

Presented at the 2nd People's Education Congress, Homi Bhabha Centre for Science Education, Mumbai, 5-9 Oct 2009, in both panels on (a) Mathematics Education and (b) History and Philosophy of Science in Science Education.

Calculus Without Limits: Report of an Experiment

C. K. Raju

Centre for Studies in Civilizations, New Delhi and
Inmantec, Ghaziabad,
ckr@ckraju.net, c.k.raju@inmantec.edu
web:ckraju.net

- 1 Aim and Summary
- 2 The new history
- 3 The new philosophy
- 4 The experiment
- 5 Results

Aim

- To report on a new system of teaching mathematics, and some recently conducted experiments on it.

Outline of the new system

- Q. Why is math difficult?

Outline of the new system

- Q. Why is math difficult?
- A. Because theology has got mixed up with it.

Outline of the new system

- Q. Why is math difficult?
- A. Because theology has got mixed up with it.
- Q. How to make math easy?

Outline of the new system

- Q. Why is math difficult?
- A. Because theology has got mixed up with it.
- Q. How to make math easy?
- A. Eliminate the theology from mathematics.

The basis of the new system

- Involves new history (“theology has got mixed with math”).

The basis of the new system

- Involves new history (“theology has got mixed with math”).
- and new philosophy of math (“Eliminate theology from math”).

The new history

- Calculus developed in India in response to needs of monsoon-driven agriculture and navigation,

The new history

- Calculus developed in India in response to needs of monsoon-driven agriculture and navigation,
- as a means of calculating accurate trigonometric values required for the calendar.

The new history

- Calculus developed in India in response to needs of monsoon-driven agriculture and navigation,
- as a means of calculating accurate trigonometric values required for the calendar.
- Calculus was transmitted to Europe in the 16th c. in response to needs of European navigation

The new history

- Calculus developed in India in response to needs of monsoon-driven agriculture and navigation,
- as a means of calculating accurate trigonometric values required for the calendar.
- Calculus was transmitted to Europe in the 16th c. in response to needs of European navigation
- which required a table of secants (for Mercator chart) and a precise calendar (for latitude determination; hence calendar reform of 1582).

Reception of calculus in Europe

- Trigonometric values used for Mercator chart.

Reception of calculus in Europe

- Trigonometric values used for Mercator chart.
- Beyond the capabilities of Tycho Brahe, Kepler.

Reception of calculus in Europe

- Trigonometric values used for Mercator chart.
- Beyond the capabilities of Tycho Brahe, Kepler.
- Rejected as disreputable by Galileo. Accepted by his student Cavalieri.

Reception of calculus in Europe

- Trigonometric values used for Mercator chart.
- Beyond the capabilities of Tycho Brahe, Kepler.
- Rejected as disreputable by Galileo. Accepted by his student Cavalieri.
- Accepted by Fermat and Pascal. Rejected as “beyond the capacity of the human mind” by Descartes.

Reception of calculus in Europe

- Trigonometric values used for Mercator chart.
- Beyond the capabilities of Tycho Brahe, Kepler.
- Rejected as disreputable by Galileo. Accepted by his student Cavalieri.
- Accepted by Fermat and Pascal. Rejected as “beyond the capacity of the human mind” by Descartes.
- Accepted by Newton and Leibniz whose work was torn to bits by Berkeley.

Lesson

- Reception of the calculus in Europe involved difficulties.

Lesson

- Reception of the calculus in Europe involved difficulties.
- These difficulties persisted until formal real numbers by Dedekind.

Lesson

- Reception of the calculus in Europe involved difficulties.
- These difficulties persisted until formal real numbers by Dedekind.
- Which used Cantor's set theory, formalised in the 1930's.

Lesson

- Reception of the calculus in Europe involved difficulties.
- These difficulties persisted until formal real numbers by Dedekind.
- Which used Cantor's set theory, formalised in the 1930's.
- Leads to present-day teaching of calculus with limits as in current school texts.

Phylogeny is Ontogeny

- Math is difficult because this historical trajectory is replayed in the classroom.

Phylogeny is Ontogeny

- Math is difficult because this historical trajectory is replayed in the classroom.
- in fast forward mode.

Lesson

- These difficulties arose because Indian *gaṇita* different from mathematics.

Lesson

- These difficulties arose because Indian *gaṇita* different from mathematics.
- The calculus developed in one cultural context and was absorbed in another.

Lesson

- These difficulties arose because Indian *gaṇita* different from mathematics.
- The calculus developed in one cultural context and was absorbed in another.
- Calculus arose as *gaṇita* and was sought to be absorbed within the religious tradition of mathematics (as science of the soul)

Philosophical objections

- Current notion of mathematical proof not rigorous.

Philosophical objections

- Current notion of mathematical proof not rigorous.
 - Why?

Philosophical objections

- Current notion of mathematical proof not rigorous.
 - Why?
 - It uses two valued logic. So theorems will change with Buddhist or Jain logic.

Philosophical objections

- Current notion of mathematical proof not rigorous.
 - Why?
 - It uses two valued logic. So theorems will change with Buddhist or Jain logic.
 - Empirical methods must be admitted in proof since they are MORE certain than metaphysics (deduction).

Calculation vs proof

- Proof is not mathematics at all. Mathematics relates to practical calculation and not proof (which is only of theological value).

Calculation vs proof

- Proof is not mathematics at all. Mathematics relates to practical calculation and not proof (which is only of theological value).
 - Why?

Calculation vs proof

- Proof is not mathematics at all. Mathematics relates to practical calculation and not proof (which is only of theological value).
 - Why?
 - Proof is not universal, and only gives cultural satisfaction. Calculation delivers practical value (as in science).

Calculation vs proof

- Proof is not mathematics at all. Mathematics relates to practical calculation and not proof (which is only of theological value).
 - Why?
 - Proof is not universal, and only gives cultural satisfaction. Calculation delivers practical value (as in science).
 - This theology of "universal" reason used to convert Muslims during Crusades, and now being used to dominate the world (globalization and soft power see my article on "Benedict's Maledicts" on ZNet and in *Indian Journal of Secularism*).

Calculation vs proof

- Proof is not mathematics at all. Mathematics relates to practical calculation and not proof (which is only of theological value).
 - Why?
 - Proof is not universal, and only gives cultural satisfaction. Calculation delivers practical value (as in science).
 - This theology of "universal" reason used to convert Muslims during Crusades, and now being used to dominate the world (globalization and soft power see my article on "Benedict's Maledicts" on ZNet and in *Indian Journal of Secularism*).
 - Racist history of science ("Euclid" etc) key part of this agenda of domination through soft power. (Children should grow up in awe of the West.)

contd

- Teach secular math. What we should to teach our children is practical value derived from ability to calculate.

contd

- Teach secular math. What we should to teach our children is practical value derived from ability to calculate.
- Easy math. Difficulties with calculus are removed by teaching it as it developed in India in response to material needs.

contd

- Teach secular math. What we should teach our children is practical value derived from ability to calculate.
- Easy math. Difficulties with calculus are removed by teaching it as it developed in India in response to material needs.
- Zeroism. This also involved a new philosophy of zeroism, similar to Nagarjuna's *śūnyavāda*.

contd

- Teach secular math. What we should to teach our children is practical value derived from ability to calculate.
- Easy math. Difficulties with calculus are removed by teaching it as it developed in India in response to material needs.
- Zeroism. This also involved a new philosophy of zeroism, similar to Nagarjuna's *śūnyavāda*.
- For more details on zeroism see my paper <http://ckraju.net/papers/Zeroism-and-calculus-without-limits.pdf>, 4th Nalanda conference, on my website.

The experiment

- Course initially tried on teacher trainees at Inmantec School of Education.

The experiment

- Course initially tried on teacher trainees at Inmantec School of Education.
- Then on 27 students at Central University of Tibetan Studies.

The experiment

- Course initially tried on teacher trainees at Inmantec School of Education.
- Then on 27 students at Central University of Tibetan Studies.
- Age group 22-55 years. Included faculty and head of department.

The experiment

- Course initially tried on teacher trainees at Inmantec School of Education.
- Then on 27 students at Central University of Tibetan Studies.
- Age group 22-55 years. Included faculty and head of department.
- Background: Students admitted after 8th std. Some have monastic education.

The experiment

- Course initially tried on teacher trainees at Inmantec School of Education.
- Then on 27 students at Central University of Tibetan Studies.
- Age group 22-55 years. Included faculty and head of department.
- Background: Students admitted after 8th std. Some have monastic education.
- Very poor performance in pre-test even on elementary arithmetic.

Pre-test

Calculus without Limits

Notes:

1. This is NOT a competition. The aim of this test is only to provide *feedback* regarding your current knowledge of mathematics.
2. Some questions may be beyond your current knowledge. Please don't be anxious about it. It is expected that you do *not* know the answers to all questions, and those questions are there only to establish the limits of your knowledge.

I : Arithmetic

1. Find $124 + 568$.
2. Find $532 - 319$.
3. Calculate 3542×213 .
4. If 2184 is divided by 17 what is the quotient and what is the remainder?
5. Which is the greatest among the following four numbers: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$?
6. Write $\frac{3}{4}$ as a decimal.
7. Write 0.4352 as a proper fraction.
8. What is the square of 23?
9. A trader bought an item for Rs 26 and sold it for Rs 38. What percentage profit did he make?
10. The Rajdhani express travels from Delhi to Mumbai in 18 hours and 30 minutes with stops of 10 minutes each at Kota, Ratlam and Baroda. If its average speed is 81 km, what is the distance from Delhi to Mumbai?
11. If 3 kg of flour sells for Rs 32 how much does 5 kg of flour sell for?

II : Alegbera

12. If $x = 5$ what is the value of x^2 ?
13. If $2x + 3 = 10$ what is the value of x ?
14. If $2x + 3y = 40$ and $x = 7$ what is the value of y ?
15. If $x^2 - x - 6 = 0$ what are the possible values of x ?

III : Geometry

16. If one angle of a right-angled triangle is 30° write the other two angles in degrees.
17. A rectangle has length 1 and width 2. What is the length of it diagonal?
18. Give an approximate figure for the circumference of a circle whose radius is 1.
19. Plot a straight line through the points (2, 3) and (2, -3).

IV : Elementary Calculus

20. What is $\frac{d}{dx} \sin(x)$?
21. What is $\int x^2 dx$?

V: Calculus questions from question bank

22. Differentiate $\sqrt{\frac{\sin x - 1}{\sin x + 1}}$ with respect to x .
23. Differentiate $\log \frac{\sqrt{1+x^2}-6}{\sqrt{1+x^2}+6}$ with respect to x .
24. Evaluate the integral $\int \frac{x^2+1}{x^2+1} dx$.
25. Evaluate the integral $\int x^2 \tan^{-1} x dx$.

Objective of experiment

- Challenge: to teach them calculus within 5 lectures, using the new philosophy.

Objective of experiment

- Challenge: to teach them calculus within 5 lectures, using the new philosophy.
- Test of learning: they should be able to solve questions drawn at random from a calculus question bank.

Objective of experiment

- Challenge: to teach them calculus within 5 lectures, using the new philosophy.
- Test of learning: they should be able to solve questions drawn at random from a calculus question bank.
- (seed supplied by Vice Chancellor).

Objective of experiment

- Challenge: to teach them calculus within 5 lectures, using the new philosophy.
- Test of learning: they should be able to solve questions drawn at random from a calculus question bank.
- (seed supplied by Vice Chancellor).
- And also solve ordinary differential equations.

Post-test
Calculus without Limits

I : Elementary computations

1. Convert 30 deg to radians.
2. Convert 2 radians into degrees.

II : Elementary Calculus

3. What is $\frac{d}{dx} \sec(x)$?
4. Evaluate $\int \cos(3x + 1) dx$
5. Find the second derivative of $x \sin x$.
6. Find

$$\int_0^1 x e^x dx$$

7. Numerically integrate

$$\int_0^{0.5} \frac{1}{\sqrt{1-x^2}\sqrt{1-x}} dx$$

III: Questions from question bank (differentiation)

Differentiate the following functions with respect to x .

8. $\sqrt{1-x^2}$.
9. $x^2 e^{\sqrt{x}}$.
10. $x^2 \sin^3 x \cos^4 x$

(continued from page 1: differentiate the following with respect to x)

11.

$$\log \sqrt{\frac{1+x \cos x}{1-x \cos x}}$$

12.

$$\tan^{-1} \left(\frac{e^{2x} + 1}{e^{2x} - 1} \right)$$

IV: Questions from question bank (integration)

Evaluate the following integrals.

13.

$$\int \frac{1}{1-x^2} dx$$

14.

$$\int \frac{1}{x^3 + x^2 + x + 1} dx$$

15.

$$\int \frac{\sqrt{x} - \sqrt{x}}{1 - \sqrt{2x}} dx$$

16.

$$\int \sec^{-1} \sqrt{x} dx$$

17.

$$\int \cot^5 x dx$$

V : Ordinary differential equations

18. Solve the differential equation $y' = 2y$, with $y(0) = 1$ and hence find $y(4)$.

19. Solve the differential equation $y' = x \sin(x)$ with $y(0) = 1$ and find the value of $y(10)$.

20. Solve the differential equation $y'' = -3y$ with $y(0) = 1$ and $y'(0) = 0$, and find the value of $y(20)$.

Results

- Above 60% — 4

Results

- Above 60% — 4
- Between 35-60% — 8

Results

- Above 60% — 4
- Between 35-60% — 8
- Below 35% — 15

Conclusions

- Test was moderately successful. In the pre-test only 1 student attempted any of the question-bank questions.

Conclusions

- Test was moderately successful. In the pre-test only 1 student attempted any of the question-bank questions.
- In the post-test this student got nearly 100%. About half the class managed to clear the test.

Conclusions

- Test was moderately successful. In the pre-test only 1 student attempted any of the question-bank questions.
- In the post-test this student got nearly 100%. About half the class managed to clear the test.
- The bottom half of the class performed poorly.

Conclusions

- Test was moderately successful. In the pre-test only 1 student attempted any of the question-bank questions.
- In the post-test this student got nearly 100%. About half the class managed to clear the test.
- The bottom half of the class performed poorly.
- As clear from the pre-test some of the students (and faculty) did not fulfil the starting criterion of knowing school math at 8th std. level. They are being given remedial coaching in school math.

Central University of Tibetan Studies

Lhasa, Tibet

Workshop on "Calculus without limits"

22nd - 28th September, 2009

