. We establish that the Jesuits

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[33] Letter by Matteo Ricci to Petri Maffei on 1 Dec 1581. *Goa* **38** I, ff 129r--30v, corrected and reproduced in *Documenta Indica*, **XII**, 472-477 (p 474). Also reproduced in Tacchi Venturi, *Matteo Ricci S.I.*, *Le Lettre Dalla Cina 1580-1610*, vol 2, Macerata, 1613.
[34] T. J. S. Patterson, 'Science and Medicine in India', in Eds Pietro Corsi and Paul Weindling, *Information Sources In The History Of Science And Medicine*, Butterworths Scientific, 1983, p 437-456.
[35] D. Sarton, 1955, *The Appreciation of Ancient and Medieval Science During the Renaissance (1450-1600)*, University of Pennsylvania, 1955, p102
[36] D. Ferroli, *The Jesuits in Malabar*, 1939, volume 2, p 407.

had close relations with the kings of Cochin, and that the latter were knowledgeable about mathematics and astronomy in the tradition of Kerala.

The kings of Cochin came from the scholarly Kshatriya, Varma Tampuran family who were knowledgeable about the mathematical and astronomical works of medieval Kerala[37]. Rama Varma Tampuran who, in 1948 (together with A.R. Akhileswara Iyer),[38] had published an exposition in Malayalam on the *Yuktibhasa* was one of the princes of Cochin already stated; this exposition which has been the basis of most subsequent work on the *Yuktibhasa* used the *TantraSangrahaVyakhya* manuscript of the Desa Mangalatta Mana (a Namputiri household, now disbanded). This manuscript was in the keeping of Rama Varma Tampuran (who belonged to that household). Moreover, various authors, from Charles Whish in the 19th c. to Rajagopal and Rangachari have acknowledged that members of the royal household were very helpful in supplying these manuscripts in their possession.

This suggests that the former royal family in Cochin, which was in possession of a large number of MSS, had not only a scholarly tradition, but also a tradition of helping other scholars. Thus, the royal family could itself have been a possible source of knowledge for the Jesuits. Indeed the Jesuits working on the Malabar coast had close relations with the kings of Cochin[39]. Furthermore, around 1670, they were granted special privileges by King Rama Varma[40] who, despite his misgivings about the Jesuit work in conversion, permitted members of his household to be converted to

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[37] C. M. Whish, *On the Hindu Quadrature of the Circle….*, Transactions of the Royal Asiatic Society of Great Britain and Ireland, 3(3), 1835, pp 509-523. Whish states here (p 521) that the author of the astronomical work the *Sadratnamalah* is Sankara Varma, the younger brother of Raja of Cadattanada near Tellicherry and further states that the Raja is a very acute mathematician. Also C. N. Srinivasiengar, *The History of Ancient Indian Mathematics*, World Press, Calcutta, 1967. Srinivasiengar also states that Sankara Varma is the author of *Sadratnamalah* and is brother of the Uday Varma, the King of Kerala (p146) . He further refers (p 145) to the Malyalam *History of Sanskrit Literature in Kerala* which identifies the King of Cochin, Raja Varma, as being aware of the chronology of the *Karana-Paddhati*.

[38] *Yuktibhasa* Part 1, with notes (Ed) Ramavarma (Maru) Tampuran and A. R. Akhileswara Aiyar, Mangalodayam Ltd, Trichur, 1948 (in Malayalam). Most subsequent work has relied on this exposition. C. T. Ragagopal and M. S. Rangachari, On an Untapped Source of Medieval Keralese Mathematics, *Archive for the History of Exact Sciences*, **18**, 1978, 89-102. Rajagopal and Rangachari state that Rama Varma Tampuran supplied them with the manuscript material [Desa Mangalatta Mana MS of *TantraSangrahaVyakhya*] relating to Kerala mathematics. Also K. V. Sarma, *A History of the Kerala School of Hindu Astronomy*, Hoshiarpur Vishveshvaranand Institute, 1972. On page 12, K. V. Sarma points to the valuable notes added to the analysis of Kerala astronomy by Rama Varma Maru Tampuran in his 1948 exposition of the first part of the *Yuktibhasa.* His subsequent translation of the *Yuktibhasa* has utilised the same material (personal communication).

[39] see for example, *Documenta Indica*, **X**, pages 239, 834, 835, 838, and 845

Christianity[41]. The close relationship between the King of Cochin and the foreigners from Portugal was cemented by King Rama Varma's appointment of a Portuguese as his tax collector[42].

Given this close relationship with the Kings of Cochin, the Jesuit desire to know about local knowledge, and the royal family's contiguity to the works on Indian astronomy, it is quite possible that the Jesuits may have acquired the key manuscripts via the royal household.

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[40] *Documenta Indica*, **XV**, p 224
[41] *Documenta Indica*, vol XV, p 7*
[42] *Documenta Indica*, vol XV, p 667