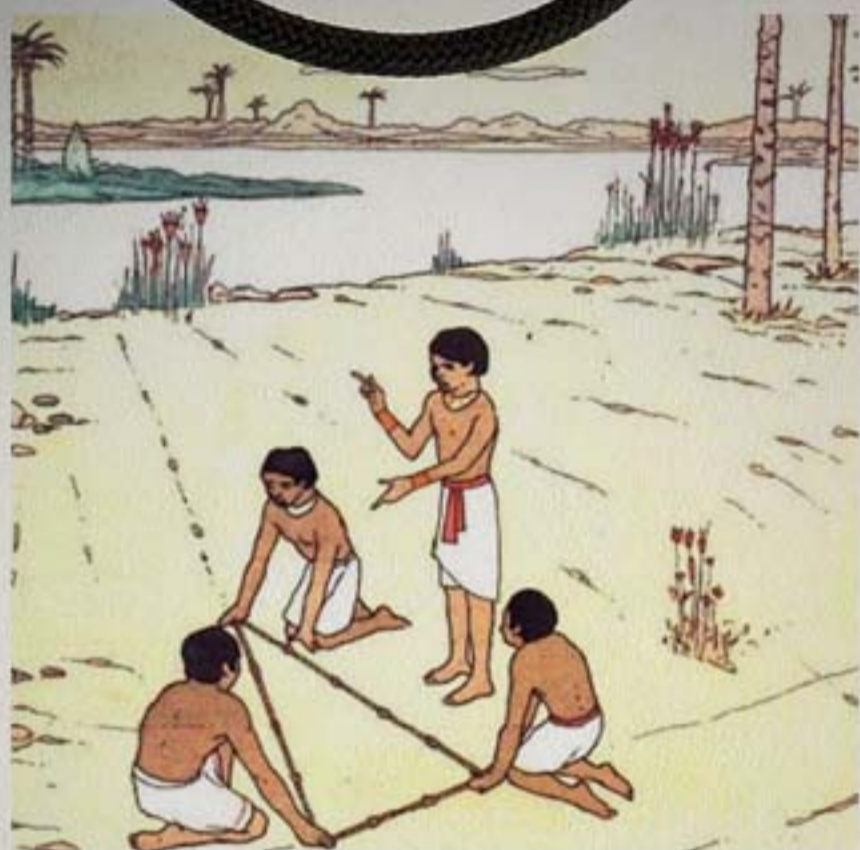
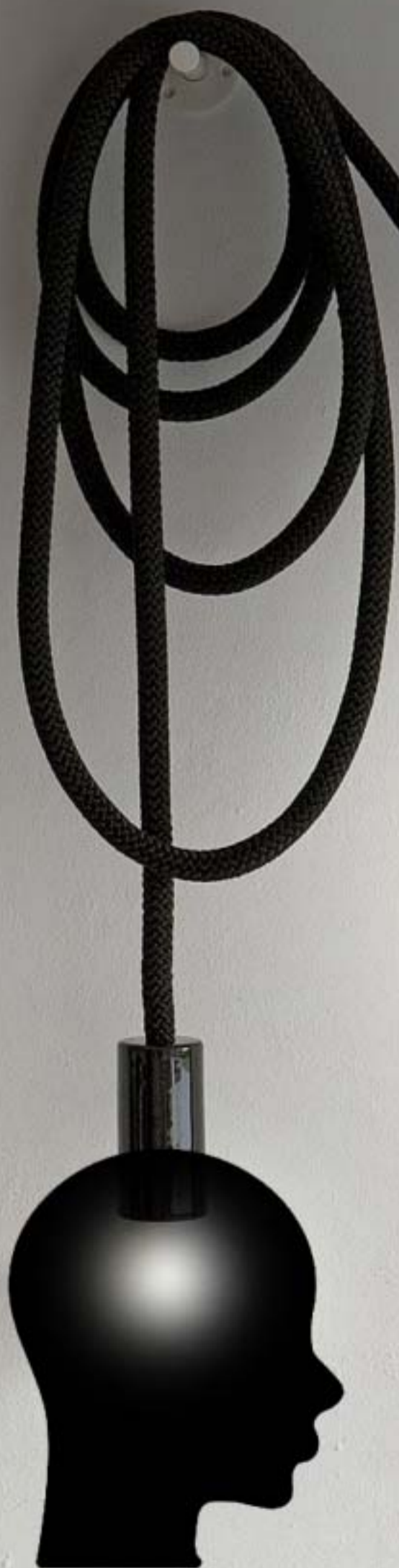


CK Raju



# Rajju Ganit

String Geometry  
for Class 9

# Rajju ganit

A text on string geometry for class 9

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## Preface

The present book aims to teach practical geometry (string geometry, cord geometry, rajju-ganit) to school students at roughly the level of the 9<sup>th</sup> std. There are two main new features. (1) The cord replaces the entire compass box. (2) Empirical methods are admitted in geometry contrary to the philosophy of formal math and using instead the philosophy of approximation called zeroism.

What are the useful new things students would learn as part of string geometry or rajju-ganit? Some of these new features are listed below. A proper understanding of these features requires reading the book.

**1. Conceptual clarity.** The greatest new features is conceptual clarity, since

1. we teach only one kind of geometry not a mixture of several incompatible geometries
2. We teach geometry empirically. E.g. dot is a point one can see.
3. We eliminate ritualistic features of the compass box, such as set squares and dividers. Actually, only ruler and compass needed.
4. The low-cost and eco-friendly string/tape can replace the entire compass box. A straight line segment is defined using a stretched string. Infinite lines are not needed.
5. A circle is drawn by keeping one end of the string fixed.
6. An ellipse is drawn by keeping two points fixed. (Can you draw an ellipse using the instruments of the compass box?)

**2. Measurement of angles.**

1. Measuring an angle requires measurement to measure the length of a curved arc.
2. Constructing a protractor.
3. We allow the use a string to measure the length of curved lines.
4. Practical measurement of lengths and areas using a cord.
5. The radian measure of angles.
6. The historical origin of the degree measure of angles in astronomy.
7. Similar triangles and the arithmetic rule of three.

**3. Simplified geometry**

1. The use of empirical proofs greatly simplifies geometry. We are able to prove all the usual geometrical results easily.
2. It also maintains consistency with superposition required for practical applications of geometry.
3. Instead of theorems we establish rules, on the understanding that these are only approximate. Only a few rules are needed.
4. Similar triangles, which are of great practical value, are included.

**4. Measurement of the circle.**

1. Why does a measured angle not depend upon the size of the protractor. Because the diameter and circumference of a circle are in proportion.
2. The calculation of  $\pi$  by three methods.
  1. Empirical method
  2. The octagon doubling method.
    1. Corollary: why the ratio of circumference to radius is constant.
  3. Monte Carlo method

#### 5. **The theory of approximation.**

Practical applications of mathematics require approximation. But the theory of approximation is never taught.

1. The Manava sulba sutra method of stating the Pythagorean proposition
2. The square root algorithm. Its difficulty, the origin of the term surd.
3. Why it does not terminate for the case of 2.
4. Meaning of savisesa.
5. Is there any place where the Pythagorean proposition holds exactly? No escape from approximation.
6. How to handle approximations and non-uniqueness using zeroism. Case of multiple lines connecting two dots.

#### 6. **Trigonometry.**

1. “Trigonometry” as the second Pythagorean calculation.
2. Circular functions defined using a circle (else no way to understand their relation to  $\pi$ )
3. Origin of the term sine.
4. The measurement of real life angles. Measuring latitude by pole star.
5. The calculation of intermediate sine values: similar triangles rule of and linear interpolation.

#### 7. **Applications to real life.**

1. The measurement of tree heights.
2. Measuring the height of a hill.
3. Measuring the radius of the earth.
4. Determining latitude by day
5. Determining longitude by solving the longitude triangle.

It should be clearly understood that the present version of the book is a DRAFT. Indeed, it is a hasty draft, prepared in a very short time. There may be typos and errors, the figures are often crudely drawn, and may include copyrighted material.

As such this draft is only for private circulation as part of the ongoing teaching experiment. Please point out any errors, typos, or improvements to the author at [ckr@ckraju.net](mailto:ckr@ckraju.net).

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The idea of string geometry has been festering since 2007. The author is particularly grateful to Prof. Murzban Jal and Dr Sandeep Deshmukh for helping this project to take off, and for pushing the writing of this text at breakneck speed.

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# Rajju Ganit: Part-2

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