Draft: Do not quote without permission

Eternity and Infinity

The Western misunderstanding of Indian mathematics, and its consequences for science today

C. K. Raju

Practical Indian math

Most students today study mathematics for its practical applications. And it was for its practical applications that *ganita* developed in India: arithmetic and algebra for commerce, permutations and combinations for the theory of metre, probability theory for the game of dice,[1] "trigonometry" and calculus, or rather the study of the circle and the sphere, for astronomy and navigation. Navigation was important for overseas trade, which stretches back 5000 years in India, and was an important source of wealth. Astronomy was needed for the calendar and to determine the seasons, since the rainy season is critical to Indian agriculture the other key source of wealth in India.[2]

Since *ganita* was done for its practical applications, Indian texts from the ancient sulba sutra-s, through the 5th c. Aryabhata to the 16th c. Yuktidipika, all admit empirical proofs.[3] For example, Aryabhata states that a plumb line is the test of verticality. Secondly, all practical applications invariably involve a tolerance level, or an "error margin". Thus, all the above three texts give the ratio of the circumference of a circle to its diameter, or the number today designated by π, as 3.1415.... The sulba sutra-s declare its value to be non-eternal (*anitya*)[4] and imperfect (*savisesa*,[5] with something left out). Aryabhata who numerically solves a differential equation, to derive his sine values precise to the first sexagesimal minute (about 5 decimal places), declares his value of π to be *asanna* (near value).[6]

Discarding insignificant quantities naturally extends to the discarding of infinitesimals, which enters essentially in the the way Indian texts treat infinity. By the 14th c., Aryabhata's method was extended in India to an infinite "Taylor" series for the sine, cosine, and arctangent functions, to derive their values accurate to the third sexagesimal minute (about ten decimal places), and Nilakantha in his commentary also explains why the near value of π is given and not the real value (*vasatavim sankhya*).[7] The 15th c., Nilakantha is also the first source for the formula for the sum of an infinite geometric series.[8]

---

1 C. K. Raju, "Probability in Ancient India", chp. 37 in *Handbook of the Philosophy of Science, vol 7. Philosophy of Statistics*, ed. Prasanta S. Bandyopadhyay and Malcolm R. Forster, General ed. Dov M. Gabbay, Paul Thagard and John Woods. Elsevier, 2011, pp. 1175-1196 (http://www.ckraju.net/papers/Probability-in-Ancient-India.pdf.)

2 For a short account, see "Cultural Foundations of Mathematics", *Ghadar Jari Hai*, **2**(1), 2007, pp. 26-29. http://ckraju.net/papers/GJH-book-review.pdf.

3 For a detailed discussion of this issue of empirical vs deductive proofs, see C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhâsâ", *Philosophy East and West*, **51**(3) (2001) pp. 325–362. http://ckraju.net/papers/Hawaii.pdf.

4 Apastamba sulba sutra 3.2,

5 Baudhayana sulba sutra 2.12

6 *Aryabhatiya*, Ganita 10.

7 Nilakantha, *Aryabhatiyabhasya*, commentary on Ganita 10, Trivandrum Sanskrit Series, 101, reprint 1977, p. 56, and its translation in C. K Raju, *Cultural Foundations of Mathematics*, Pearson Longman, 2007, pp. 125-26.

8 Nılakantha, *Aryabhatıyabhasya*, cited earlier, commentary on Ganita 17, p. 142.

Religious Western math

However, in the West mathematics was valued for its religious links. The very word mathematics derives from mathesis, which means learning.[9] In Plato's *Meno*, Socrates explains that learning is achieved by arousing the soul, for "all learning is recollection" of the eternal ideas in the soul. Having demonstrated a slave boy's innate knowledge of mathematics, he claims he has proved the existence of the soul and its past lives: for, he argues, if the slave boy did not learn mathematics in this life, he must have learnt it in a previous life.[10] The Greeks had imported Egyptian mystery geometry which had the spiritual aim of arousing the soul by turning the mind inward.[11] The belief was that math contains eternal truths hence arouses the eternal soul by sympathetic magic. This notion of soul became unacceptable to the post-Nicene church which cursed the belief in past lives,[12] and banned mathematics, in the 6th c. However, those "Neoplatonic" beliefs survived in Islam as part of what Muslim scholars called "the theology of Aristotle", and were influential in the *aql-i-kalam* or Islamic theology of reason.

When the wealthy Khilafat of Cordoba splintered and became weak, in the 11th c., the church saw an opportunity, and launched the Crusades with a view to conquer and convert Muslims by force, the way Europe was earlier Christianised by force. However, the Crusades failed militarily (beyond Spain and after the first Crusade). Nevertheless, Muslim wealth was so tempting that the church changed its entire theology to the Christian theology of reason promoted by Aquinas and his schoolmen. Reason was declared universal, since Muslims too accepted it so it helped to persuade them. However, not wishing to acknowledge that this major change in theological beliefs arose from an adaptation of Islamic beliefs, and not finding any sources in the Bible to support rational theology, the church claimed ownership of reason by attributing its origin to an early Greek called Euclid. Alongside it reinterpreted the *Elements* and its geometry as concerned solely with metaphysical (deductive) proofs to align it with the post-Crusade theology of reason.

From concocted Euclid to formalism

There is no evidence for "Euclid". While my book *Euclid and Jesus* goes into all the details of this spurious myth, to avoid having to do so repeatedly, I instituted the "Euclid" prize of USD 3300 for serious evidence about "Euclid". Needless to say, the challenge has not been met.

---

9 Contrary to the wrong derivation of mathematics from "mathema", given currency by the Wikipedia, Proclus clearly derives mathematics from "mathesis". "This, then, is what learning (μάθεσις) [mathesis] is, recollection of the eternal ideas in the soul; and this is why the study that especially brings us the recollection of these ideas is called the science concerned with learning (μαθέματική) [mathematike]). Its name thus makes clear what sort of function this science performs. It arouses our innate knowledge. . . takes away the forgetfulness and ignorance [of our former existence] that we have from birth,. . . fills everything with divine reason, moves our souls towards Nous,. . . and through the discovery of pure Nous leads us to the blessed life." Proclus, *Commentary on the Elements* [Corrected title], trans. Glenn R. Morrow, Princeton University Press, Princeton, 1992, 47, p. 38.

10 Plato, *Meno*, In: *The Dialogues of Plato*, trans. B. Jowett, Encyclopedia Britannica, Chicago, 1994, pp. 179–180.

11 C. K. Raju, *Euclid and Jesus: How and why the church changed mathematics and Christianity across two religious wars*, Mulitversity, 2012.

12 C. K. Raju, "The curse on 'cyclic' time", chp. 2, in *The Eleven Pictures of Time*, Sage, 2003.

Post-Crusade, the belief in the eternal truths of mathematics persisted for new theological reasons. Western theologians who always understood how God worked, said that logic bound God who could not create an illogical world, but was free to create the facts of his choice. Hence it came to be believed in the West that mathematics, as truth which binds God, or eternal truth, must be "perfect" and cannot neglect even the tiniest errors (which are bound to surface some time during eternity!) It was further believed, that this "perfection" could be achieved only through metaphysics: a "perfect" mathematical point is never a real dot on a piece of paper, howsoever much one may sharpen the pencil.

Carried away by the story that this metaphysical understanding of "real" math originated with "Euclid" and his "irrefragable" proofs, for 7 centuries, no European scholar noticed the fact that the very first proposition of the *Elements* uses an empirical proof! When this was finally admitted, in the 19th c. that empirical proofs are essential to the *Elements,* what happened was even more amusing. For metaphysicians, the story naturally proved to be stronger than the facts! Instead, of accepting that the story was false, those empirical proofs were attributed to an error by the supposed "Euclid" in executing his purported intentions. Bertrand Russell and David Hilbert then rewrote the *Elements* to correct "Euclid" and make his book 100% metaphysical! That rewriting does not fit the *Elements*, but it led to the present-day formal mathematics of Russell and Hilbert which makes all mathematics 100% metaphysics.[13]

Transmission of Indian math and its European misunderstanding

The two streams of mathematics, religious and practical, collided when the West started importing Indian mathematics for its practical applications from the 10th c.[14] Earlier, on the "Neoplatonic" belief that knowledge is virtue, the Baghdad House of Wisdom had imported numerous texts from all over the world, especially India, in the 9th c. Muslims frankly acknowledged those imports as in al Khwarizmi's *Hisab al Hind*. When these new techniques travelled to Europe, they were called algorismus (after al Khwarizmi's Latinized name) Again, the algebra from Brahmagupta[15] came to be known as algebra after al Khwarizmi's *Al jabr waa'l Muqabala*. These arithmetic and algebraic techniques were adopted by Florentine merchants because of their practical advantage for commerce.

Transmission of knowledge often results in misunderstanding, and the hilarious story of the persistent European misunderstanding of imported Indian math is told by the very words like "zero", "surd", "sine", "trigonometry" etc. in common use today. Zero (from cipher, meaning mysterious code) created conceptual difficulties for Europeans for centuries, since it involved the sophisticated place value system, different from the primitive Greek and Roman numerals which were additive and adapted to the abacus. Thus, in 976, Gerbert, who later became the infallible pope Sylvester, got constructed a special abacus for "Arabic numerals" which he imported from Cordoba, for he thought the abacus was the only way to do arithmetic![16] Due to these conceptual difficulties, the practical algorismus entered the Jesuit math syllabus only around 1572.

Similar hilarious European confusion underlies the term "surd" from the Latin *surdus* meaning deaf, applied today to the square root of 2. That was calculated since the sulba sutra as the diagonal (*karna*)

13 C. K. Raju, "Euclid and Hilbert", chp. 1 in *Cultural Foundations of Mathematics*, Pearson Longman, 2007.
14 C. K Raju, "Math wars and the epistemic divide in mathematics", chp 8 in *Cultural Foundations of Mathematics*, cited above. Also at
15 See, e.g., *Algebra....from the Sanscrit of Brahmegupta and Bhascara* trans. H. T. Colebrooke, John Murray, London, 1817.
16 For a picture of this abacus, see *Euclid and Jesus*, cited above.

of the unit square, and the term *surdus* is a mistranslation of bad *karna*, meaning bad diagonal but misunderstood as bad ear, for the word *karna* also means ear. Similarly the term sine from the Arabic *jaib* meaning pocket is a misreading of *jiba* from the vernacular *jiva* from the Sanskrit *jya* meaning chord. Since the chord relates to the circle, not the triangle, the word "trigonometry" indicates a European conceptual misunderstanding for what should properly be called circlemetry, and was studied in Indian texts in chapters on the circle.

The problem of infinite series

While early imports of Indian mathematics in Europe came indirectly via Arabs from Cordoba and Toledo, probability and calculus went directly through Jesuits from Cochin in the 16th c. [17] The maximum confusion and misunderstanding attended the transmission of the infinite series of the Indian calculus to Europe. As already indicated, the most elementary circlemetric ratio, the ratio of the circumference of a circle to its diameter, necessarily involves an infinite series, as in $\pi = 3.1415....$, which decimal representation is an infinite sum $3 + 1/10 + 4/100 + 1/1000 + 5/10000 +....$ These infinite series were used in India to derive sine values accurate to the third sexagesimal minute (about ten decimal places).[18] These values ("tables of secants") were of great practical importance to the (specifically) European navigational problem of determining loxodromes, and also latitude, longitude at sea, navigation then being the principal scientific challenge facing Europe which dreamt of wealth through overseas trade. This overwhelming practical value meant that the infinite series could not simply be abandoned.

Now for practical purposes, related to navigation and astronomy, a precision of, say, 8 decimal places was ample. But the infinite series presented a conceptual difficulty on the unfounded European faith in mathematics as perfect. Thus, the infinite series of the imported Indian calculus could not be "perfectly" summed. Practically speaking, even today, one typically states the number π only to a few decimal places as $\pi = 3.14$. Usually, one stops after at most 100 or 1000 places after the decimal point with the understanding that one can go on further if one really needs to do so. The resulting tiny error is of no practical consequence. While this process is adequate for *all* practical purposes, the infinite sum is nevertheless not "perfect", since some tiny error would still be neglected. On the other hand, it is evidently impossible to sum the series "perfectly" by adding all terms, for that would take an eternity of time.

Hence, on the deep-seated Western faith in mathematics as perfect, Descartes[19] declared that the ratio of curved and straight lines was beyond the human mind! Coming from a leading Western mind, this was hilarious, since, from the days of the sulba sutra, Indian children were taught to measure curved lines using a string, and to compare them with straight lines just by straightening the string. This was not Descartes' individual problem. Galileo in his letters to Cavalieri[20] concurred with Descartes, and

---

17 For detailed documentation of this claim of transmission of calculus from India to Europe, see C. K. Raju, *Cultural Foundations of Mathematics: the nature of mathematical proof and the transmission of the calculus from India to Europe*, Pearson Longman, 2007.

18 For a complete tabulation of the sine values and errors involved, see Tables 3.1 and 3.2 in Chp 3, "Infinite series and π" in *Cultural Foundations of Mathematics*, cited above.

19 R. Descartes, *The Geometry*, trans. David Eugene and Marcia L. Latham, Encyclopaedia Britannica, Chicago, 1996, Book 2, p. 544.

20 For a short account of Galileo's letters to Cavalieri, see Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, Oxford University Press, Oxford, 1996, pp. 118–122.

Newton's posthumous opponent Berkeley[21] thought that this was good reason to reject the calculus.

Indeed, though calculus began as circlemetry, this Cartesian difficulty with curved lines is still part of Western mathematical indoctrination today; the ritual compass box which every child carries to school has nothing with which to measure curved lines.[22] Newton himself thought that Descartes objection could be met by making time "flow" metaphysically,[23] a statement explicitly recognized as meaningless by Indians at least since Sriharsa.[24] Newtonian physics failed just because of this conceptual error about the nature of time.[25]

Infinity and eternity

Though Newton's fluxions were eventually abandoned, the West still maintained that a metaphysical understanding of infinity was the only solution to the specifically European problem of "perfectly" summing an infinite series. In the 19th c the Western solution to the problem of infinite sums moved towards metaphysical "real numbers", or the continuum, an uncountable infinity of numbers constructed using Cantorian set theory and its transfinite cardinals. That is how calculus is taught in schools and universities today, by appealing to the continuum and the metaphysical "limits" that make it possible to "perfectly" sum infinite series. Actually, all that metaphysics is too difficult to teach in high school and even most undergraduate courses for non-mathematics majors, so students are only told about it, not actually taught. That is, not only was the calculus wrongly attributed to Newton and Leibniz, in a persistent act of falsehood, it is today taught in universities and schools claiming that its infinite series can be summed "rigorously" only by using a particular metaphysics of infinity.

It should be clearly noted that here there is nothing unique or "universal" about metaphysical notions such as infinity and eternity. The notion of *atman* in the Upanishads, so fundamental to Hinduism, is embedded in an underlying *physical* belief[26] in quasi-cyclic time: eternity is not "linear". The same is true of the notion of soul according to Egyptians, Socrates, or early Christians. Indeed, the primary conflict in Christian theology was over the nature of eternity, whether it is quasi-cyclic as Origen thought, or whether it is metaphysical and apocalyptic as believed in post-Nicene theology.[27] It was this fundamental religious conflict over the nature of eternity which culminated in the church ban on mathematics (for "pagans" like Hypatia and Proclus still understood mathematics as concerning the

21 George Berkeley, *The Analyst or a Discourse Addressed to an Infidel Mathematician*, 1734, ed. D. R. Wilkins, available online at http://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/Analyst.html.

22 C. K. Raju, "Towards Equity in Math Education 2. The Indian Rope Trick", *Bharatiya Samajik Chintan* (New Series) 7 (4) (2009) pp. 265–269.

23 I. Newton, *The Mathematical Principles of Natural Philosophy*, A. Motte's translation revised by Florian Cajori, Encyclopedia Britannica, Chicago, 1996, "Absolute, true, and mathematical time...flows equably without relation to anything external." For a detailed analysis of how Newton made time metaphysical, see C. K. Raju, "Time: what is it that it can be measured", *Science & Education*, **15**(6) (2006) pp. 537–551.

24 Sriharsa, *KhandanaKhandaKhadya*. IV.142. For a discussion in the context of McTaggart's paradox, see "Philosophical time", chp. 1 in *Time: Towards a Consistent Theory*, Kluwer Academic, 1994.

25 C. K. Raju, *Time: Towards a Consistent Theory*. Kluwer Academic, 1994. For a quick summary, see, C. K. Raju, "Retarded gravitation theory", in: Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro ed., *Sixth International School on Field Theory and Gravitation*, American Institute of Physics, New York, 2012, pp. 260-276. http://ckraju.net/papers/retarded_gravitation_theory-rio.pdf.

26 C. K. Raju, "Atman, Quasi-Recurrence and paticca samuppada", in *Self, Science and Society, Theoretical and Historical Perspectives*, ed. D. P. Chattopadhyaya, and A. K. Sengupta, PHISPC, New Delhi, 2005, pp. 196-206. http://ckraju.net/papers/Atman-quasi-recurrence-and-paticca-samuppada.pdf.

27 C. K. Raju, *The Eleven Pictures of Time*, cited above.

soul). This conflict over the nature of eternity was also the basis of the subsequent curse on "cyclic" time called the anathemas against pre-existence. It is also reflected in the first creationist controversy, which concerned the nature of eternity not evolution. Thus, Proclus stated, in *his* book, also called *Elements*, that eternity turns back on itself, as in the uroburos, or a snake eating it own tail. This was the ancient Egyptian symbol for quasi-cyclic time and is still the modern symbol for infinity, ∞ . In contrast, John Philoponus[28] maintained that would make one time creation, as in the Bible, impossible, and also make apocalypse impossible, depriving the church of a valuable weapon of terror ("doomsday is round the corner").

The metaphysical continuum not essential for calculus

Nevertheless, those who study calculus today from school and university texts are taught to believe that the only way to do calculus rigorously is to use the continuum (and the underlying beliefs about infinity). Contrary to this indoctrination, calculus and the summation of infinite series can be done using number systems both smaller and larger than the continuum. Formally speaking, the continuum or the field of real numbers, *R*, is the largest "Archimedean" ordered field. Therefore, any ordered field, *F*, larger than *R* must be non-Archimedean. The failure of the Archimedean property in *F* means that *F* must have an element x such that $x > n$ for all natural numbers n. (Any ordered field must contain a copy of the natural numbers, and also fractions or "rational" numbers.) Such an x may be called an infinite number. Since F is a field, the positive element x must be invertible, and the inverse too must be positive, so we must have $0 < 1/x < 1/n$ for all natural numbers n. Such a number may be called an infinitesimal. Note that, unlike non-standard analysis, where such infinities and infinitesimals appear only at an intermediate stage, the infinities and infinitesimals in a non-Archimedean field are "permanent".

On the other side, of a smaller number system, a computer can only work with a finite number system, but is indispensable for all practical applications of the calculus such as sending a rocket to Mars. A computer cannot handle infinity or the continuum and uses instead floating point numbers resulting from rounding or discarding small numbers. These numbers do not even obey the associative "law", hence do not constitute a field.

Both approaches (with a larger or smaller number system) fit into the *sunyavada* philosophy, which I call zeroism, which tells us that in representing an entity (any real entity, not merely a "real" number or integers) we are compelled to discard or "zero" some small aspect as "non-representable" on the grounds that "we don't care". This happens because any real entity constantly changes, though we usually neglect those changes as to tiny to care about. Likewise, when we speak of "2 dogs" we do not thereby imply that the two dogs are identical but only that we don't care to describe the differences.

Indians used both rounding and discarding of infinitesimals, which processes are similar but not identical. The formula for an infinite geometric series was first developed using exactly such non-Archimedean arithmetic. From the time of the 6^{th} c. Brahmagupta, Indians used polynomials, which they called unexpressed numbers. This naturally led to "unexpressed fractions" corresponding to what are today called "rational functions", and are an example of non-Archimedean arithmetic.[29] What are today called "limits" were determined in that non-Archimedean arithmetic using order counting or

28 Ioannes Philoponus, *De aeternitate mundi contra Proclum,* B. G. Teubner, Leipzig, 1899.

29 An elementary construction of the non-Archimedean field of rational functions is given in E. Moise, *Elementary Geometry from an Advanced Standpoint*, Addison Wesley, Reading, Mass., 1963.

discarding infinitesimals very similar to discarding small numbers.[30] (Formally speaking, limits in a non-Archimedean field are not unique and involve discarding infinitesimals.) Obviously, this was too sophisticated for Western mathematicians of the 17th to understand: who lacked even a precise idea of infinitesimal and naively thought of it as a very tiny quantity.

It is well known that constructing the continuum required Cantor's set theory which was full of holes exposed by paradoxes such as Russell's paradox. While the axiomatisation of set theory resolved Russell's paradox, others paradoxes like the Banach-Tarski paradox still persist, though they are not so well known. According to this paradox, using set theory, one ball of gold can be cut into a finite number of pieces which can be reassembled into two balls of gold of the exact same size! Western mathematicians believe that to be a form of truth higher than empirical truth, hence one on which they base present-day math! More fundamentally, the consistency of set theory is maintained by using double standards typical of theology: adopting separate standards of proof for metamathematics and mathematics. If transfinite induction were permitted in metamathematics, as it is in mathematics, that would make set theory decidable, hence inconsistent. If transfinite induction is not solid enough for metamathematics, why should it be acceptable in mathematics? Thus, it is only an agreement between Western scholars, an agreement which is sustained by a system of "authorised knowledge" which suppresses dissent, through traditional church processes such as secretive refereeing, designed for that purpose.

To reiterate, this Western metaphysics of infinity in present-day mathematics does not affect any practical applications of mathematics to science and engineering, which must all be done in the old way. For example, as already noted, all practical applications of the calculus are done on a computer which cannot handle the continuum. The practical way to do calculus still involves Aryabhata's method of numerical solution of differential equations.[31]

Spreading religious biases through math

However, this cocktail of Indian practical mathematics and Western metaphysics was declared "superior" to the original, and returned to India, and globalised through colonial education. The claim of "superiority" is a fake one: one could, with stronger reason, maintain that empirical proofs are more reliable than the metaphysical claims of Western theologians, and reject formalism.

However, it would be naïve to suppose that this claim of "superiority" is confined to racist and colonial historians: it is *the* mainline story of the West. This story of "superiority" is central to the church propaganda that Christians (and their beliefs) are superior to all others. If philosophers like Hume, Kant, Carlyle or Johnson were racists, that was no individual aberration.

Indeed, along with the practical value of mathematics, children today learn at an early age that empirical proofs are inferior. Now, all systems of Indian philosophy accept the *pratyaksa*, or empirically manifest, as the first means of *pramana*. This applies also to Indian *ganita*, which accepts empirical proofs. Therefore, along with mathematics, children today are taught in school that all systems of Indian philosophy are "inferior" compared to "superior" church metaphysics. Since Islamic

30 *Cultural Foundations of Mathematics*, cited earlier.

31 Aryabhata did solve differential equations using what is today called Euler's method. (Euler studied Indian texts.) Specifically, his method of obtaining sine differences cannot be understood as an algebraic equation. *Cultural Foundations of Mathematics,* cited earlier, chp. 3,

philosophy too accepts *tajurba* as a means of proof, this bias is against all non-Christian beliefs. Note that science too prefers empirical proofs to metaphysics, so accepting empirical proofs does not damage any practical applications of mathematics to science. It just gives our children the sense that foolish sense of "superiority" which accompanies the blind imitation of church metaphysics. It teaches them that they must reject Indian tradition as "inferior". Protests are suppressed.

Indian philosophers too have swallowed the story that mathematics involves some superior kind of knowledge ("binding on God", true in all possible worlds which God could create, true in all possible worlds on possible-world semantics). "As certain as 2+2=4", as even the late Daya Krishna once said to me. But why exactly is deduction a "superior" form of proof? One understands that this claim is indispensable to Christians, but why should others accept it? When Western theologians spread the superstition that logical proofs are "superior" since logic binds God, they neglected to ask "which logic"? Like all Western theological claims, this one too flounders on an elementary empirical fact. Logic is not universal. The Buddhist logic of *catuskoti* or the Jain logic of *syadavada* incorporates systems of logic which are not 2-valued, and not even truth-functional. Therefore, the theorems of mathematics are at best cultural truths relative to a cultural biased axiom set[32] and a cultural biased logic: in other words, mathematical theorems may be Christian truths, but they are far from being universal truths.

Critically re-examining Western math

So, it is important to carefully examine the "superior" way of doing 2+2=4 as metaphysics, the links of this metaphysics to church theology, and whether that really is a superior way of doing mathematics or just an inferior misunderstanding which should be abandoned. The "superior" way to do 2+2=4 is to prove it as a theorem starting from Peano's axioms. This brings in infinity by the backdoor (a computer can never do Peano arithmetic). That infinity links to the Western theology of eternity.

Such a critical examination was never done. Fed on the story that math is universal, or that Western math is "superior", most people wrongly think that their empirical understanding of the number 2 as a natural abstraction derived from observations of 2 dogs, 2 cats etc. is the one used in present-day math. These are typically people who never heard of Peano's axioms.

Recently, a serious challenge to the Western philosophy of mathematics as metaphysics did come up, through my philosophy of zeroism, Western mathematicians, their followers, and Western philosophers of mathematics are reduced to silence: for there is no answer to these potent objections to Western superstitions. More people need to be informed about about this cocktail of practical value and religious belief which indoctrinates millions of children into religious biases, though they come to school to learn the practical applications of mathematics.

As a matter of fact, Western education, originally designed for missionaries, makes it almost impossible for anyone to carry out such a critical examination. The ordinary way of doing 2+2=4 is to point to two pairs of apples to make four apples. This is erroneous on the "perfect" or "superior" Western way of deducing 2+2=4 as a consequence of Peano's axioms or set theory. Most people never learn this

---

32 The continuum is a culturally biased axiom set as pointed out by Naqib al Atas, for Islamic philosophers preferred an atomic number system. See, further, C. K. Raju, "Teaching Mathematics with a Different Philosophy. 1: Formal mathematics as biased metaphysics". *Science and Culture* **77**(7–8) (2011) pp. 275–80. http://www.scienceandculture-isna.org/July-aug-2011/03%20C%20K%20Raju.pdf.

"perfect" and "superior" way, because it is too complicated to teach axiomatic set theory at the high-school level. Thus, when it comes to mathematics, for even a simple thing like 2+2=4, the Western-educated have no option but to rely on authority which is located in the West.

It does not strike people that it is possible to separate the practical value of mathematics from its add on metaphysics. Computer arithmetic is adequate for all practical tasks, but a computer can never do Peano arithmetic since that "superior" and "perfect" way of adding 2 and 2 involves an underlying metaphysics of infinity. That is a computer can do ordinary integer arithmetic to some point beyond which we don't care; a Java program will give a negative number only if we add 2 billion + 2 billion. Moreover, the limit of 2 billion can be extended. But "perfect" Peano arithmetic is impossible for a computer.[33]

What is theoretically needed to apply calculus to science?

So far as practical applications of mathematics to science are concerned, we have seen that the continuum is a redundant piece of metaphysics. However, the theoretical defects in the Western misunderstanding of the Indian calculus, even from within formalism, were exposed long ago. On university-text calculus, a differentiable function must be continuous, so a discontinuous function cannot be differentiated. However, long before the axiomatisation of set theory, which supposedly made this approach to calculus "rigorous" by giving an acceptable basis to the continuum, Heaviside was merrily differentiating discontinuous functions in his operational calculus, for the need to do so arises in science and engineering. The formalised version of Heaviside's theory is known as the Schwartz theory of distributions. This permits a discontinuous function to be infinitely differentiated. So what exactly is the definition of the derivative one must use in physics? The one on which a discontinuous function is not differentiable, or the one on which it is infinitely differentiable? "Choose what you like" is the typical response of a formal mathematician. This may sit well with the belief that mathematics is metaphysics, but the slightest reflection shows that this "choose what you like" makes physics irrefutable, hence unscientific in a Popperian sense.

Worse, we *cannot* choose what we like, since *both* definitions are inadequate. The inadequacy of the Schwartz theory was established even before its birth, for products of distributions arise in the S-matrix expansion in quantum field theory, and such products are not defined on the Schwartz theory.[34] Many equations of physics, such as the equations of fluid dynamics, or of general relativity, are nonlinear partial differential equations. Shocks arise naturally, and represent a (hyper)surface of discontinuity. If we use university-text calculus, based on the continuum, then the derivative of a discontinuous function is not defined, so the "laws of physics" break down, as for example, in Stephen Hawking's creationist claim that a singularity represents the moment of Christian creation when the "laws of physics" break down.[35] If we use the Schwartz theory, then derivatives are define, but not products, so there is again a problem. (A similar problem arises in quantum field theory, and is known as the renormalization problem.)

To be sure there are umpteen definitions of the product of Schwartz distributions, including one that I

33 The theory. is explained in my classroom notes on computer programming, put up at http://ckraju.net/hps2-aiu/ints.pdf and http://ckraju.net/hps2-aiu/floats.pdf.

34 C. K. Raju, "Distributional Matter Tensors in Relativity." In: *Proceedings of the 5th Marcel Grossmann meeting on general relativity*, D. Blair and M. J. Buckingham (ed), R. Ruffini (series ed.), World Scientific, Singapore, 1989, pp. 421–23. arxiv: 0804.1998.

35 A detailed analysis of Stephen Hawking's singularity theory and its linkages to Christian theology may be found in *The Eleven Pictures of Time* cited earlier.

proposed long ago, using non-standard analysis.[36] The problem is which one to choose. There are two ways of deciding: (1) consult an authoritative Western mathematician, and (2) choose the definition which best fits the widest possible practical applications, where the "best fit" is to be decided by empirical proof or an empirical test of the consequences. Most mathematicians will prefer the first method, for formal mathematics, like theology, is all about authority. But this method does not suit science, so I prefer the second one of relying on practical applications. This selects out my definition, which is the only one which works for both classical physics[37] and quantum field theory.[38]

The interesting thing is this. While my definitions initially used non-standard analysis, it can all be done just as easily using a non-Archimedean ordered field.[39] That brings us back full circle to the original Indian understanding of the calculus as best suited even to present-day science. So, we should discard formalist mathematics as merely a biased metaphysics of infinity, based on Western notions of eternity, and a Western misunderstanding of Indian calculus which does not properly fit applications to current science.

---

36 C. K. Raju, "Products and compositions with the Dirac delta function", *J. Phys. A: Math. Gen.* **15** (1982) pp. 381–96.

37 C. K. Raju, "Junction conditions in general relativity", *J. Phys. A: Math. Gen.* **15** (1982) pp. 1785–97. See also "Distributional matter tensors in relativity" cited above for new shock conditions. For a more recent exposition of a link between renormalization theory and a modification of Maxwell's equations, see C. K. Raju, "Functional differential equations. 3: Radiative damping", *Physics Education* (India), **30**(3), article 7, Sep 2014. http://www.physedu.in/uploads/publication/15/263/7.-Functional-differential-equations.pdf.

38 C. K. Raju, "On the square of x^{-n}". *J. Phys. A: Math. Gen.* **16** (1983) pp. 3739–53. C. K. Raju, "Renormalization, extended particles and non-locality", *Hadronic J. Suppl.* **1** (1985) pp. 352–70.

39 "Renormalization and shocks", appendix to *Cultural Foundations of Mathematics*, cited above.