TRANSMISSION OF THE CALCULUS FROM KERALA TO EUROPE[1]

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Introduction

It is by now widely recognised[3] that the calculus had already developed in India in the works of the mathematicians and astronomers of the Aryabhata school: Madhava, Nilakantha (*Tantrasangraha*), Jyeshtadeva (*Yuktibhasa*) etc, between the 14th and 16th centuries CE. These developments included infinite "Gregory/Taylor" series for sine, cosine and arctan functions,[4] with accurate remainder terms, and a numerically efficient algorithm, leading to a 9 decimal-place precision table for sines and cosines stated in sexagesimal *katapayadi* notation in two verses found also in the widely distributed *KaranaPaddhati* of Putumuna Somayaji.[5] The development also included the calculation of complex derivatives like that of arcsin ($p$sin x) (*Tantrasangraha* V.53-54), and p sin x/(1+$p$cos x) (*Sphutanirnaya* III.19-20), to calculate the instantaneous velocities of the sun and the moon, and infinite series expansions, and high-precision computations of the value of π correct to 9, 10 (and later 17) decimal places.[6] (As already noted by Benjamin Heyne[7] in 1805 these developments were probably not confined to Kerala but were available also in Tamil Nadu, Telangana, and Karnataka, though this possibility has not yet been properly investigated.)

A key point, that has not been noticed earlier, is this: these developments *cannot* be dismissed as "pre-calculus", the way the works of Fermat, Pascal, Wallis, Torricelli, Roberval, etc. usually are. Thus, the traditional Indian number system, similar to the floating point numbers used in present-day computers, together with sharp estimates of the remainder or error term, enabled the Indian mathematicians to provide a rigorous rationale for the infinite series and the infinitesimal calculus.[8] This was quite unlike the case of Newton, etc. who, lacking also the notion of real number, used "fluxions" or "infinitesimals"[9], the exact meaning of which remained a mystery until the development of mathematical analysis and the clarification of the notion of "proof" in the late 19th century and early 20th century CE. Since the Indian mathematicians had a rigorous rationale which Newton could not possibly have had, the *Yuktibhasa* exposition should, *a fortiori*, count as calculus.

It is true that the *Yuktibhasa* ideas of mathematics and proof differ from the Platonic and Hilbertian idea that mathematics must be divorced from the empirical; however, it is hard to see, from either a theoretical or a practical point of view, why acceptance of the Platonic point of view and Platonic authority *ought* to be a key ingredient of mathematics. In particular, the Platonic insistence on a divorce from the empirical leaves hanging in the air the question of what *logic* ought to underlie a proof,[10] whereas acceptance of the empirical would mean a change in the notion of proof, since different criteria are used to validate a physical theory. Since this paper is concerned with transmission, rather than epistemology, we do not pursue this any further.

Prior to Vasco da Gama there is ample evidence of the import of Indian mathematical knowledge into Europe.[11] The history of Indian arithmetical techniques imported into Europe via the Arabs as "algorismus" texts is now well known. (Algorismus is the Latinized version of Al Khwarizmi (9th century CE), who translated the arithmetical and astronomical texts of the 7th century CE Brahmagupta.) These "algorismus" techniques were first introduced into Europe by Gerbert (Pope Sylvester III) in the 10th century CE, but it is only in the 16th century CE that their final triumph over abacus techniques started being depicted on the covers of arithmetical texts.[12] On the other hand, there is also, from *after* the late 17th century, ample evidence of the large scale import of Indian texts and manuscripts, tens of thousands of which are today housed in European libraries[13]. Our primary hypothesis for investigation is that this process of importing Indian texts continued also during the unstudied intervening period of the 16th and 17th century. *Our general hypothesis is that the arrival of Vasco da Gama in Calicut not only short-circuited the traditional Arab route for spices, it also short-circuited the traditional Arab route for knowledge of Indian mathematics and astronomy.*

Further, it is our hypothesis that the epistemological difficulties encountered with infinitesimals in Europe from the 17th to the 19th centuries CE arose, exactly like the difficulties with *sunya* (zero), due to the import of techniques with a different epistemological base.[14] This has important pedagogical implications. However, in this paper we will set aside the epistemological and pedagogical issues and focus on the question of transmission.

What is the evidence for transmission? Before addressing this question we need to address a meta-question: what is an acceptable *standard* of evidence for transmission? This meta-question seems not to have been addressed at all in the literature on the history of mathematics.[15] In the past there have been far too many claims of transmission, where the evidence produced is farcical; for example, in support of the widespread claim of the transmission of Ptolemaic astronomy from Alexandria to India, one line of evidence, proposed by Thibaut, is that Varahamihira's use of "Paulisha" suggests that it could have been derived from "Paul" (rather than Pulisha or Pulastya, one of the seven sages forming the constellation known as the Great Bear). If this be the standard of evidence, there is nothing for us to prove. For the works of Paramesvara, Madhava, Nilakantha, and Jyeshtadeva, clearly precede those of Fermat, Pascal, Gregory, Wallis, Newton, and Leibniz, and India was clearly known (and actively linked) to Europe by the 16th century CE.

However, we are aware that, for some unfathomable reasons, the standard of evidence required for an acceptable claim of transmission of knowledge from East to West is different from the standard of evidence required for a similar claim of transmission of knowledge from West to East. Priority and the possibility of contact *always* establish a socially acceptable case for transmission from West to East, but priority and definite contact *never* establish an acceptable case for transmission from East to West, for there always is the possibility that similar things could have been discovered independently. Hence we propose to adopt a legal standard of evidence good enough to hang a person for murder. Briefly, we propose to test the hypothesis on the grounds of (1) motivation, (2) opportunity, (3) circumstantial evidence, and (4) documentary evidence.

(1) Motivation

The motivation for import of knowledge derived from the needs of greater accuracy in (a) navigation, (b) the calendar, and (c) practical mathematics (used for everyday financial calculations). Christoph Clavius, a key figure in the transitional period, exhibits all three concerns: since he (a) authored a book on practical mathematics, (b) headed the calendar reform committee, and (c) was a student of the famous navigational theorist Pedro Nunes.

Navigation was clearly the key motivation, being then a matter of the greatest strategic and economic importance for Europe. Early navigators like Columbus, and Vasco da Gama did *not* know stellar navigation,[16] and dead reckoning was of little use in "uncharted" seas unless, like Columbus, one was aimed at so massive a shore line, that one could hardly hope to miss it! Various European governments officially acknowledged both (a) the European ignorance of navigational techniques, and (b) the enormous interest in learning more. Many governments also widely publicised this official acknowledgment by instituting huge prizes for anyone who could provide an accurate technique of navigation. These included the Spanish government (1567, 1598) the Dutch government (1632), the French government (1670), and the British government (1711), which last prize was finally claimed in 1762 by Harrison with his chronometer (which came into general use only by the 19th century). The value of these prizes gives an indication of the importance that various governments attached to the problem of navigation: to get an idea of the true value of these prizes we observe that e.g. the British government's prize of 1711 was more than 300 times Newton's *annual* fellowship.

Consequently, not only kings and parliaments, but also very many of the leading scientists of the times were involved in these efforts. Galileo, for example, unsuccessfully competed for the revised Spanish prize for 16 years, before shifting his attention to the Dutch prize. Colbert wrote personally to all the leading scientists of Europe, offering large rewards, and selected from the replies received to start the French Royal Academy "to improve maps, sailing charts, and advance the science of navigation". One of the stated aims of the newly founded Royal Society was: Finding the longitude. Newton testified before the British parliament in connection with this problem of navigation.

However, the popular accounts of the history of navigation have focussed on the development of the marine chronometer, which relates to the 18th century CE. Our concern is with navigation in the 16th and 17th centuries CE, when the focus was on stellar and celestial navigation, apart from geodesy and geography. Indeed the enormous growth of interest in mathematics and astronomy in Europe in the 16th and 17th centuries CE, is directly attributable to the practical benefit that these studies were expected to have on the navigational problem.

Secondly, the popular account of navigation history has focussed almost exclusively on the problem of *longitude* determination, whereas in the 16th century CE, *latitude* determination was the key problem, because of an inaccurate calendar, and the then lack of knowledge of celestial navigation in Europe. Latitude determination also

involved a problem of timekeeping, but this timekeeping pertained to the *calendar* rather than the *chronometer*.
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Finally, we do need to ask: how did the European attack on the problem of navigation and the related problems of astronomy and mathematics proceed? Did Europe abandon a 500 year old tradition of importing knowledge and books from Arabs and India and suddenly switch to a completely autonomous path of knowledge development? Did Europeans turn a blind eye to the knowledge resources of the areas with which they were hoping to establish new trade routes? The slightest acquaintance with European texts of the 16th and 17th centuries CE shows that exactly the contrary was the case. The "knowledge of the ancients" was most highly valued, and was sought after by such disparate personalities as Stevins, Mersenne and Fermat. All of them actively sought out knowledge from ancient texts, regarding this as one of the superior ways of attacking the problem, and as we shall explicitly see later on in this paper, in "ancient" texts they clearly included texts from other lands.

What sorts of texts might the Europeans have searched for to learn more about navigation? As we saw above, stellar/solar navigation naturally related to the study of astronomy and timekeeping, which was, then, inseparably linked to mathematics. Most Indian texts on mathematics, too, were located in the context of astronomy and timekeeping. (In particular, this is true for all the texts on the calculus, mentioned above.) The term *jyotisa*, as in *Vedanga Jyotisa*, meant timekeeping.[17] Stellar astronomy and mathematics was used for timekeeping, and also for constructing the calendar.

Moreover, the navigational and calendrical knowledge that the Europeans needed and sought in ancient texts *was*, in fact, available in Indian mathematical and astronomical texts. The widely distributed *Laghu Bhaskariya* (abridged work of Bhaskara) and *Maha Bhaskariya* (extensive work of Bhaskara) of the first Bhaskara (629 CE) explicitly detailed methods of determining the local latitude *and* longitude, using observations of solar declination, or pole star altitude, and simple instruments like the gnomon, and the clepsydra.[18] Since local latitude could easily be determined from solar declination by day and e.g. pole star altitude at night (using a common instrument like the *kamal*), an accurate sine table was just what was required to determine local *longitude* from a knowledge of latitudinal differences and course angle. The simplest method was to solve a plane triangle. The *Laghu Bhaskariya* already states the criticism that determination of longitude by a calculation involving *plane* triangles is not adequate because of the roundness of the earth. Later texts like the *Siddhanta* of Vatesvara (904 CE) pointed out that these techniques needed to be corrected by applying spherical trigonometry. Al Biruni, the 10th-11th century scholar, who visited India on behalf of Mahmud of Ghazni, and systematically studied and translated Indian mathematical and astronomical texts, explicitly used these techniques of spherical trigonometry to determine local latitudes and longitudes in his treatise on mathematical geography.[19]

Exactly this technique started being tried in Europe in the 16th century, when the centre of navigational excellence in Europe had shifted from Florence to Coimbra. In the first half of the 16th century, Pedro Nunes studied motion on the sphere along a given rhumb line, or a given course[20]. Evaluating such a path or a loxodromic (Gr.

loxos = oblique, *dromos* = course) curve is exactly equivalent to the fundamental theorem of calculus: [21] given the tangent at every point, to determine the curve passing through these points which has those tangents. It is very interesting that these loxodromic curves were, in fact, studied by Pedro Nunes and Simon Stevin[22] *using sine tables, and the above stated technique*,[23] with a solution in spherical triangles, though the triangles involved were not strictly spherical, as Stevin observed. (The exact technique by which Mercator obtained the loxodromic curves for his famous chart is not known, but was probably similar.) The conjectures of Pedro Nunes were tested in a voyage to Goa, in the 1540's during which they reportedly failed, presumably due to inaccurate techniques of calculation, inadequate sine table, and other factors listed below.

Though the Europeans, motivated by navigation, were actively seeking the knowledge of determining local latitude and longitude through stellar astronomy, and though this knowledge was available in Indian mathematical texts, there were three things that impeded their search. They lacked: (a) knowledge of practical and mental arithmetic, (b) an accurate calendar, and (c) an accurate estimate of the size of the earth. An accurate estimatew of the size of the earth was needed for the calculation of longitudes/departures, from a knowledge of only latitude differences and course angle.[24] While Caliph al Mamun had confirmed through empirical observations in the 9th centruy CE, the estimates of the equatorial radius of the earth given in Indian astronomical texts, and al Biruni had implemented a cheaper and easier technique to confirm these,[25] Columbus undid this. To sell his idea of sailing West to reach the East, he underestimated the size of the earth by 40%. Presumably due to Columbus' "success" this error persisted, with, for example, Newton's initial estimates being off by 25%, until Picard's accurate re-determination of the size of the earth, in 1671, funded by the French Royal Academy as its first scientific effort.

Secondly, in the first part of the 16th century lacking even an accurate calendar, and lacking techniques of calculation, the difficulty was with determining *latitude* correctly. It has been overlooked in the popular history of navigation that the longitude problem was preceded by a latitude problem. Well before the attempt to construct an accurate chronometer to solve the longitude problem, the attempt was to construct an accurate calendar to solve the latitude problem.

Thus, Vasco da Gama was unacquainted with instruments like the *kamal* or *rapalagai* used to determine latitude by measurement of pole-star altitude. This instrument was used by the Indian pilot who navigated him across the Arabian sea from Melinde in Africa to Calicut in India. Since the instrument has a string, which is held with the teeth, and since the instrument uses the pole star, called *kau* in Malayalam, a term which also means teeth, Vasco da Gama thought the pilot was telling the distance by his teeth! The instrument has a string on which are tied knots in harmonic proportion: not realising this, Vasco da Gama carried back a copy of this instrument to get it graduated in inches!

While this technique was presumably mastered by the Europeans by the mid-16th century CE, this was useful only at night. The standard technique for determining latitude in day time was to use solar declination. While a great variety of instruments were available for measuring solar declination, linking solar declination to the local latitude required an accurate calendar. The European calendar, then in use, was unchanged since Roman times, and was inaccurate because it was a solar calendar which assumed that the length of the year was 365 ¼ days. Making the calendar accurate was a problem which involved the date of the equinoxes, identical with the ritual concern with the date of the Easter festival. By 1545 CE this problem had been recognised by both navigators (like Nunes) and by the church which set up a committee to review the date of Easter.

Thus, it was in this direction of correcting the calendar that the European efforts to gather knowledge were first focussed. As already stated above, the calendar, in India, was constructed by *jyotisi-s*, who used *jyoti* manuals that also explicitly stated techniques of determining local latitude and longitude through observations using simple instruments and possibly complex calculations.

How did the Europeans gather local knowledge? Were they familiar with local languages? Like al Biruni was to Mahmud of Ghazni, the Jesuits were to the Portuguese an intelligence gathering arm. The Jesuits learned the local languages like Malayalam, Telegu and Tamil[26] easily enough, and Valignano declared that it was more important for Jesuits to learn the local language than to learn philosophy. However, the Jesuits were (a) deficient in knowledge of mathematics, and (b) constrained by an inaccurate ritual calendar. Christoph Clavius, who had studied under the famous Pedro Nunes at Coimbra, realised this handicap. He reformed the Jesuit mathematical curriculum at the Collegio Romano in the 1570's, and later went on to head the committee which reformed the Gregorian Calendar to which the Pope gave his assent in 1582[27].

Clavius also wrote a text on practical mathematics, and compiled and published a tables of sines[28] which could be looked up without the need for any mental calculation. These tables, presumably, were intended to replace the tables of Regiomontanus, taken from Arabic sources, and those of Rheticus, who perhaps also obtained his information from Arabic sources, like Copernicus.[29] Thus, Clavius recognized and strove to remove all the drawbacks listed above. The need for more accurate sine tables for navigational purposes was stressed also by Clavius' contemporary, Simon Stevins, in his criticism of the work of Pedro Nunes. Stevins explicitly states Aryabhata's value of π, observing that it is more accurate than that of Regiomontanus, who came nearly a thousand years after Aryabhata.[30] (As is well known, Stevins contemporary, Ludolph von Ceulen devoted a lifetime to getting increasingly accurate values of π.)

(2) Opportunity

The famous Matteo Ricci was in the first batch of Jesuits trained in the new mathematics curriculum introduced in the Collegio Romano by Clavius[31]. He also went to Lisbon to study cosmography and nautical science. Ricci was then sent to India in 1578. While the Portuguese had shifted their headquarters to Goa, the Jesuits maintained a large presence in Cochin (until the Protestant Dutch closed down the Cochin College around 1670). Subsequently many other scientist Jesuits trained both by Clavius or Grienberger were sent to India. Most notable of these, in terms of their scientific activity in India, were Johann Schreck[32] and Antonio Rubino[33]. The former had studied with the French mathematician Viete, well known for his work in algebra and geometry.

At some point in their stay in India these Jesuits went the Malabar region including the city of Cochin, the epicentre of developments in the infinitesimal calculus. Further, in order to not only aid conversions but also to collect local knowledge, the Jesuits learned the local languages like Malayalam, Telegu and Tamil[34].

As mentioned above, Ricci was in search of Indian calendrical knowledge. He and the other Jesuits could not but have noticed the discrepancy between their calendar and the local calendar. For example, the Jesuits were accustomed to the idea that festivals like Christmas fell on a fixed day of their calendar so they could not have failed to notice that the major Indian festivals like Dussehara, Diwali, Holi, Sankranti etc., did not fall on the same days of the Julian calendar. However, the typical Jesuit before Matteo Ricci probably did not know enough about astronomy to have known the difference even between the sidereal year (the basis of the Indian calendar) and the tropical year (the basis of the Julian/Gregorian calendar); so they could hardly have been expected to understand the complexities of the Indian calendar. However they could easily have acquired the knowledge in manuscript form and sent it back to either Maffei in Portugal or to Clavius in the Collegio Romano for analysis. Let us assess the likelihood that this did, in fact, happen.

In India, preparing the calendar (*pancanga*) was and remains to this day the task of the *jyotishi*. The typical *jyotishi* relied (and still relies), like a clerk, on a handbook of rules, without bothering to go into too many details of how the rules were derived. The standard treatises that were consulted and are today still consulted for this purpose are the *Laghu Bhaskariya*, and, in Kerala, the *KaranaPaddhati*. These are in the nature of calendrical manuals, and so are widely distributed throughout the country, since they are used every year to determine the dates of a large number of festivals. Depending upon differences of religion, caste, and region, each group of people only accept as authoritative a *pancanga* (almanac) prepared by a particular family. Thousands of families of *pancanga* makers were hence involved in this process of calendar making, across the country.[35] So, if Matteo Ricci did try to find out about the calendar from a Brahmin source in Cochin, in the heart of Kerala, as he explicitly stated he was trying to do, it is difficult to conceive that he did not run into these verses incorporated in manuscripts like the *Tantrasangraha, Yuktibhasa*, *Kriyakramakari*, and the *KaranaPaddhati*. It may help to reiterate that the *KaranaPaddhati*, etc incorporate Madhava's sine table, in a single verse, along with the cosine table in another verse.

We also emphasise that the Jesuits had much more than a casual interest in the calendar. For at just about that time, Matteo Ricci's teacher, Christoph Clavius was busy heading the commission that ultimately reformed the Gregorian calendar in 1582, an event that had been preceded by centuries of controversy. The 1545 Council of Trent had already acknowledged the error in the Julian calendar, and had authorised the Pope to correct it. So Matteo Ricci's interest in the Indian calendar was not a casual one, but was an effort preceded by years of preparation and study, and came at a time when the Jesuit interest in both India and the calendar was at a peak.

Finally, we emphasise that Matteo Ricci's mathematical preparation was most suited to the task at hand. Christoph Clavius had written a commentary on the *Sphere* of

Sacro Bosco, (after studying the *Sphere* of Pedro Nunes) and had published in 1580 a large 645 page book on *Gnomonices*. The sphere (*gola*) and the gnomon (*shanku*) were the two key topics needed to understand Indian astronomy and timekeeping: Aryabhata devotes a chapter to the sphere, while Vatesvara has a whole book on it. Ricci had studied nautical science along with the *Cosmographia* of Apian, so he could hardly have missed the significance and importance of precise sine values. The latter Jesuits such as Schreck and Rubino were just as qualified to undertake this mission to acquire knowledge[36].

Ricci went to Cochin after taking his orders. He remained in touch with the Dean of the Collegio Romano. Writing from Cochin to Maffei, he explicitly acknowledged that he was trying to find out about the calendar from Indian sources, both Hindus and Muslim. (For details and references, see Section 4 below.)

We emphasise that the Jesuits had much more than a casual interest in India. For at just about the time that Matteo Ricci was in Cochin, in 1580, the Mughal emperor Akbar invited the Jesuits to his court. This was represented in Rome as a sign of his imminent conversion, an event of the greatest importance, which could bring along with it all the political and material benefits that the Roman church obtained from Constantine; three high-level missions were sent to Akbar's court. Matteo Ricci was, at the same time, writing back sending details of the Mughal army. So Matteo Ricci's interest in the Indian calendar was not only not a casual one, preceded as it was by years of preparation and study, but came at a time when the Jesuit interest in both India and the calendar was at a peak.

(3) The Scenario of Transmission

Before we move on to the documentary evidence for transmission to Europe of the calculus that had developed in India, in the works of the mathematicians and astronomers of the Aryabhata school between the 14th and 16th centuries, it is apposite to recapitulate. There clearly was strong motivation for the transmission in the needs of navigation and the calendar reform, which were recognised as the most important scientific problems of that age in Europe. Europeans, particularly Jesuits, had ample opportunity to access the texts and calendrical almanacs in which this information was to be found, not only in the straightforward sense that they knew the local languages well, but also in the sense that trained mathematicians were sent for the express purpose of collecting the knowledge available in these texts.

Let us now turn to detail the circumstantial and documentary evidence, for it helps also to understand how this knowledge, after arriving in Europe, diffused in Europe. (Apart from historical curiosity, this is also a matter of contemporary pedagogical significance, since it enables us to assess, in a non-destructive way, the possible impact of introducing today the study of calculus with a different epistemological basis.)

We know too little even to conjecture anything about the accumulation of knowledge in Coimbra, nor exactly what happened to this accumulated knowledge after Jesuits took over control of the university after about 1560. After 1560, in line with the above reasoning, our working hypothesis, or scenario, is that over a 50 year period, say from 1560 to 1610, knowledge of Indian mathematical, astronomical and calendrical techniques accumulated in Rome, and diffused to nearby universities like Padova and Pisa, and to wider regions through Cavalieri and Galileo, and through visitors to Padova, like James Gregory[37]. Subsequently, it also reached Paris where, through the agency of Mersenne, and his study circle, it diffused throughout Europe.

Mersenne, though a minim monk, had received a Jesuit education, and was closely linked to Jesuits. Mersenne's correspondence reveals that Goa and Cochin were famous places in his time,[38] and Mersenne writes of the knowledge of Brahmins and "Indicos",[39] and mentions the orientalist Erpen and his "les livres manuscrits Arabics, Syriaques, Persiens, Turcs, Indiens en langue Malaye".[40] Mersenne's study circle included Fermat, Pascal, Roberval etc., and Mersenne's well-known correspondence with leading scientists and mathematicians of his time, could have helped this knowledge diffuse throughout Europe. (Newton, as is well known,[41] followed Wallis, and Leibniz himself states, he followed Pascal.) Of course, acquisition of knowledge of Indian mathematics could hardly have been a controlled process, so that many others, like the Dutch and French, for instance, could have simultaneously acquired this knowledge directly from India, without the intervention of Rome.

(3) Circumstantial Evidence

With this scenario as the background, we can ask: what sort of circumstantial evidence can we *hope* to find? Certainly it would be absurd to expect citations in published work! The tradition in Europe of that time was that mathematicians did not reveal their sources. When they could get hold of others' sources, they copied them without compunction. The case of e.g. Cardan is well known, and there are well documented cases against, e.g. Copernicus,[42] Galileo[43], Descartes[44], etc., of copying from others, whether or not such copying amounted to "plagiarism". Under these circumstances, mathematicians naturally kept their sources a closely guarded secret: they published only problems, not their solutions, and challenged other mathematicians to solve them.

(a) Fermat and Pell's Equation.

One such challenge problem was proposed by Fermat, and has come to be known as Pell's equation (for no fault of Pell). "In a letter of February 1657 (*Oeuvres*, II, 333-335, III, 312-313) Fermat challenged all mathematicians (thinking in the first place of John Wallis in England) to find an infinity of integer solutions of the equation $x^2 - Ay^2 = 1$, where A is any nonsquare integer."[45] Mathematicians in Europe were unable to solve Fermat's challenge problem for over 75 years, until Euler published a general solution in 1738. In February 1657, Fermat also wrote a letter to Frenicle, where he elaborated upon this problem:[46] "What is for example the smallest square which, multiplied by 61 with unity added, makes a square?"

As Struik further notes, Indian mathematicians had a solution to this problem. In fact, strangely enough, *exactly* the case of A=61 is given as a solved example in the *BeejGanita* text of Bhaskara II. This coincidence is not trivial when we consider that the solution x = 1766319049, y = 226153980 involves rather large numbers.[47] A similar problem had earlier been suggested by the 7th century Brahmagupta, and Bhaskara II provides the general solution with his *chakravala* method. Thus, Fermat's challenge problem, strongly suggests a connection of Fermat with Indian mathematics: Fermat probably had access to some Indian mathematical texts like the *BijaGanita*.

Euler certainly knew about Indian astronomy (hence mathematics), for Giovanni Cassini, then the most reputed astronomer of France, had already published an account of "Hindu astronomy" in 1691, and Euler wrote on the "Hindu year" [48] (sidereal year; the pope's bull was still not acceptable e.g. to Protestant Britain, until 1752). Since numerous Indian astronomical texts deal with "Pell's" equation, Euler had presumably learnt about this as well. However, we are not aware that he acknowledged this when he published his solution to what he called Pell's equation.

This suggestion that Fermat knew something about Indian mathematics is reinforced by Fermat's friendship not only with Mersenne, but with Jacques de Billy (1602-1669), a Jesuit teacher of mathematics in Dijon. Fermat also had the habit (then a general proclivity) of searching for knowledge in ancient books.[49]

(b) Fermat, Pascal and the Calculus

"One of Fermat's most stunning achievements," continues Aczel, "was to develop the main ideas of calculus, which he did thirteen years before the birth of Sir Isaac Newton [in 1642]." Fermat's and Pascal's approach to the calculus reinforces the belief in a connection with Indian mathematics. It was at Mersenne's place that Pascal met Descartes who remarked in his *La Geometrie* of 1637 about the impossibility of measuring the circumference of a circle: "The ratios between straight and curved lines are not known, and I even believe cannot be discovered by men, and therefore no conclusion based upon such ratios can be accepted as rigorous and exact." Therefore, there was not, at this point of time, anything that could be called an indigenous or acceptable tradition of calculus.

However, in Indian mathematics, from the time of the *sulba sutra*s, because of the different epistemological base, measuring the length of a curved line by laying a rope (*sulba*) along it, and straightening it (or measuring a general area by triangulation) has been quite an acceptable process, used to obtain a value of π. In fact, Aryabhata states that the area of a general plane figure should be obtained by triangulation,[50] before going on to give an improved measure of the (curved) circumference of a circle in terms of the diameter (a straight line), in the next verse.[51] In the very next verse he explains how his sine table is derived by approximating small arcs by line segments.[52] It is therefore quite natural to find this eventually developing into the calculus in the *Tantrasangraha*, whose author Nilakantha belonged to the Aryabhata school, and wrote wrote a lengthy commentary on the *Aryabhatiya*, almost exactly a thousand years after it.

Nilakantha's younger contemporary, Jyeshtadeva, author of the Malayalam *Yuktibhasa* ("Discourse on Rationales"), has a chapter on the circle, explaining the method of deriving the improved sine table stated in the *Tantrasangraha*. The key step in this derivation[53] is the evaluation of:

$$\lim_{n\to\infty} \frac{1}{n^{k+1}}\sum_{i=1}^{n} i^k = \frac{1}{k+1}, \quad k = 1,2,3,\ldots$$

This is also exactly the approach to calculus adopted by Fermat, Pascal, Wallis, etc. to evaluate the area under the parabolas $y = x^k$, or, equivalently, calculate $\int x^k \, dx$. As Pascal remarked, about this formula, it serves to solve all sorts of problems of the calculus. "Any person at all familiar with the doctrine of indivisibles will perceive the results that one can draw from the above for the determination of curvilinear areas. Nothing is easier, in fact, than to obtain immediately the quadrature of all types of parabolas and the measures of numberless other magnitudes." This formula has been attributed to Fermat in 1629; Roberval, another member of Mersenne's circle, also worked on it. Earlier Cavalieri, a student of Galileo, had stated this formula, without proof, in 1635, after waiting five years for Galileo to write on infinitesimals. John Wallis, who visited Pisa, verified the formula for a few values of *k*, and obtained his value of π using similar series expansions.

The curious thing is that though so many European mathematicians seem to have suddenly "discovered" this formula at about the same time, the formula had no natural epistemological basis in European mathematics, either of that time or for the next two centuries, for European mathematics was oriented towards "proof" rather than "calculation", and shared the Greek "horror of the infinite". Even today, despite the compelling changes of technology, due to widespread use of supercomputers, the situation has not entirely changed, and as in the time of Clavius, calculation continues to be regarded as "inferior" to "proof".

Though the European mathematicians were unable to prove the above formula or provide a rigorous rationale for it within their epistemology,[54] even the techniques by which they attempted to prove the formula suggests transmission. For example, Pascal tried to establish[55] the formula using the so-called Pascal's triangle, for the binomial coefficients. The triangle appears as the *meru prastara*, in Pingala's Chandahsutra of (-3 century CE), and another 1200 years later in the work of his 10th century CE commentator Halayudha.[56] It was known to the Arabs and the Chinese.[57] Among Renaissance European mathematicians it is found in the arithmetic of Apian, and in the work of authors like Bombelli[58].

(c) The Ahargana and the Julian Day-Number System

The use of Julian day numbers is another kind of evidence. These day numbers, used in scientific specification of dates, are named, somewhat ambiguously, after Julius Scaliger, the father of Joseph Scaliger. Joseph Scaliger was a well-known opponent of Christoph Clavius, and he, too, introduced his numbering system from 1582.

Now, from at least the time of Aryabhata, all dates in Indian astronomy are specified in this way, using day numbers. This eliminates any possible ambiguity due to calendrical differences; such ambiguities did exist because of the variety of calendars in use. These day numbers are specified as *Ahargana* or "heap of days". Understanding the first stanza of the *Aryabhatiya* requires us to know about this system; in fact, the day number system could have been transmitted by absolutely any Indian astronomical text.

The difference between *Ahargana* and Julian day numbers is only this: the *Ahargana* count starts from the beginning of the Kaliyuga, (17 Feb -3102 CE) whereas the

Julian day-number count starts from 1 Jan - 4713 CE (an astronomically convenient date, presumably related to the date of Biblical creation). Thus, the *Ahargana* differs from the Julian day number by exactly 588,465 which is the Julian day number for the start of the Kaliyuga. Of course, the system is simple enough and could have been invented by anyone at any time. The strange thing is that the system was allegedly invented in Europe at *exactly* the time, in 1582, when it could have been transmitted through a stated earlier desire to learn about Indian calendrical techniques. If our conjecture about transmission of the day-number system is true, it would seem that well before their conquest of Cochin, the Dutch had independent sources of information from India.

(d) Planetary Models and Elliptic Orbits

There are many other key instances that should count as circumstantial evidence. For example, Nilakantha's planetary model, in the *Tantrasangraha*, is exactly the "Tychonic" model (Tycho was a contemporary of Clavius), except that it involves elliptical orbits. (It is now known that Tycho's student, Kepler, obtained his elliptical orbits by computing his "observations".[59]) We do not go into these for reasons of space, since we first need to give an exposition of all the relevant planetary theories.

(4) **Documentary Evidence of the Role of the Jesuits**

The period after Vasco da Gama's arrival in Calicut in 1498 and the establishment, shortly thereafter, of a Portuguese colony with bases in Cochin, Cannanore and Goa, by Afonso de Albuquerque in 1510, laid the foundations for Catholic missionary work in the Malabar coast.

Among the various missions, the Jesuit one was the most important in respect of transmitting local knowledge to Europe. While there is a paucity of literature on this subject (mainly due to the, as yet, uncatalogued nature of a vast quantity of oriental manuscripts in Portugal), this is not the case with French missionary work.[60] That the French Jesuits were actively engaged in the acquisition of Indian astronomy is reported by Otto Spies.[61] Spies makes explicit reference to the Jesuit Calmette's study of local astronomy.

The earlier Jesuits in the Malabar Coast were interested in arithmetic, astronomy and timekeeping is indicated by many references in the *Documenta Indica*[62]. Their interest in local science seems motivated also indirectly by their policy to master vernacular languages such as Malaylam and Tamil[63] so much so that they were encouraged to speak to each other in the vernacular to progress this policy. Prominent Jesuits who became fluent native speakers included De Nobili who spoke Sanskrit and Tamil (the language spoken in Trichur) and the Portuguese Diogo Gonsalves who spoke Malayalam fluently. The attempt by Jesuits to learn the vernacular was so widespread that they frequently used Malayalam to sign their names in letters to the Society of Jesus headquarters in Rome.[64] The rationale for learning the vernacular languages was to aid their work in converting the local populace to Jesuit Catholicism by understanding their science, culture and customs and, of course, by facilitating communication. The former was important for the Jesuits and this included, at the very least, awareness of *jyotisa* - de Nobili, for instance, in 1615, wrote[65] a strong polemic against the Vedanga Jyotisa, a work that had been discarded as obsolete by Varahamihira, a thousand years earlier. To formalise the policy of educating Jesuit workers in the local culture, 'local' subjects such as astrology or *jyotisa* were included in the curriculum of the Jesuit colleges in the Malabar Coast.[66] The Jesuits' study of the vernacular languages was not merely intended to facilitate their work in conversions. It was also intended to enable work on transmitting this knowledge back to Europe. This is supported by the acquisition of translations of Malyalam and Sanskrit manuscripts[67]. Further evidence of this knowledge acquisition is contained in the ARSI collections *Goa* **38**, **46** and **58**. The last being the work of Diogo Gonsalves and contains detailed notes on the judicial system and on the sciences and mechanical arts of the Malabar region. In addition the Jesuit Luis Frois who worked in the Malabar region is referred to in *Goa* **46** was active in information acquisition[68]. Then there is de Menses who, writing from Kollam in 1580, reports that, on the basis of local knowledge, the European maps have inaccuracies[69].

The arrival of Matteo Ricci in Goa in September 13, 1578, as pointed out earlier, was significant in respect of Jesuit acquisition of local knowledge. His specialist knowledge of mathematics, cosmography, astronomy and navigation made him a candidate for discovering the knowledge of the colonies and he had specific instructions to investigate the science of India. The Jesuit historian Henri Bernard states that Ricci

"…had resided in the cities of Goa and of Cochin for more than three years and a half (September 13, 1578-April 15, 1582): he had been requested to apply himself to the scientific study of this new and imperfectly know country, in order to document his illustrious contemporary, Father Maffei, the 'Titus Livius' of Portuguese explorations."[70]

Bernard reports that Ricci had begun his task prior to arriving in India by setting about a study of nautical science. It is also known that Ricci had enquiries about Indian calendrical science; in a letter to Maffei he states that he requires the assistance of an "intelligent Brahmin or an honest Moor" to help him understand the local ways of recording and measuring time (lit. *jyotisa*). [71]

There were other later Jesuits who report of scientific findings on such diverse things as calendrical sciences and inaccuracies in the European maps and mathematical tables. Antonio Rubino writes, in 1610, similarly about inaccuracies in European mathematical tables[72]. Then there is the letter from Schreck, in 1618, of astronomical observations intended for the benefit of Kepler[73] - the latter had requested the eminent Jesuit mathematician Paul Guldin to help him to acquire astronomical knowledge from India to support his theories[74]. Further research is needed on the activities of these latter Jesuits as little is literature is available on this subject.

Further, the Jesuits established printing presses all over the Malabar region; in 1550 in Goa which used Roman types, in 1577 in Vaipicota using Tamil and Malyalam types, in 1602 Vaipicota using Syro-Chaldic, and in 1578 in Tuticorin with Tamil types. The aim of these presses was to publish the catechism so essential for missionary work; for example, St Francis Xavier's catechism was published in 1557 by the Goa press. The aim was also to translate the local science into Portuguese prior to transmission to

Europe; for example, Garcia da Orta's *Colloquios dos simples e drogas he cousas mediçinas da India* published in Goa in 1563[75]. There were many other publications of this type but they remain obscure because, as Sarton points out "A Portuguese book printed in Goa could not attract much attention outside the Portuguese world."[76] Nevertheless, subsequent Jesuits such as Schreck followed on from Garcia da Orta in botany[77].

All available evidence points out that the Jesuits were active in acquiring local scientific knowledge including astronomy and calendrical science. Thus they would have been aware of the astronomy of the Madhava school and would have sought it out. But how might the Jesuits have obtained key manuscripts of Indian astronomy such as the *Tantrasangraha* and the *Yuktibhasa*? We present a plausible conjecture. Such manuscripts would require the Jesuits being in close contact with scholarly Brahmins; there is concrete evidence that they were in contact with such people. There is plenty of evidence of the Jesuits being in contacts with kings across the country. We establish that the Jesuits had close relations with the kings of Cochin, and that the latter were knowledgeable about mathematics and astronomy in the tradition of Kerala.

The kings of Cochin came from the scholarly *kshatriya* Varma 'Tampuran' family who were knowledgeable about the mathematical and astronomical works of medieval Kerala[78]. Rama Varma Tampuran who, in 1948 (together with A.R. Akhileswara Iyer),[79] had published an exposition in Malayalam on the *Yuktibhasa* was one of the princes of Cochin already stated; this exposition which has been the basis of most subsequent work on the *Yuktibhasa* used the *TantraSangrahaVyakhya* manuscript of the Desa Mangalatta Mana (a Namputiri household, now disbanded). This manuscript was in the keeping of Rama Varma Tampuran (who belonged to that household)[80]. Moreover, various authors, from Charles Whish in the 19th century to Rajagopal and Rangachari have acknowledged that members of the royal household were very helpful in supplying these manuscripts in their possession.

This suggests that the former royal family in Cochin, which was in possession of a large number of MSS, had not only a scholarly tradition, but also a tradition of helping other scholars. Thus, the royal family could itself have been a possible source of knowledge for the Jesuits. Indeed the Jesuits working on the Malabar Coast had close relations with the kings of Cochin[81]. Furthermore, around 1670, they were granted special privileges by King Rama Varma[82] who, despite his misgivings about the Jesuit work in conversion, permitted members of his household to be converted to Christianity[83]. The close relationship between the King of Cochin and the foreigners from Portugal was cemented by King Rama Varma's appointment of a Portuguese as his tax collector[84].

Given this close relationship with the Kings of Cochin, the Jesuit desire to know about local knowledge, and the royal family's contiguity to the works on Indian astronomy, it is quite possible that the Jesuits may have acquired the key manuscripts via the royal household. In addition after the 1580 annexation of Portugal by Spain and subsequent loss of funding from Lisbon, the rationale for transmission acquired another dimension, that of profit[85]. Given the financial rewards for accurate navigational methods the motivation for such acquisition must have been overwhelming.

Conclusion

At the beginning of this paper we proposed to take a legal view of the evidence for the transmission to Europe of the calculus that had developed in the works of the mathematicians and astronomers of the Aryabhata school between the 14^{th} and 16^{th} centuries. We established strong motivation for the transmission in the needs of navigation and the calendar reform., which were recognised as the most important scientific problems of that age in Europe. We also established that Europeans had ample opportunity to access the texts and calendrical almanacs in which this information was to be found not only in the straightforward sense that they knew the local languages well, but also in the sense that trained mathematicians were sent for the express purpose of collecting the knowledge available in these texts. The circumstantial evidence for transmission would appear while the work on documentary evidence has only just started.

ENDNOTES

[1] This paper is based on a presentation at the 'International Conference on 1500 years of the Aryabhateeyam', Thiruvanthapuram, India, 12-16 Jan, 2000.

[2] The Aryabhata Group acknowledges financial support from the School of Education, University of Exeter, in the work that led to this paper.

[3] A.P. Jushkevich, *Geschichte der Matematik im Mittelater* German translation, Leipzig, 1964, of the original, Moscow, 1961. Victor J. Katz, *A History of Mathematics: An Introduction*, HarperCollins*CollegePublishers*, 1992. Srinivasiengar, *The History of Ancient Indian Mathematics*, World Press, Calcutta, 1967, A. K. Bag. *Mathematics in Ancient and Medieval India*, Chaukhambha Orientalia, Delhi, 1979. A popular account may be found in G. G. Joseph, *The Crest of the Peacock: The Non-European Roots of Mathematics*, Princeton University Press, 2000. But it needs to be pointed out that there is 'inertia' in some quarters; for example, L. Fiegenbaum ('Brook Taylor and the Method of Increments, *Arch Hist Ex Sci*, 34(1), 1986, 1-140, p72) makes no ackowledgement of the work of the Aryabhata School "….it is known today that Taylor was not the first to have discovered (the Taylor theorem), and that he was anticipated by at least five others: James Gregory, Newton, Leibniz, Johann Bernoulli and Abraham de Moivre. Nor is it certain that the list ends here since it reflects only our current awareness of the published and unpublished papers of Taylor's predecessors."

[4] The version of the *TantraSangraha* which has been recently serialised (K. V. Sarma ed) together with its English translation (V. S. Narasimhan Tr.) in the *Indian Journal of History of Science* (issue starting Vol. 33, No. 1 of March 1998) is incomplete and does not contain the relevant passages. We have used the verion of the *TantraSangraha* as found in the *TantraSangrahaVyakhya*, Palm Leaf MS No 697 and its transcript No. T 1251, both of the Kerala University MS Library, Trivandrum. The missing verses are after II.21a of the Trivandrum Sanskrit Series MS. The same verses are also found on pp 68—69 of of the transcript No., T-275 of the *TantraSangrahaVyakhya* at Trippunitra Sanskrit College Library, copied from a palm leaf manuscript of the Desa Mangalatta Mana. We found Pl-697/T-1251 more useful since the commentary in Malayalam is clearly separated from the original text in Sanskrit. Needless to say these verses are also found in the YuktiBhasa etc.

[5] P. K. Koru (ed) *Karana-paddhati of Putumuna Somayaji*, Astro Printing and Publishing Co., Cherp (Kerala), 1953, p 203; S. K. Nayar (ed) *Karana Paddhati of Putumuna Somayaji*, Govt. Oriental Manuscript Library, Madras, 1956, pp 189--193. For an exposition, see C. K. Raju, "Kamal or Rapalagai", Paper presented at the Xth Indo-Portuguese Conference on History, Indian National Science Academy, New Delhi, Dec 1998.

[6] K. V. Sarma (Ed and Tr) *GanitaYuktiBhasa of Jeyshtadeva (Analytical Exposition of the Rationales of Indian Mathematics and Astronomy)*, unpublished typescript , p 33-34. The precise number of decimal places actually calculated should not be used as a yardstick of *development*, since the method used was such higher precision values could be obtained with only slightly greater effort. Thus, the number of correct decimal places actually obtained in the expansiion of π should be taken as an index of requirement rather than capability.

[7] J. Warren, Kala Sankalita, Madras 1825, pp 93, 309--310. This observation of Heyne precedes the better known paper of Charles Whish, presented in 1832: "On the Hindu quadrature of the circle and the infinite series of the proportion of the circumference to the diameter exhibited in the four Shastras, the Tantrasamgraham, Yukti-Bhasa, Carana Padhati, and Sadratnamala." Tr. Royal Asiatic Society of Gr. Britain and Ireland, **3** (1835) p 509-523.

[8] C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhasa", Plenary talk: Session on Technology, Education, and Changing Conceptions of Knowledge, 8th East-West conference, Hawai'i, January 2000.

[9] For example in J.F.Scott, The Mathematical Work of John Wallis, Chelsea, New York, 1981, P 66 we see that "Wallis [in A Defense of the Treatise of the Angle of Contact] introduced the idea of 'inceptive quantities', and his definition of these – prima principia quod sic – no doubt provided Newton with a hint for the 'nascent quantities' which are at the root of the Method of Fluxions. 'There are some things', said Wallis, 'which tho' as to some kind of Magnitude, they are nothing; yet are in the next possibility of being somewhat. They are not in it, but *tantum non*; they are in the next possibility to it; and the beginning of it….'"

[10] This is not an empty question since the logic employed in other cultural traditions, like the empirical logic of quantum mechanics, need be neither 2-valued nor even truth functional; see, C. K. Raju, "Mathematics and Culture", in: *History, Time and Truth: Essays in Honour of D. P. Chattopadhyaya*, (eds) Daya Krishna and K. Satchidananda Murty, Kalki Prakash, New Delhi 1998.

[11] Suzan Rose Benedict, *A Comparative Study of the Early Treatises Introducing into Europe the Hundu Art of Reckoning*, Ph.D. Thesis, University of Michigan, April, 1914, Rumford Press.

[12] The victory of algorismus over abacus was depicted by a smiling Boethius using Indian numerals, and a glum Pythagoras to whom the abacus technique was attributed. This picture first appeared in the *Margarita Philosophica* of Gregor Reisch, 1503, and is reproduced e.g. in Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, (Tr) Paul Broneer, MIT Press, Cambridge, Mass., 1970, p 350. According to the periodisation suggested by Menso Folkerts, the abacus period commenced by the 12th century, though the use of the abacus is obviously much older. Menso Folkerts, Lecture at the Second Meeting of the International Laboratory for the History of Science, Max Planck Institute for the History of Science, Berlin, 19-26 June 1999.

[13] An early descriptive catalogue of the Indian manuscripts in the Vatican is P. Paulino A. S. Bartholomaeo, *Historico-Criticum Codicum Indicorum*, Rome, 1792.

[14] For the different epistemology underlying the notion of *sunya*, see C. K. Raju, "Mathematical Epistemology of *Sunya*", summary of interventions at the Seminar on the Concept of *Sunya*, Indian National Science Academy and Indira Gandhi Centre for the Arts, Delhi, Feb, 1997. To appear in Proceedings.

[15] See, however, C. K. Raju, "India's Interaction with China, Central and West Asia in Mathematics and Astronomy" in A. Rahman (ed) *India's Interaction with China, Central and West Asia*, PHISPC, New Delhi, 2000.

[16] For references and other details see C. K. Raju, "Kamal or Rapalagai" Paper presented at the Xth Indo-Portuguese meeting on History, Indian National Science Academy, Dec 1998, to appear in Proc. The rapalagai was the instrument used by the Indian pilot who brought Vasco da Gama to India, from Melinde.

[17] Later on the term *jyotisa* came to mean the determination of more specifically auspicious or "lucky" times for performing rituals, so that *jyotisa* is today often mistranslated as astrology, "astral science", etc.

[18] Specifically, the method of determining longitude using a clepsydra is detailed in Laghu Bhaskariya , II.8 (Ed and Tr. K. S. Shukla), Department of Mathematics and Astronomy, Lucknow University, 1963, p 53.

[19] E. S. Kennedy, *A Commentary Upon Biruni's Kitab Tahdid al Amakin, An 11th Century Treatise on Mathematical Geography*, American University of Beirut, Beirut, 1973.

[20] J. Carvalho, *Pedro Nunes – Defensaõ Do Tratado Da Rumacao Do Globo Para A Arte De Navegar,* Coimbra, 1952

[21] D. J. Struik, *A Source Book in Mathematics, 1200--1800,* Harvard University Press, Cambridge, Mass., 1969, p 253.

[22] *The Principal Works of Simon Stevin, Vol. III, Astronomy and Navigation* (eds) A Pannekoek and Ernst Crone, Amsterdam, Swets and Seitlinger, 1961.

[23] For a figure etc., see C. K. Raju, "Kamal..." cited above. Stevins, *The Haven Finding Art*, cited above, p 481 et. seq. Nunes ascribed to the loxodrome a particular property, viz. That the sines of the polar distances of the points of intersection with meridians at equal differences of longitude form a continued proportion. Stevins,(p 491) refuted this by direct calculation.

[24] For more details, see C. K. Raju, "Kamal..." cited above

[25] For details of Al Biruni's technique, and for the connection to the Indo-Arabic navigational measure of zam, see C. K. Raju, "Kamal or Rapalagai," cited earlier.

[26] See, for example, D. Ferroli, *The Jesuits in Malabar*, Bangalore, 1939, Volume 2, page 402.
[27] It is noteworthy that Nunes had, in 1577 received an invitation to join the commission on the reform of the calendar. See J. Carvalho, *Pedro Nunes – Defensaõ Do Tratado Da Rumacao Do Globo Para A Arte De Navegar*, Coimbra, 1952, p x.

[28] Christophori Clavii Bambergensis, *Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000 ...*, Ioannis Albini, 1607. As the title suggests, this table concerns not sine values proper, as today understaood, but RSine values which are what are given in Indian manuscripts. Stevins follows the same practice for his secant tables, The Haven Finding Art, cited above, p 483.

[29] George Saliba, *A History of Arabic Astronomy in the Golden Age of Islam*, New York University Press, 1994.

[30] *The Principal Works of Simon Stevins*, Vol III, cited above, p 603.

[31] V.Cronin, in *The Wise Man From The West – Matteo Ricci and His Mission To China*, Fount, Collins, 1984, p 22, states that ". ….. in 1575 Ricci entered a new phase of his studies; philosophy and mathematics, Aristotle and Euclid. The advanced course was taught by a young German, Christopher Clavius, the most brilliant mathematician of his day. …He showed special aptitude for this course winning notice as a mathematician of promise."

[32] Isaia Iannaccone, *Johann Schreck Terrentius*, Instituto Universitario Orientale, Napoli, 1998

[33] See, for example, Ugo Baldini, Studi Su Filosofia E Scienza Dei Gesuiti In Italia 1540 - 1632, Bulzoin Editore, 1992

[34] See, for example, D. Ferroli, *The Jesuits in Malabar*, Bangalore, 1939, Volume 2, page 402.

[35] Printing has somewhat simplified this process. For a listing of 60 different currently printed *pancanga*-s, see Report of the Calendar Reform Committee, Govt of India, CSIR, 1955.

[36] Ugo Baldini, *Studi su filosofia e scienza dei gesuiti in Italia 1540 - 1632*, Bulzoni Editore, 1992, p70 "Si può ricordare che molti dei migliori allievi gesuiti di Clavio e Geienberger (iniziando con Matteo Ricci e proseguendo con C. Spinola, G. Aleni, G. A. Rubino, S. De Ursis, Schreck, G. Rho) divennero missionari nelle Indie orientali. Questa scelta, se li fece protagonisti di un interscambio tra la tradizione europea e quelle indiana e cinese, particolarmente in matematica ed astronomia, che fu di per sé un fenomeno di grande significato storico, limitò certamente la loro produttività scientifica."

[37] See H W Turnbull, *James Gregory Tercentenary Memorial Volume*, London , 1939. "Gregory spent three or four years (1664-1668) in Italy....having stayed most of the time in Padua, where Galileo taught." (p 4). That Gregory acquired his knowledge through other sources is made plain by A. Prag, "On James Gregory's Geometriae Pars Universalis", pp 487-509, in H W Turnbull, *James Gregory Tercentenary Memorial Volume*, London , 1939: "James Gregory published *Geometriae Pars Universalis* at the end of his visit to Italy in 1668. This book is the firs attempt to write a systematic text-book on what we should call the calculus. Gregory does not suggest that he is the actual author of all the theorems in this work …We cannot judge exactly how much Gregory borrowed from other authors, because we do not know which books he may have read and to what extent he had knowledge of the unprinted work of his contemporaries."

[38] *Correspondance du P. Marin Mersenne*, 18 volumes, Presses Universitaires de France, Paris, 1945-. ; A letter from the astronomer Ismael Boulliaud to Mersenne in Rome, Vol XIII, p 267-73.

[39] *Correspondance*, Vol XIII, p 518-521.

[40] *Correspondance*, Vol II, p 103-115.

[41] E.g. Carl B. Boyer, *A History of Mathematics*, Wiley, 1968, p 424; C. H. Edwards, *The Historical Development of the Calculus*, Springer, 1979, p 113.

[42] George Saliba, cited earlier.

[43] William Wallace, *Galileo and his Sources – The heritage of the Collegio Romano in Galileo's Science*, Princeton University Press, 1984, has direct evidence of Galileo borrowing heavily from Jesuit sources in the Collegio Romano.

[44] Fauvel and Gray, *The History of Mathematics – A Reader*, Macmillan, 1987, p 291, describe Wallis's strong claim that Descartes plagiarized a treatise of Thomas Harriot (Britain's greatest mathematician before Newton).

[45] D. Struik, *A Source Book in Mathematics*, cited earlier, p 29.

[46] D. Struik, *A Source Book in Mathematics*, cited above, p 30.

[47] For an modern introduction to details of Bhaskara's *chakravala* method, see I. S. Bhanu Murthy, A Modern Introduction to Ancient Indian Mathematics, Wiley Eastern, new Delhi, 1992. Also see C.Selenius, 'Rationale of the Chakravala Process of Jayadeva and Bhaskara II', *Historia Mathematica*, **2**, 1975, 167-184

[48] Euler's paper on the "Hindu year" was an appendix to *Historia Regni Graecorum Bactriani* by T. S. Bayer; see G. R. Kaye, *Hindu Astronomy*, 1924, reprinted, Cosmo Publications, 1981, p 1.

[49] Amir D. Aczel, *Fermat's Last Theorem*, Penguin, p 5.

[50] *Ganita*, 9 a-b. Aryabhatiya of Aryabhata,(ed and Tr.) K. S. Shukla and K. V. Sarma, Indian National Science Academy, Delhi, 1976. It may be recollected here, that area is not defined anywhere in any well-known version of the Elements, and that Hilbert's synthetic approach to the Elements, which defines neither length nor area, hence fails beyond Proposition 1.35 of the Elements.

[51] *Ganita*, 10. *Aryabhatiya*, cited above.

[52] *Ganita*, 11. *Aryabhatiya*, cited above.

[53] C. K. Raju, "Approximation and proof in the Yuktibhasa derivation of Madhava's sine series", cited earlier.

[54] E..g. Cavalieri's statement is now termed a conjecture, while Wallis is said to have stated the formula without proof. Edwards, cited above, p 114; Boyer, cited above, p 417.

[55] E.g. Edwards, cited above, p 109-113.

[56] S. N. Sen, "Mathematics", in D. M. Bose, S. N. Sen, and B. V. Subbarayappa (eds), A Concise History of Science in India, Indian National Science Academy, Delhi, 1971, pp 156-157.

[57] Photographs of a 1303 Chinese depiction of Pascal's triangle have been provided by Joseph Needham, *The Shorter Science & Civlisation in China*, Vol 2, Cambridge University Press, 1981, p 55.

[58] Bombelli acknowledged the transmission of Indian mathematics to the West. From J. Fauvel and J. Gray, *The History of Mathematics*, Macmillan, 1987, we have (page 264) from the preface of Bombelli's *Algebra* " …a Greek work on this discipline has been discovered in the Library of our Lord in the Vatican, composed by a certain Diophantus of Alexandria, a Greek author, who lived at the time of Antoninus Pius. When it had been shown to me by Master Antonio Maria Pazzi, from Reggio, public lecturer in mathematics in Rome…..(we) set ourselves to translate it … in this work we have found that he cites Indian authors many times, and thus I have been made aware that this discipline belonged to the Indians before the Arabs."

[59] "Planet fakery exposed. Falsified data: Johannes Kepler" *The Times* (London) 25 January 1990, 31a, including large excerpts from the article by William J. Broad, "After 400 years, a challenge to Kepler: He fabricated his data, scholars say". *New York Times* 23 January 1990, C1, 6. The key article is William Donahue, "Kepler's fabricated figures: Covering up the mess in the *New Astronomy" Journal for the History of Astronomy*, **19** (1988) p 217-37.

[60] E.g. Gérard Colas, *Les manuscrits envoyé de l'Inde par les jésuites français entre 1729 et 1735*, Bibliothèque nationale de France, Paris.

[61] O. Spies, "Il P. Calmette e le sue Conoscenze Indologiche", *Studia Indologica*, Bonn, 1955, p 53-64.

[62] See, for example, *Documenta Indica*, **IV** p 293 and **VIII** p 458.

[63] *Documenta Indica*, **XIV** p 425 and **XV** p 34*, and D. Ferroli, *The Jesuits in Malabar*, Bangalore, 1939.

[64] see, e.g., the letters contained in the manuscript collection *Goa* **13** at the Archivum Romanicum Societate Iesu in Rome.

[65] V. Cronin, *A Pearl to India – The Life of Roberto de Nobili*, Darton, Longman and Todd, 1966, p 178-180.

[66] Documenta Indica, **III**, p 307

[67] For example, D. Ferroli, *The Jesuits in Malabar*, 1939, Vol 2, p402, states "In Portuguese India, hardly seven years after the death of St. Francis Xavier the fathers obtained the translation of a great

part of the 18 Puranas and sent it to Europe. A Brahmin spent eight years in translating the works of Veaso (Vyasa)……several Hindu books were got from Brahmin houses, and brought to the Library of the Jesuit college. These translations are now preserved in the Roman Archives of the Society of Jesus. (*Goa* **46**)"

[68] Luis Pina, *As Ciencias Na Historia Do Imperio Colonial Portugueses, (seculos XV a XIX)*, Lisbon, 1956, p 127, " Ao jesuita luis frois cabe lugar especial neste trabalho; a este, como a outros padres da compangia; basta lermos o valioso capitulo ALEM-MAR, CIENCAS E LETRAS."

[69] *Documenta Indica* **XI**, P185, a letter of de Menses, Coulano, dated 31/12/1580 states "Lá mando a V.P. huma descripçam de todo o mundo por muitos astrologos apurada e pilotos, e quanto ás cousas da india, sem nenhum erro nas alturas e schalas miliharias, por se fazer por astrologos e pilotos que cada dia correm estas terras, porque as cartas de lá sam todas erradas nas alturas indicas, como eu claramente vi."

[70] Henri Bernard, *Matteo Ricci's Scientific Contribution to China*, Hyperion Press, Westport, Conn., 1973, p38. It is relevant to point out that it was Maffei (I. Petri Maffei, *History of the Indies*, in book 1, Venice 1589) who details the navigational help received by da Gama to cross the Arabian sea and arrive at Calicut. Ricci, furthermore, was not alone in scientific exploration of India; for example in *Documenta Indica*, Vol 15, p 185, De Menses reports 'Lá mando a V.P. huma descripçam de todo o mundo por muitos astrologos apurada e pilotos, e quanto ás cousas da india, sem nenhum erro nas alturas e schalas miliharias, por se fazer por astrologos e pilotos que cada dia correm estas terras, porque as cartas de lá sam todas erradas nas alturas indicas, como eu claramente vi. Em os santos sacrificios de V.P. muito me encomendo.'

[71] "Com tudo não me parece que sera impossivel saberse, mas has de ser por via d`algum mouro honorado ou brahmane muito intelligente que saiba as cronicas dos tiempos, dos quais eu procurarei saber tudo." Letter by Matteo Ricci to Petri Maffei on 1 Dec 1581. *Goa* **38** I, ff 129r--30v, corrected and reproduced in *Documenta Indica*, **XII**, 472-477 (p 474). Also reproduced in Tacchi Venturi, *Matteo Ricci S.I., Le Lettre Dalla Cina 1580-1610*, vol 2, Macerata, 1613.

[72] Ugo Baldini, *Studi su filosofia e scienza dei gesuiti in Italia 1540 - 1632*, Bulzoin Editore, 1992, p214reports of Rubino's letter "…….scritta da una località indiana il cui nome è di lettura incerta, egli informa di fare rilevazíoni per cartografare il territorio, per determinare alcune posizioni si è servito di eclissi, confrontando i tempi locali reali con quelli desumibili dalle efemeridi di Magini e riscontrandovi grandi inesattezze; chiede perciò altre efemeridi, oppure le *Tabulae prutenicae*."

[73] See, Isaia Iannaccone, *Johann Schreck Terrentius*, Instituto Universitario Orientale, Napoli, 1998, p58 "Durante la sosta a Goa, Schreck fu altresi impegnato in campo astronomico: assieme a Kirwitzer, Schall von Bell, Rho e Rubino annotò le osservazioni di alcune comete visibili verso la fine del 1618; i risultati di queste osservazioni furono poi inviate in Europa ai confratelli Ziegler e Decker del Collegio di Ingolstadt, affinché le trasmettessero a Kepler

[74] C. Baumgardt, *Johannes Kepler: Life and Letters*, Philosophical Library, NY, 1951, p 153, (in Kepler's letter to Paul Guldin Jesuit professor of mathematics in Vienna, from Linz, Feb7,1626) Kepler writes "If I could only obtain soon the observations on the eclipse from India or from any other place (where it occurs)…"

[75] T. J. S. Patterson, 'Science and Medicine in India', in Eds Pietro Corsi and Paul Weindling, *Information Sources In The History Of Science And Medicine*, Butterworths Scientific, 1983, p 437-456.

[76] D. Sarton, 1955, *The Appreciation of Ancient and Medieval Science During the Renaissance (1450-1600)*, University of Pennsylvania, 1955, p102

[77] See Isaia Iannaccone, *Johann Schreck Terrentius*, Instituto Universitario Orientale, Napoli, 1998

[78] C. M. Whish, *On the Hindu Quadrature of the Circle….*, Transactions of the Royal Asiatic Society of Great Britain and Ireland, 3(3), 1835, pp 509-523. Whish states here (p 521) that the author of the astronomical work the *Sadratnamalah* is Sankara Varma, the younger brother of Raja of Cadattanada near Tellicherry and further states that the Raja is a very acute mathematician. Also C. N. Srinivasiengar, *The History of Ancient Indian Mathematics*, World Press, Calcutta, 1967. Srinivasiengar also states that Sankara Varma is the author of *Sadratnamalah* and is brother of the Uday Varma, the King of Kerala (p146) . He further refers (p 145) to the Malyalam *History of Sanskrit Literature in Kerala* which identifies the King of Cochin, Raja Varma, as being aware of the chronology of the *Karana-Paddhati*.

[79] *Yuktibhasa* Part 1, with notes (Ed) Ramavarma (Maru) Tampuran and A. R. Akhileswara Aiyar, Mangalodayam Ltd, Trichur, 1948 (in Malayalam). Most subsequent work has relied on this exposition. C. T. Ragagopal and M. S. Rangachari, ‘On an Untapped Source of Medieval Keralese Mathematics’, *Archive for the History of Exact Sciences*, **18**, 1978, 89-102. Rajagopal and Rangachari state that Rama Varma Tampuran supplied them with the manuscript material [Desa Mangalatta Mana MS of *TantraSangrahaVyakhya*] relating to Kerala mathematics. Also K. V. Sarma, *A History of the Kerala School of Hindu Astronomy*, Hoshiarpur Vishveshvaranand Institute, 1972. On page 12, K. V. Sarma points to the valuable notes added to the analysis of Kerala astronomy by Rama Varma Maru Tampuran in his 1948 exposition of the first part of the *Yuktibhasa.* His subsequent translation of the *Yuktibhasa* has utilised the same material (personal communication).

[80] In a personal communication with, Mukunda Marar, the eldest son of the last king of Cochin the Aryabhata Group has established that the former kings were, at the least, all aware of the astronomical methods of the astrological prediction and of the MS that contained these methods. Several were scholarly enough to publish commentaries of the mathematical/astronomical works. Mukunda Marar, himself, worked with C T Rajagopal and published a work *On the Hindu Quadrature of the Circle*, Journal of the Royal Asiatic Society (Bombay branch), 20, 1944, 65-82.

[81] see for example, *Documenta Indica*, **X**, pages 239, 834, 835, 838, and 845

[82] *Documenta Indica*, **XV**, p 224

[83] *Documenta Indica*, vol XV, p 7*

[84] *Documenta Indica*, vol XV, p 667

[85] “Most of the Jesuit missionaries set to work to master the vernaculars…. some of their number studies Indian books and Indian philosophy, not merely with the idea of refuting it, but with the desire of profiting by it.” D. Ferroli, *The Jesuits in Malabar,* 1939, volume 2, p 93.