From Johnson and Walker's article on "Sir Michael Atiyah's Einstein Lecture..."

additional 6 (or 7) dimensions are hidden from our normal experience of reality.

2. Whereas more classically, the basic objects such as electrons, protons, and quarks, could be thought of as point particles, string theory interprets such objects as being very small "strings". This allows for a "smoothing out" of the singularities that arise in the classical picture when such particles come close together. Even the singularities associated with mixing quantum mechanical and gravitational forces are resolved in this manner.
3. Very sophisticated geometry is used, involving a vast amount of mathematics, both old and new.
4. No unique model or picture has emerged out of string theory, but rather several versions exist. These different theories are now known to be different facets of the same theory. What has happened to the "real world"?
5. Quantum mechanics remains the basic framework.

String theory has had a remarkable and mysterious impact on pure mathematics, leading to many new concepts and results. In some cases, such results have been given proofs in the traditional mathematical sense. In other situations, the "results" merely fit well with known mathematical results or accepted features of string theory. In particular, string theory has had an impact in

1. algebraic geometry, by addressing enumerative questions concerned with counting algebraic curves satisfying certain conditions;
2. knot theory, by construction of new topological invariants of knots that can sometimes distinguish a knot from its mirror image;
3. four-dimensional geometry, by giving new, unexpected, and very deep results that are unique to four dimensions; and
4. various branches of algebra.

A New Paradigm?

If a "theory of everything" emerges from string theory, we will discover a universe built on *fantastically intricate mathematics*. In particular, the Calabi-Yau manifolds that make up the hidden dimensions are extremely complicated. Atiyah suggested that it is not satisfying that the true theory would be so complicated—even writing down the terms of the theory requires a vast amount of background.

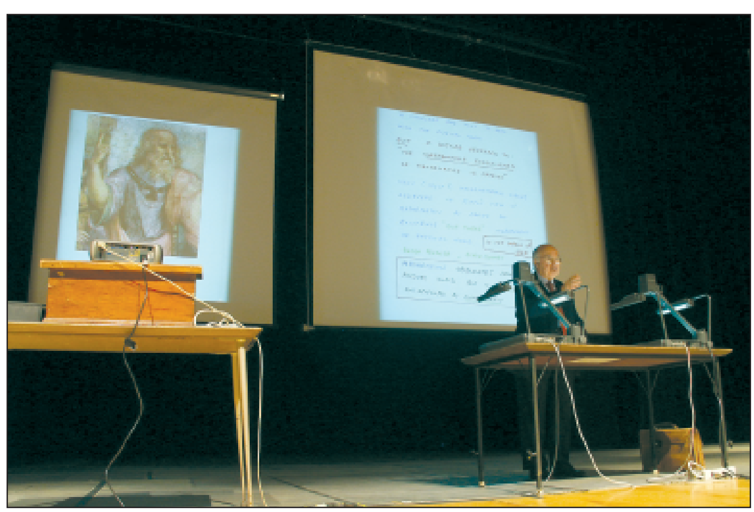
Perhaps, according to Atiyah, a new paradigm is needed; perhaps the complicated mathematics appearing in string theory is merely "in the eye of the beholder". That is, maybe we do not understand the fundamental nature of reality well enough, and this misunderstanding is leading to such exceptionally complicated mathematics. String theory, from this point of view, is only our method of approximating a simple reality. Perhaps, Atiyah suggested, we should follow Einstein and question quantum mechanics.

In order to make progress, we might need to dispense with some piece of accepted dogma. Relativity, quantum mechanics, and string theory have already dispensed with many previously held tenets, and so one might ponder whether there remains any such dogma left to throw away. Atiyah noted that all physical models since Newton, including even quantum mechanics, have assumed one basic premise—that we can *predict the future from full knowledge of the present*. Atiyah suggested an alternative to this paradigm: Perhaps we need full knowledge of the present *and the past* in order to predict the future. That is, maybe the universe has memory. As a simple example, the notion of the velocity of an object is viewed as being a property of the present, but, in reality, to measure velocity one needs to know not only where the object is now but where it was a moment earlier.

Atiyah's hypothesis possibly leads to several interesting consequences:

1. The mathematics used in physical theory would become more difficult, since all previously used mathematics in physics assumes that knowledge of the present suffices. With the new paradigm, for example, retarded (or delay) differential equations would become necessary.
2. Since we do not have complete knowledge of the past, uncertainty would arise. This might shed light on the uncertainty inherent in quantum mechanics.
3. Perhaps the complicated mathematics of string theory arises from our attempt to understand the full implications of the theory of general relativity without incorporating the knowledge of the past.

Atiyah does not promote discarding older, time-tested physical theories. Rather, such a new



paradigm ought to build on the old theories, much as relativity builds on Newtonian mechanics.

Speculations and Questions

There are various attitudes among physicists toward string theory. Some dismiss it as fancy mathematics that is unrelated to the real world, since string theory makes no testable predictions. Others believe the mathematical applications of string theory give confidence in the physical insights and indicate that the theory is on the right track. From this point of view, mathematical applications become a kind of alternative to experimental evidence. A third point of view is that we should continue to push forward with string theory in the hope that the new results and ideas that emerge will serve as a guide for finding a final unified theory.

Atiyah concluded his talk by speculating on the meaning of all this—quantum field theory, string theory, and their mathematical applications. What will the future physical theory look like? The aim is to unify quantum mechanics, the physics of the very small, with general relativity, the physics of the very large. Supersymmetry is a symmetry in which physical laws are unchanged when bosons and fermions are interchanged. Superstring theory, a supersymmetric string theory, is a perturbative approach, one that Atiyah compared with the theory of epicycles developed by Ptolemy. But what is the real theory; that is, what is being perturbed? Is it M-theory, a currently incomplete theory unifying all five versions of string theory? Is the universe really built using all this sophisticated machinery or is this an example of mathematics imposed by us? Perhaps the real physics is simpler and one should adhere to the dictate of Occam's razor—concepts should not be multiplied beyond necessity. Do we need to modify quantum mechanics? Atiyah closed by saying "This is for young people: Go away and explore it. If it works, don't forget I suggested it. If it doesn't, don't hold me responsible."

The second Einstein Public Lecture in Mathematics was delivered on April 29, 2006, in conjunction with the AMS Spring Western Sectional Meeting at San Francisco State University. Benoît Mandelbrot of Yale University spoke on "The nature of roughness in mathematics, science, and art".

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