Transmission of the Calculus from Kerala to Europe
Part 1: Motivation and Opportunity

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It is by now widely recognised[2] that the calculus had already developed in India in the works of the mathematicians and astronomers of the Aryabhata school: Madhava, Nilkantha (*Tantrasangraha*), Jyeshtadeva (*Yuktibhasa*) etc, between the 14th and 16th centuries CE. These developments included infinite "Gregory/Taylor" series for sine, cosine and arctan functions,[3] with accurate remainder terms, and a numerically efficient algorithm, leading to a 9 decimal-place precision table for sines and cosines stated in sexagesimal *katapayadi* notation in two verses found also in the widely distributed *KaranaPaddhati* of Putumuna Somayaji.[4] The development also included the calculation of complex derivatives like that of arcsin (*p* sin *x*) (*Tantrasangraha*

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[1] The Aryabhata Group acknowledges financial support from the School of Education, University of Exeter, in the work that led to this paper.

[2] A.P. Jushkevich, *Geschichte der Matematik im Mittelater* German translation, Leipzig, 1964, of the original, Moscow, 1961. Victor J. Katz, *A History of Mathematics: An Introduction*, HarperCollins*CollegePublishers*, 1992. Srinivasiengar, *The History of Ancient Indian Mathematics*, World Press, Calcutta, 1967, A. K. Bag. *Mathematics in Ancient and Medieval India*, Chaukhambha Orientalia, Delhi, 1979. A popular account may be found in G. G. Joseph, *The Crest of the Peacock: non-European Roots of Mathematics*, Penguin, 1992.

[3] The version of the *TantraSangraha* which has been recently serialised (K. V. Sarma ed) together with its English translation (V. S. Narasimhan Tr.) in the *Indian Journal of History of Science* (issue starting Vol. 33, No. 1 of March 1998) is incomplete and does not contain the relevant passages. We have used the verion of the *TantraSangraha* as found in the *TantraSangrahaVyakhya*, Palm Leaf MS No 697 and its transcript No. T 1251, both of the Kerala University MS Library, Trivandrum. The missing verses are after II.21a of the Trivandrum Sanskrit Series MS. The same verses are also found on pp 68—69 of of the transcript No., T-275 of the *TantraSangrahaVyakhya* at Trippunitra Sanskrit College Library, copied from a palm leaf manuscript of the Desa Mangalatta Mana. We found Pl-697/T-1251 more useful since the commentary in Malayalam is clearly separated from the original text in Sanskrit. Needless to say these verses are also found in the YuktiBhasa etc. For detailed quotations, translations, and a mathematical exposition, collected in one place, see C. K. Raju, "Approximation and Proof in the Yuktibhasa derivation of Madhava's Sine series". Paper presented at the National Seminar on Applied Sciences in Sanskrit Literature, Various Aspects of Utility, Agra 20--22 Feb 1999. To appear in Proc.

[4] P. K. Koru (ed) *Karana-paddhati of Putumuna Somayaji*, Astro Printing and Publishing Co., Cherp (Kerala), 1953, p 203; S. K. Nayar (ed) *Karana Paddhati of Putumuna Somayaji*, Govt. Oriental Manuscript Library, Madras, 1956, pp 189--193. For an exposition, see C. K. Raju, "Kamal or Rapalagai", Paper presented at the Xth Indo-Portuguese Conference on History, Indian National Science Academy, New Delhi, Dec 1998. To appear in Proc.

V.53-54), and $p \sin x/(1+p \cos x)$ (*Sphutanirnaya* III.19-20), to calculate the instantaneous velocities of the sun and the moon, and infinite series expansions, and high-precision computations of the value of π correct to 9, 10 (and later 17) decimal places.[5] (As already noted by Benjamin Heyne[6] in 1805 these developments were probably not confined to Kerala but were available also in Tamil Nadu, Telangana, and Karnataka, though this possibility has not yet been properly investigated.)

A key point, that has not been noticed earlier, is this: these developments *cannot* be dismissed as "pre-calculus", the way the works of Fermat, Pascal etc. usually are. Thus, the use of traditional floating point numbers, enabled the Indian mathematicians to provide a rigorous rationale for the infinite series and the infinitesimal calculus. This was quite unlike the case of Newton etc. who, lacking also the notion of real number, used "fluxions" or "infinitesimals", the exact meaning of which remained a mystery until the development of mathematical analysis and the clarification of the notion of "proof" in the late 19th century and early 20th century CE. Since the Indian mathematicians had a rigorous rationale which Newton could not possibly have had, the *Yuktibhasa* exposition should, *a fortiori*, count as calculus.

It is true that the *Yuktibhasa* ideas of mathematics and proof differ from the Platonic and Hilbertian idea that mathematics must be divorced from the empirical; however, it is hard to see, from either a theoretical or a practical point of view, why acceptance of the Platonic point of view and Platonic authority *ought* to be a key ingredient of mathematics. In particular, the Platonic insistence on divorce from the empirical leaves hanging in the air the question of what *logic* ought to underlie a proof,[7] whereas acceptance of the empirical would mean a change in the notion of proof, since different criteria are used to validate a physical theory. Since this paper is

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[5] K. V. Sarma (Ed and Tr) *GanitaYuktiBhasa of Jeyshtadeva (Analytical Exposition of the Rationales of Indian Mathematics and Astronomy)*, unpublished typescript , p 33-34.

[6] J. Warren, *Kala Sankalita*, Madras 1825, pp 93, 309--310. This observation of Heyne precedes the better known paper of Charles Whish, presented in 1832: "On the Hindu quadrature of the circle and the infinite series of the proportion of the circumference to the diameter exhibited in the four Shastras, the Tantrasamgraham, Yukti-Bhasa, Carana Padhati, and Sadratnamala." *Tr. Royal Asiatic Society of Gr. Britain and Ireland*, **3** (1835) p 509-523.

[7] This is not an empty question since the logic employed in other cultural traditions, like the empirical logic of quantum mechanics, need be neither 2-valued nor even truth functional; see, C. K. Raju, "Mathematics and Culture", in: *History, Time and Truth: Essays in Honour of D. P. Chattopadhyaya*, (eds) Daya Krishna and K. Satchidananda Murty, Kalki Prakash, New Delhi 1998. Reproduced *in Philosophy of Mathematics Education*, **11**, available at http://www.ex.ac.uk/~PErnest.

concerned with transmission, rather than epistemology, we do not pursue this any further.

Prior to Vasco da Gama there is ample evidence of the import of Indian mathematical knowledge into Europe.[8] The history of Indian arithmetical techniques imported into Europe via the Arabs as "algorismus" texts is now well known. (Algorismus is the Latinized version of Al Khwarizmi (9th century CE), who translated the arithmetical and astronomical texts of Brahmagupta (7th century CE.) These "algorismus" techniques were first introduced into Europe by Gerbert (Pope Sylvester III) in the 10th century CE, but it is only in the 16th century CE that their final triumph over abacus techniques started being depicted on the covers of arithmetical texts.[9] On the other hand, there is also, from *after* the late 17th century, ample evidence of the large scale import of Indian texts and manuscripts, tens of thousands of which are today housed in European libraries[10]. Our primary hypothesis for investigation is that this process of importing Indian texts continued also during the unstudied intervening period of the 16th and 17th century. *Our hypothesis is that the arrival of Vasco da Gama in Calicut not only short-circuited the traditional Arab route for spices, it also short-circuited the traditional Arab route for knowledge of Indian mathematics and astronomy.*

Further, it is our hypothesis that the epistemological difficulties encountered with infinitesimals in Europe from the 17th to the 19th centuries CE arose, exactly like the difficulties with *sunya*/zero, due to the import of techniques with a different epistemological base.[11] This has important pedagogical implications. However, in this

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[8] Suzan Rose Benedict, *A Comparative Study of the Early Treatises Introducing into Europe the Hundu Art of Reckoning*, Ph.D. Thesis, University of Michigan, April, 1914, Rumford Press.

[9] The victory of algorismus over abacus was depicted by a smiling Boethius using Indian numerals, and a glum Pythagoras to whom the abacus technique was attributed. This picture first appeared in the *Margarita Philosophica* of Gregor Reisch, 1503, and is reproduced e.g. in Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, (Tr) Paul Broneer, MIT Press, Cambridge, Mass., 1970, p 350. According to the periodisation suggested by Menso Folkerts, the abacus period commenced in the 12th century. Menso Folkerts, Lecture at the Second Meeting of the International Laboratory for the History of Science, Max Planck Institute for the History of Science, Berlin, 19-26 June 1999.

[10] An early descriptive catalogue of the Indian manuscripts in the Vatican is P. Paulino A. S. Bartholomaeo, *Historico-Criticum Codicum Indicorum*, Rome, 1792.

[11] For the different epistemology underlying the notion of *sunya*, see C. K. Raju, "Mathematical Epistemology of *Sunya*", summary of interventions at the Seminar on the Concept of *Sunya*, Indian National Science Academy and Indira Gandhi Centre for the Arts, Delhi, Feb, 1997. To appear in Proceedings.

paper we will set aside the epistemological and pedagogical issues and focus on the question of transmission.

In the past there have been far too many claims of transmission, where the evidence produced is farcical; for example, in support of the widespread claim of the transmission of Ptolemaic astronomy from Alexandria to India, one line of evidence, proposed by Thibaut, is that Varahamihira's use of "Paulisha" suggests that it could have been derived from "Paul" (rather than Pulisha or Pulastya, one of the seven sages forming the constellation known as the Great Bear). If this be the standard of evidence, there is nothing for us to prove. For the works of Paramesvara, Madhava, Nilkantha, and Jyeshtadeva, clearly precede those of Fermat, Pascal, Gregory, Wallis, Newton, and Leibniz, and India was clearly known (and actively linked) to Europe by the 16th century CE.

However, we are aware that, for some unfathomable reasons, the standard of evidence required for an acceptable claim of transmission of knowledge from East to West is different from the standard of evidence required for a similar claim of transmission of knowledge from West to East. Priority and the possibility of contact *always* establish a socially acceptable case for transmission from West to East, but priority and definite contact *never* establish an acceptable case for transmission from East to West, for there always is the possibility that similar things could have been discovered independently. Hence we propose to adopt a legal standard of evidence good enough to hang a person for murder. Briefly, we propose to test the hypothesis on the grounds of (1) motivation, (2) opportunity, (3) circumstantial evidence, and (4) documentary evidence.

(1) Motivation

The motivation for import of knowledge derived from the needs of greater accuracy in (a) navigation, (b) the calendar, and (c) practical mathematics (as in algorismus texts). Navigation was clearly the key motivation, being then a matter of the greatest strategic and economic importance for Europe. Early navigators like Columbus, and Vasco da Gama did *not* know stellar navigation,[12] and dead reckoning was of little use in "uncharted" seas unless, like Columbus, one was aimed at so massive a shore line, that one could hardly hope to miss it! Official acknowledgment of both the ignorance of navigational techniques, and the great need to learn more, came from various European governments which instituted huge prizes for anyone who could provide an accurate technique of navigation. These included the Spanish government (1567, 1598) the Dutch government (1632), the French government (1670), and the British government (1711), which last prize was finally claimed in 1762 by Harrison with his chronometer (which came into general use only by the 19th century). To get an idea of the true value of these prizes we observe that e.g. the British government's prize was more than 300 times Newton's *annual* fellowship.

Consequently, not only kings and parliaments, but also very many of the leading scientists of the times were involved in these efforts. Galileo, for example, unsuccessfully competed for the revised Spanish prize for 16 years, before shifting his attention to the Dutch prize. Colbert wrote personally to all the leading scientists of Europe, offering large rewards, and selected from the replies received to start the French Royal Academy "to improve maps, sailing charts, and advance the science of navigation". This was also a key motivating problem for starting the British Royal Society. Prior to the Royal Society, groups of scientists began meeting in London and Oxford from 1645, the longitude problem being their main object of concern. The importance of the longitude problem, and the related difficulty about the size of the earth, both, are captured in a 1661 poem describing the work going on at Gresham College:

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[12] For references and other details see C. K. Raju, "Kamal or Rapalagai" Paper presented at the Xth Indo-Portuguese meeting on History, Indian National Science Academy, Dec 1998, to appear in Proc. The rapalagai was the instrument used by the Indian pilot who brought Vasco da Gama to India, from Melinde.

The Colledge will the whole world measure,
Which most impossible conclude,
And Navigators make a pleasure
By finding out the longitude.
Every Tarpalling shall then with ease
Sayle any ships to th'Antipodes.

[Tarpalling here means a sailor] The group from Gresham College include John Wallis and Robert Hooke, which later joined other groups to became the Royal Society of London for the Promotion of Natural Knowledge. Christopher Wren, also a member of the Gresham College group, wrote the preamble to the Royal Society's charter. One of the stated aims of the newly founded Royal Society was: Finding the longitude. Newton testified before the British parliament in connection with this problem of navigation.

Stellar navigation naturally involved the study of astronomy and timekeeping, which was, then, inseparably linked to mathematics. Most Indian texts on mathematics were located in the context of astronomy and timekeeping (*jyotisa*). Moreover, the navigational knowledge that the Europeans sought *was*, in fact, available in Indian mathematical and astronomical texts. The widely distributed *Laghu Bhaskariya* (abridged work of Bhaskara) and *Maha Bhaskariya* (extensive work of Bhaskara) of the first Bhaskara (629 CE) explicitly detailed methods of determining the local latitude *and* longitude, using observations of solar declination, or pole star, and simple instruments like the gnomon, and the clepsydra.[13] Since local latitude could easily be determined from solar declination by day and e.g. pole star altitude at night (using an instrument like the *kamal*), an accurate sine table was just what was required to determine local longitude from a knowledge of latitudinal differences and course angle. The *Laghu Bhaskariya* already states the criticism that determination of longitude by a calculation involving plane triangles is not adequate because of the roundness of the earth. Later texts like the *Siddhanta* of Vatesvara (904 CE) pointed

[13] For detailed quotations, and an exposition, see C. K. Raju, “Kamal or Rapalagai”, cited earlier. Specifically, the method of determining longitude using a clepsydra is detailed in Laghu Bhaskariya ,

out[14] that these techniques needed to be corrected by applying spherical trigonometry. Al Biruni, the 10th-11th century scholar, who visited India on behalf of Mahmud of Ghazni, and systematically studied and translated Indian mathematical and astronomical texts, explicitly used these techniques of spherical trigonometry to determine local latitudes and longitudes in his treatise on mathematical geography.[15]

Exactly this technique started being tried in Europe in the 16th century, when the centre of navigational excellence in Europe had shifted from Florence to Coimbra. In the first half of the 16th century, Pedro Nunes studied motion on the sphere along a given rhumb line, or a given course. Evaluating such a path or a loxodromic (Gr. *loxos* = oblique, *dromos* = course) curve is exactly equivalent to the fundamental theorem of calculus: [16] given the tangent at every point, to determine the curve passing through these points which has those tangents. It is very interesting that these loxodromic curves were, in fact, studied by Pedro Nunes and Simon Stevin[17] *using sine tables, and the above stated technique*,[18] with a solution in spherical triangles, though the triangles involved were not strictly spherical, as Stevin observed. (The exact technique by which Mercator obtained the loxodromic curves for his famous chart is not known, but was probably similar.) The conjectures of Pedro Nunes were tested in a voyage to Goa, in the 1540's during which they reportedly failed, presumably due to inaccurate techniques of calculation, inadequate sine table, and other factors listed below.

Though the Europeans, motivated by navigation, were actively seeking the knowledge of determining local latitude and longitude through stellar astronomy, and though this knowledge was available in Indian mathematical texts, there were three things that impeded their search. They lacked: (a) knowledge of practical and mental mathematics, (b) an accurate calendar, and (c) an accurate estimate of the size of the

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II.8 (Ed and Tr. K. S. Shukla), Department of Mathematics and Astronomy, Lucknow University, 1963, p 53.

[14] For detailed quotations, see C. K. Raju, “Kamal...” cited above.

[15] E. S. Kennedy, *A Commentary Upon Biruni's Kitab Tahdid al Amakin, An 11th Century Treatise on Mathematical Geography*, American University of Beirut, Beirut, 1973.

[16] D. J. Struik, *A Source Book in Mathematics, 1200--1800*, Harvard University Press, Cambridge, Mass., 1969, p 253.

[17] *The Principal Works of Simon Stevin, Vol. III, Astronomy and Navigation* (eds) A Pannekoek and Ernst Crone, Amsterdam, Swets and Seitlinger, 1961.

[18] For a figure etc., see C. K. Raju, “Kamal...” cited above. Stevins, *The Haven Finding Art*, cited above, p 481 et. seq. Nunes ascribed to the loxodrome a particular property, viz. That the sines of the

earth. The last was needed for the calculation of longitudes/departures, from a knowledge of only latitudes and course angle.[19] While Caliph al Mamun had confirmed through empirical observations in the 9th centruy CE, the estimates of the equatorial radius of the earth given in Indian astronomical texts, and al Biruni had implemented a cheaper and easier technique to confirm these,[20] Columbus undid this. To sell his idea of sailing West to reach the East, he underestimated the size of the earth by 40%. This error persisted, with e.g. Newton's initial estimates being off by 25%, until Picard's accurate re-determination of the size of the earth, in 1671, funded by the French Royal Academy as its first scientific effort. However, in the first part of the 16th century lacking even an accurate calendar, and lacking techniques of calculation, the difficulty was with determining *latitude* correctly!

Like al Biruni was to Mahmud of Ghazni, the Jesuits were to the Portuguese an intelligence gathering arm. While the Jesuits learned the local languages like Malayalam and Tamil easily enough, they were (a) deficient in knowledge of mathematics, and (b) constrained by an inaccurate ritual calendar. Christoph Clavius, who had studied under the famous Pedro Nunes at Coimbra, realised this handicap. He reformed the Jesuit mathematical curriculum at the Collegio Romano in the 1570's, and later went on to head the committee which reformed the Gregorian Calendar to which the Pope gave his assent in 1582.

Clavius also wrote a text on practical mathematics, and compiled and published a tables of sines[21] which could be looked up without the need for any mental calculation. These tables, presumably, were intended to replace the tables of Regiomontanus, taken from Arabic sources, and those of Rheticus, who perhaps also obtained his information from Arabic sources, like Copernicus.[22] Thus, Clavius recognized and strove to remove all the drawbacks listed above. The need for more

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polar distances of the points of intersection with meridians at equal differences of longitude form a continued proportion. Stevins,(p 491) refuted this by direct calculation.

[19] For more details, see C. K. Raju, "Kamal..." cited above.

[20] For details of Al Biruni's technique, and for the connection to the Indo-Arabic navigational measure of zam, see C. K. Raju, "Kamal or Rapalagai," cited earlier.

[21] Christophori Clavii Bambergensis, *Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000 ...*, Ioannis Albini, 1607. As the title suggests, this table concerns not sine values proper, as today understaood, but RSine values which are what are given in Indian manuscripts. Stevins follows the same practice for his secant tables, The Haven Finding Art, cited above, p 483.

accurate sine tables for navigational purposes was stressed also by Clavius' contemporary, Simon Stevins, in his criticism of the work of Pedro Nunes. Stevins explicitly states Aryabhata's value of π, observing that it is more accurate than that of Regiomontanus, who came nearly a thousand years after Aryabhata.[23] (As is well known, Stevins contemporary, Ludolph von Ceulen devoted a lifetime to getting increasingly accurate values of π.)

(2) Opportunity

The famous Matteo Ricci was in the first batch of Jesuits trained in the new mathematics curriculum introduced in the Collegio Romano by Clavius. He also went to Lisbon to study cosmography and nautical science. Ricci was then sent to India. While the Portuguese had shifted their headquarters to Goa, the Jesuits maintained a large presence in Cochin (until the Protestant Dutch closed down the Cochin College around 1670). So it was to Cochin that Ricci went after taking his orders. He remained in touch with the Dean of the Collegio Romano. Writing from Cochin to Maffei, he explicitly acknowledged that he was trying to find out about the calendar from Indian sources, both Brahmins and Moors. (For details, see Section 4 below.)

We emphasise that the calendar is necessary for synchronising social activities, so the Jesuits could not but have already noticed the discrepancy between their calendar and the local calendar. For example, the Jesuits were accustomed to the idea that festivals like Christmas, or Easter came on a fixed day of their calendar. They could not have failed to notice that the major Indian festivals like Dussehara, Diwali, Holi, Sankranti etc., did not fall on the same days of the Julian calendar. However, the typical Jesuit before Matteo Ricci probably did not know enough about astronomy to have known the difference even between the sidereal year (the basis of the Indian calendar) and the tropical year (the basis of the Julian/Gregorian calendar); so they could hardly have been expected to understand the complexities of the Indian calendar.

Now, in India, preparing the calendar (*pancanga*) was and remains to this day the task of the *jyotishi*. The typical *jyotishi* relied (and still relies), like a clerk, on a handbook

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[22] George Saliba, *A History of Arabic Astronomy in the Golden Age of Islam,* New York University Press, 1994.

of rules, without bothering to go into too many details of how the rules were derived. The standard treatises that were consulted and are today still consulted for this purpose are the *Laghu Bhaskariya*, and, in Kerala, the *KaranaPaddhati*. These are in the nature of calendrical manuals, and so are widely distributed throughout the country, since they are used every year to determine the dates of a large number of festivals. Depending upon differences of religion, caste, and region, each group of people only accept as authoritative a *pancanga* (almanac) prepared by a particular family. Thousands of families of *pancanga* makers were hence involved in this process of calendar making, across the country.[24] So, if Matteo Ricci did try to find out about the calendar from a Brahmin source in Cochin, in the heart of Kerala, as he explicitly stated he was trying to do, it is difficult to conceive that he did not run into the *KaranaPaddhati*. It may help to reiterate that the *Kriyakramakari*, the *KaranaPaddhati* etc. incorporate Madhava's sine table, in a single verse, along with the cosine table in another verse.

We emphasise that the Jesuits had much more than a casual interest in India. For at just about the time that Matteo Ricci was in Cochin, in 1580, the Mughal emperor Akbar invited the Jesuits to his court. This was represented in Rome as a sign of his imminent conversion, an event of the greatest importance, which could bring along with it all the political and material benefits that the Roman church obtained from Constantine; three high-level missions were sent to Akbar's court. Matteo Ricci was, at the same time, writing back sending details of the Mughal army.

We also emphasise that the Jesuits had much more than a casual interest in the calendar. For at just about that time, Matteo Ricci's teacher, Christoph Clavius was busy heading the commission that ultimately reformed the Gregorian calendar in 1582, an event that had been preceded by centuries of controversy. The 1545 Council of Trent had already acknowledged the error in the Julian calendar, and had authorised the Pope to correct it. So Matteo Ricci's interest in the Indian calendar was not a casual one, but was an effort preceded by years of preparation and study, and came at a time when the Jesuit interest in both India and the calendar was at a peak.

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[23] *The Principal Works of Simon Stevins*, Vol III, cited above, p 603.
[24] Printing has somewhat simplified this process. For a listing of 60 different currently printed *pancanga*-s, see Report of the Calendar Reform Committee, Govt of India, CSIR, 1955.

Finally, we emphasise that Matteo Ricci's mathematical preparation was most suited to the task at hand. Christoph Clavius had written a commentary on the *Sphere* of Sacro Bosco, (after studying the *Sphere* of Pedro Nunes) and had published in 1580 a large 645 page book on *Gnomonices*. The sphere (*gola*) and the gnomon (*shanku*) were the two key topics needed to understand Indian astronomy and timekeeping: Aryabhata devotes a chapter to the sphere, while Vatesvara has a whole book on it. Ricci had studied nautical science along with the *Cosmographia* of Apian, so he could hardly have missed the significance and importance of precise sine values.