A proposed experiment to test theories of the flyby anomaly

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Abstract

We use Lorentz covariant retarded gravitation theory (RGT), without simplifications, to validate the earlier calculations for the flyby anomaly as a gravitational effect of Earth's rotation at the special relativistic ($\frac{v}{c}$) level. Small differences persist between the theoretical predictions of RGT and the data reported by Anderson et al. That reported data, however, is not direct observational data but consists of un-modeled residues. To settle doubts, we propose a 3-way experimental test to discriminate between RGT, Newtonian gravitation (no flyby anomaly), and Anderson et al.'s formula. This involves two satellites orbiting Earth in opposite directions in the equatorial plane in eccentric orbits. For these orbits, Earth's rotation should not affect velocity on (1) Newtonian gravitation and (2) the formula of Anderson et al. However, (3) on RGT, one satellite gains and the other loses velocity, by typically a few cm/s/day, which is easily measurable by satellite laser ranging.

1 Introduction

The flyby anomaly is a tiny anomalous effect noticed when five spacecraft (four from NASA and one from ESA) flew past Earth using the technique of Earth-gravity assist to gain or lose heliocentric velocity. However, there was an anomalous gain or loss of geocentric velocity (at perigee).[1] This was minute (a few mm/s) compared to the perigee velocities of the spacecraft (~ 10 km/s). Nevertheless, a net gain or loss of geocentric velocity, howsoever small, is impossible on Newtonian gravitation. The effect cannot just be set aside since spacecraft velocities were tracked to an accuracy of 1 mm/s, and Doppler ranging data confirmed a small but definite anomalous change in velocities at perigee.

Indeed, as Anderson et al. argued, the extrapolated pre-perigee trajectory did not accurately fit the post-perigee trajectory. The NASA orbit determination program they used, for extrapolation, incorporated a variety of factors such as Earth atmosphere, ocean tides, solid Earth tides, the effects of spacecraft charging, magnetic moments, Earth albedo, solar wind, even general relativistic effects. (On general relativity the Kerr geometry around a rotating object results in frame dragging, but that effect of Earth's rotation is a million times smaller than the flyby anomaly.)

Something unexpected happened at perigee.

1.1 The theoretical prediction

It may still be debated whether such a minute effect was actually observed. However, it is theoretically predicted by retarded gravitation theory (RGT) as a $\frac{v}{c}$ gravitational effect of Earth's rotation.[2]

1.2 RGT

RGT is a Lorentz covariant reformulation of Newtonian gravitation. Since Lorentz covariance is a theoretical necessity,[3] RGT is superior to Newtonian gravitation, irrespective of how it compares with general relativity theory (GRT). Pragmatically, it is surely more *convenient* than GRT for the many body problem, which is easily do-able in RGT. The flyby anomaly itself is an example of a problem hard to solve with GRT—in a century, no one used GRT to predict special-relativistic effects of the kind easily brought out by RGT. Lorentz-covariant RGT does *not* suffer[4] from the instabilities of naive retarded theories of gravitation, formulated in a pre-relativistic way, in the 19th and early 20th centuries, to explain the anomalous perihelion advance of Mercury.

In RGT, it is meaningful to speak of the gravitational force. But the force is required to be Lorentz covariant, so it cannot be purely position dependent, but must depend also on velocity, unlike Newtonian gravitation. Hence, on RGT, unlike Newtonian gravitation, the rotation of the Earth affects the motion of nearby spacecraft and satellites at the $\frac{v}{c}$ level.

1.3 The simplifications used

Our earlier calculation for the flyby anomaly used a simplified expression for the RGT force

$$F \approx \frac{k}{r^2}\left(\frac{X}{r} + \frac{V}{c}\right). \tag{1}$$

Here F is the 4-force acting on the particle at time t, $k = GMm$, m is the (rest) mass of the "attracted" particle, while M is the (rest) mass of the "attracting" particle, and V its 4-velocity at *retarded time* t_r. This is the time at which the backward null cone from the position of the "attracted" particle at t intersects the world-line of the "attracting" particle. X is the relative 4-position vector of the "attracting" particle, also at retarded time, and r is the corresponding retarded distance. As usual, c denotes the speed of light.

The earlier calculation involved a second simplification: it took r as just the instantaneous distance.

The results obtained with these simplifications were qualitatively correct: most of the gain or loss of energy takes place close to perigee, something no other theory has explained so far. The quantitative results too were very close, but there was no neat and exact fit to data. Thus, for Galileo's first Earth flyby (Galileo-1), the calculated gain on RGT was 5.96 mm/s compared to Anderson et al.'s reportedly observed figure of 3.9 mm/s. Likewise, for Cassini, the calculated loss was -3.2 mm/s compared to the figure of -2 mm/s reported by Anderson et al.

These calculated figures are so close that they strongly suggest that the RGT explanation is valid: the flyby anomaly is indeed a $\frac{v}{c}$ effect due to the rotation of the Earth, as predicted by RGT.

2 The revised calculation

Nevertheless, we have now redone the calculation using the full RGT force:

$$F = -\frac{kc^3}{(X.V)^3}X + \frac{kc^3}{(X.V)^3}\frac{(X.U)}{(V.U)}V. \tag{2}$$

Here F, k, c, X, V, are as before, and U is the 4-velocity of the attracted particle. Since the dot products $X.V$, $X.U$, $V.U$ are scalars or Lorentz invariant, the RGT expression for the 4-force is Lorentz covariant and can be used in any Galilean reference frame.

The full force law may be rewritten in a manner similar to (1) (but without any simplification) as

$$F = \frac{k}{r_r^2} * h_1 \left(\frac{X}{r_r} + \frac{V}{c} * h_2 \right) \tag{3}$$

where the notation r_r emphasizes the use of retarded distance, and

$$h_1 = [\gamma_v(1 + \frac{v}{c}\cos(w, v))]^{-3} \tag{4}$$

$$h_2 = \frac{1}{\gamma_v}\frac{1 + \frac{u}{c}\cos(w, u)}{1 - \frac{uv}{c^2}\cos(v, u)}. \tag{5}$$

Here $\vec{w}$, $\vec{u}$, $\vec{v}$ are the 3-vectors corresponding respectively to the 4-vectors X, U, V. Explicitly, $X = (ct,\ \vec{w})$, $U = \gamma_u(c,\ \vec{u})$, and $V = \gamma_v(c,\ \vec{v})$. Further, $\gamma_v = \gamma(\vec{v}(t_r))$ is the Lorentz γ factor for $\vec{v}(t_r)$, and $\vec{v}(t_r)$ and $\vec{u}(t)$ are the 3-velocity vectors respectively of the attracting particle (at retarded time t_r), and attracted particle (at current time t). Finally, $v = ||\vec{v}||$, $r_r = ||w||$, and $\cos(w, v) = \cos(\vec{w}, \vec{v})$ is the cosine of the angle between the 3-vectors $\vec{w}$, and $\vec{v}$.

For actual calculations, the terms $\frac{v}{c}\cos(w, v)$ etc. are conveniently calculated using

$$\frac{v}{c}\cos(w, v) = \frac{\vec{w}}{r_r} \cdot \frac{\vec{v}}{c} = \frac{1}{r_r c}\vec{w} \cdot \vec{v}, \tag{6}$$

$$\frac{u}{c}\cos(w, u) = \frac{1}{r_r c}\vec{w} \cdot \vec{u}, \tag{7}$$

$$\frac{uv}{c^2}\cos(v, u) = \frac{1}{c^2}\vec{v} \cdot \vec{u} \tag{8}$$

We continue to ignore the effects of the spacecraft on the Earth, as in the simplified calculation.[2] In this 1-body case, the functional (delay) differential equations of motion in RGT reduce to ordinary differential equations. We solved these for the six flybys as before using the well known DOPRI code. The initial data were obtained from NASA Horizons interface in state vector format, for a geocentric frame with Earth mean equator and equinox of reference epoch, as displayed below (Table 1).

Table 1: Initial data from NASA Horizons used for trajectory calculations. For each spacecraft the first row contains the Julian day number and Gregorian date. The second and third rows contain the position and velocity vectors in units of km and km/s.

Galileo-1	2448234.000694444	= 1990-Dec-08 12:01:00.0000
X = 2.473779667197228E+04	Y = 2.813889802133052E+05	Z = 6.905794938512032E+04
VX = −4.941514457055581E-01	VY = −8.870912489618423E+00	VZ = −1.967648249648285E+00
Galileo-2	2448965.083333333	= 1992-Dec-08 14:00:00.0000
X = 3.533466991876042E+04	Y = 1.725196573562905E+04	Z = 2.099339477769745E+04
VX=−6.357173143445935E+00	VY=−5.098983436463288E+00	VZ=−5.502425976370658E+00
NEAR	2450836.500000000	= 1998-Jan-23 00:00:00.0000
X = 3.228651780539691E+04	Y = 1.824150970351419E+05	Z = 8.220032102022604E+04
VX=−1.018551223818951E+00	VY=−6.596090066205002E+00	VZ=−2.513531686187036E+00
Cassini	2451408.541666667	= 1999-Aug-18 01:00:00.0000
X =−1.338142479401416E+05	Y = 5.520901141486376E+04	Z = 2.971150587397455E+04
VX= 1.421307835849869E+01	VY=−6.833512053283764E+00	VZ=−3.615965629625016E+00
Rosetta	2453434.208333333	= 2005-Mar-04 17:00:00.0000
X =−9.443459287333746E+04	Y = 3.821864878059387E+04	Z = 1.506916188567711E+04
VX= 4.622615071909428E+00	VY=−1.111383971910594E+00	VZ=−2.124678157771555E-01
Messenger	2453584.958333333	= 2005-Aug-02 11:00:00.0000
X =−4.061872527992503E+04	Y = 1.331170620795427E+05	Z =−7.266170750518485E+04
VX= 1.566976521677327E+00	VY=−3.612768891021895E+00	VZ= 2.457281285018382E+00

The geocentric velocity gain or loss, calculated using RGT, for the various spacecraft are shown in Table 2, and the graphs of detailed trajectories are in Fig. 1.

To summarise, we get back the original results because the use of two simplifications (use of instantaneous distance and a simplified formula) compensated for each other, so our earlier results were correct!

2.1 Discussion

All factors have now been taken into account, in the RGT calculation. This includes the oblateness of the Earth, which is a mere confounding effect, and does not add anything to the energy gain or loss at infinity. Even possible discretization errors have been checked by performing a more analytical calculation (Suvrat Raju, personal communication). So the above figures are valid predictions according to RGT. On the

Table 2: Velocity gain or loss for various spacecraft flybys. $\vec{v}_r$ is the velocity calculated on RGT, and $\vec{v}_n$ that on Newtonian gravitation. Column 2 shows the difference of scalar velocities. Comparing it with column 3 shows how much $\vec{v}_r$ and $\vec{v}_n$ differ in direction. The last two columns are the "observed" figures for the scalar difference of velocities (in mm/s) as reported by Anderson et al. and those calculated using their formula (9).

Spacecraft	$\|\|\vec{v}_r\|\| - \|\|\vec{v}_n\|\|$ (mm/s)	$\|\|\vec{v}_r - \vec{v}_n\|\|$ (mm/s)	"Observed"	Anderson et al.
Galileo-1	5.96	7.4	3.92	4.12
Galileo-2	7.97	7.98	−4.6	−4.67
Near	3.73	5.68	13.46	13.28
Cassini	−3.32	3.53	−2	−1.07
Rosetta	4.58	6.22	1.80	2.07
Messenger	6.85	9.17	0.02	0.06

other hand, it seems unlikely that figures so close to the stated values are simply wrong.

3 The formula of Anderson et al.

Now, Anderson et al., too, attempted to link the flyby anomaly to Earth's rotation, in another way, through the phenomenological formula

$$\frac{\Delta V_\infty}{V_\infty} = K(\cos\delta_i - \cos\delta_o), \tag{9}$$

where ΔV_∞ was the difference between the incoming and outgoing asymptotic velocity in a geocentric frame. (Conceptually, V_∞ is the hyperbolic excess velocity at infinity of an osculating Keplerian trajectory, so the difference ought to have been zero on the Newtonian theory.) Further, δ_i and δ_o were the declinations of the incoming and outgoing asymptotic velocity vectors. The constant $K = 3.099 \times 10^{-6}$ was expressed in terms of the Earth's angular rotational velocity ω_E (7.292115×10^{-5} rad/s), its mean radius R_E (6371 km) and the speed of light c by

$$K = \frac{2\omega_E R_E}{c}.$$

Anderson et al. do not attempt any theoretical explanation for how the Earth's rotation might affect the motion of spacecraft. This is in sharp contrast to RGT which readily explains the gravitational effect of Earth's rotation as a special relativistic effect.

Further, the formula (9) is wrong since it has no distance dependence. The Rosetta spacecraft in its second flyby showed no such flyby effect. Anderson et al. explain this was because it was very far away (height 5322 km) at perigee. However, their formula gives no indication of the distance at which it fails or why.

It is therefore paradoxical that the formula nevertheless provides a neat and seemingly exact fit with the data for the other six flybys, where the height of perigee varies all the way from 303 km (Galileo-2) to 2347 km (Messenger). **How could observations so neatly fit a wrong formula?**

3.1 Un-modeled residues

In fact the data fitted by Anderson et al. is not direct observational data, but consists of un-modeled residues. This reconstructed data involves various speculative estimates used in modeling, which can be tweaked. Undoubtedly the NASA orbit determination program is very sophisticated, but there are difficulties with the way Anderson et al. have used it.

3.1.1 Case of Galileo-2

For example, consider the case of Galileo-2. According to RGT, there should have been a velocity gain, not the velocity loss calculated by Anderson et al.'s formula. (This, incidentally, is a clear difference between the two theories.)

Further, the perigee of Galileo-2 was very low at 303 km. At this height, atmospheric drag may lead to significant velocity loss. These atmospheric effects at high altitude depend upon unpredictable factors such as solar storms. Hence, retrospective modeling of atmospheric drag is not reliable. Further, we are not informed about the exact details of this modeling by Anderson et al., but are just presented with a speculative estimate.

3.1.2 Case of Cassini

Again, a spacecraft is not a passive point mass but is a powered vehicle. In the case of Cassini, Anderson et al. note how effects of the flyby may have been masked by the effects of thruster firing near perigee. We have no information on the exact details of these thruster firings and have not taken them into account.

3.1.3 Case of NEAR

Anderson et al.'s formula is only for the difference of *scalar* velocities. However, the flyby affects not only the speed, but also direction: in our RGT calculations in some cases the norm of the vector velocity difference (column 3 of Table 2) is significantly larger than the difference of scalar velocities (column 2 of Table 2). In the case of NEAR this difference itself amounts to 1.95 mm/s, which is twice the extrapolation errors expected by Anderson et al.

Anderson et al. provide no way to estimate such change of direction. Further, even continuous observational data was not always available: as Anderson et al. note, in the case of NEAR there was a blackout of 3 hours 39 minutes in the data near perigee. In this time, the un-modeled change of direction at perigee could have easily been amplified by other effects, such as lunar and solar perturbations.

3.1.4 Choice of osculating orbital elements

We accept the observation by Anderson et al. that the post-perigee trajectory cannot be simply extrapolated from the pre-perigee Keplerian trajectory. But Anderson et al.'s calculation of the exact gain or loss of velocity at perigee depends upon the fitting of osculating orbital elements to pre-perigee and post-perigee data.

In contrast, in the above calculations with RGT, we consistently chose orbital elements several hours before perigee ("past infinity") based on NASA Horizons data. Because details of thruster firing etc. are not available in the ephemeris data, that osculating trajectory at past infinity is not necessarily the best fit: for that we may need to fit osculating orbital elements close to perigee. Table 3 shows the difference between our data and Anderson et al.'s choices. Had we fitted osculating elements to the RGT trajectory, (a better approximation to the real trajectory) close to perigee, that change of initial data could significantly change the velocity gain or loss on RGT.

That is, the gain or loss in Anderson et al.'s calculations also depends on the exact fitting of osculating orbital elements before and after perigee. This is not a trivial matter, since the real trajectory is subject to relatively huge effects such as those of earth oblateness, or solar and lunar perturbations etc.

Table 3: Incoming declinations of velocity vectors of various spacecraft as used in RGT calculations compared with the data of Anderson et al.

Spacecraft	NASA Horizons data (Table 1)	Anderson et al.
Galileo-1	-12.487	-12.52
Galileo-2	-34.027	-34.26
Cassini	-12.914	-12.92
Rosetta	-2.559	-2.81
NEAR	-20.636	-20.76
Messenger	31.964	31.44

3.2 Discriminating between the two theories

Such doubts about the un-modeled residues cannot be settled by mere retrospective arguments. The best way forward is by performing a fresh controlled experiment to test between the two theories.

Such a controlled experiment is also important to settle any residual doubts about whether the flyby anomaly, a fundamental departure from Newtonian gravitation, is actually observed.

We suggest one such experiment below.

4 The two-satellite experiment

First, we note that RGT is a perfectly general replacement for Newtonian gravitation, and not limited to flybys. It predicts theoretical departures from Newtonian gravitation which can be tested all the way from the laboratory to the galaxy. In particular, experiments to test the gravitational effects of Earth's rotation can be performed not only with spacecraft but also satellites. This is likely to be more cost-effective.

Such a test between RGT and Newtonian gravitation should also include a test of Anderson et al.'s formula. This can be achieved as follows. Anderson et al.'s formula (9) differs from RGT on the effects of Earth's rotation—this is already clear from the case of Galileo-2. But these differences are most prominent in the case of an equatorial orbit. Thus, the formula (9) involves a factor of $(\cos\delta_i - \cos\delta_o)$, where δ_i and δ_o are the declinations of the incoming and outgoing asymptotic velocity vectors. When the orbit is in the equatorial plane, $\cos\delta_i - \cos\delta_o = 0$, so there should be no flyby effect on their formula. In contrast, on RGT the velocity effect will persist in the equatorial plane. This suggests the following experimental design.

Suppose two satellites are established in orbits in the equatorial plane; one co-rotates with the Earth, and the other counter-rotates. On the Newtonian theory the sense of rotation should make no difference to the satellite orbit.

Suppose, further, that the orbits are highly eccentric, so that the approach to perigee can be regarded as effectively a flyby. On Anderson et al.'s formula, since the trajectory is in the equatorial plane, there should again be no flyby effect and no difference from the Newtonian theory.

However, on RGT, the velocity dependent force should accelerate the co-rotating satellite and decelerate the counter-rotating satellite even in the equatorial plane.

To calculate the amount of such gain or loss on RGT, for each orbit, we need the two state vectors, at one instant of time, say apogee. Since the orbit is in the equatorial plane, the z coordinate and velocity is zero. Let us suppose that the apogee is in the x-direction, so that, at apogee, the x-coordinate is just the apogee distance and the y coordinate is zero. Likewise the x velocity is zero at apogee, and the y velocity is, say, v_y.

To get some definite numbers, let us start off as usual with an initial Newtonian orbit. For this orbit, we take the perigee distance $r_p = 8500$ km corresponding to a perigee at an altitude of around 2129 km where the atmospheric drag may be neglected. Further, let us take the eccentricity to be $e = 0.5$. If a is the semi-major axis, $r_p = a(1-e)$, so that $a = 2r_p = 17000$ km. The time period of the orbit $T = 2\pi\sqrt{\frac{a^3}{\mu}}$ where $\mu = GM_E \approx 3.98 \times 10^{14}\text{m}^3\text{s}^{-2}$. Thus $T = 2\pi\sqrt{\frac{17^3}{3.98}} \times 10^2\text{s}$ $= 553.352 \times 10^2\text{s}$, or around 15.37 hours. Accordingly, we ran the simulation for around 33 hours, starting from apogee.

From the energy equation $\frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$, at apogee ($r = a$) we have $\frac{1}{2}v^2 = \frac{\mu}{2a}$ or $v = \sqrt{\frac{\mu}{a}} = \sqrt{\frac{3.98\times10^{14}}{17\times10^6}} = 0.4838 \times 10^4\text{m/s} = 4.838$ km/s. This velocity is in the y direction, either positive or negative, depending upon the sense of rotation of the satellite in its orbit. Thus, initial data are (in units of km, and km/s)

- case 1:
  - position: (17000, 0, 0),
  - velocity: (0, 4.838, 0), and

- case 2:
  - position: (17000, 0, 0),
  - velocity: (0, -4.838, 0).

On Newtonian gravitation, the orbit stays on the initial ellipse. On RGT, it deviates slightly, as follows. In case 1, using a best linear fit, the satellite gains about 4.2 cm/s velocity in 33.33 hours, or about 3.02 cm/s/day. In case 2, it loses velocity at about the same rate. Such a difference is easily measurable with modern satellite laser ranging systems which have millimeter accuracy.

These are just illustrative figures: two counter-rotating satellites in exactly the same orbit would obviously collide! The design will need to be optimised for an actual experiment.

References

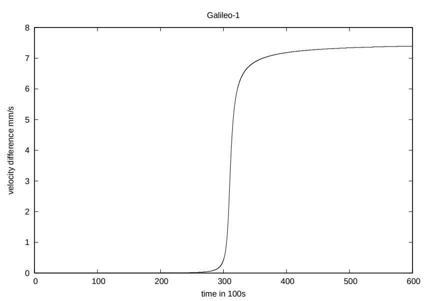
[1] John D. Anderson and et al. Anomalous orbital-energy changes observed during spacecraft flybys of earth. *Physical Review Letters*, 100:091102, 2008.

[2] C. K. Raju. Retarded gravitation theory. In Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro, editors, *Sixth International School on Field Theory and Gravitation*, pages 260–276, New York, 2012. American Institute of Physics. 1102:2945v3.

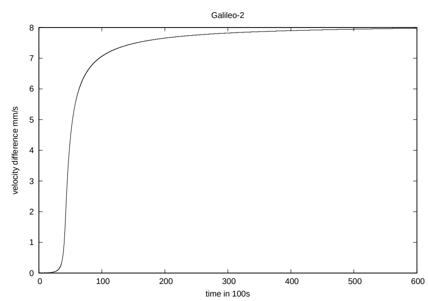
[3] C. K. Raju. *Time: Towards a Consistent Theory*, volume 65 of *Fundamental Theories of Physics*. Kluwer Academic, Dordrecht, 1994.

[4] C. K. Raju. Functional differential equations-4. retarded gravitation. *Physics Education*, 31(2), April-June 2015. `http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-(2).pdf`.

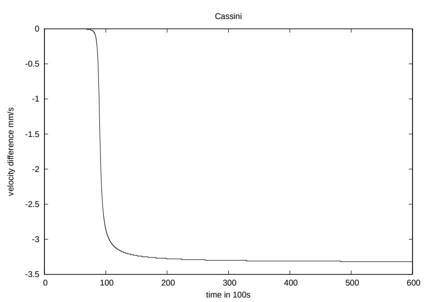
Figure 1: Graphs of velocity gain or loss for the six flybys. In (c) there was a velocity loss



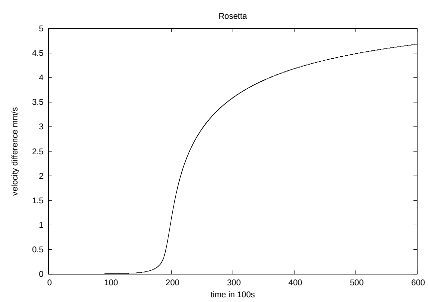
(a) Galileo-1



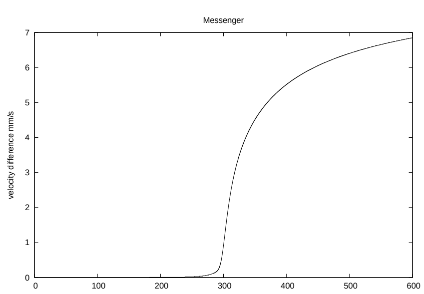
(b) Galileo-2



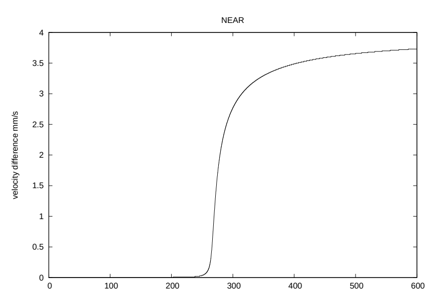
(c) Cassini



(d) Rosetta



(e) Messenger



(f) NEAR