Ganita vs formal mathematics

Re-examining the philosophy of mathematics, its pedagogy, and the implications for science

REVISED DRAFT

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Preface

This book collects together various strands of my long-standing critique of formal mathematics, and explains the alternative of ganita and its advantages for both pedagogy and a better science. The critique, the details of the pedagogy, as also the applications to science, articulated across the past two decades, are widely scattered. Hence, it is important to collect everything together in one place, to make it easy to understand it as a coherent whole, or at least get an overview of the entire effort.

This is especially important since colonial education has rendered the vast majority mathematically illiterate, and taught them to trust the wrong “experts” such as professors of mathematics who are trained solely in formal mathematics (Western ethnomathematics), and alienated from other ways of doing math. Since I taught formal mathematics to post-graduate students for several years, I understand that formal mathematicians are naturally unhappy to see their subject destroyed, but they have been unable to articulate, in public, a single reasoned response to my critique in two decades. Effectively formal mathematics is dead, and is not going to come back to life by launching personal attacks against me.

Some of my earlier articulations were not spelt out clearly enough. For example, the use of Brahmagupta’s avyakta ganita (polynomial arithmetic) by Indians to sum infinite series, was so far explained in detail only in my lectures, and only once in print. This is a key feature of the Indian calculus: for Brahmagupta’s polynomial arithmetic is “non-Archimedean” [no connection to Archimedes, just the usual Western chauvinistic terminology]. Hence, it brings in infinities and infinitesimals, hence the absence of unique limits. Because of my long familiarity with this kind of arithmetic, from non-standard analysis, I inadvertently assumed most people would easily understand this, but have realized that they find it very puzzling. In fact, it is this idea which gives the name "calculus without limits" to my new course, which makes the point that neither formal (=unreal) real numbers, nor limits are essential to the calculus.

Again, the use of numerical methods in the Indian calculus, starting from Aryabhata, is commonly confounded with “numerical analysis”. This is, of course, merely the confused reaction of people who are educated to believe in the binary of symbolic vs numerical, and whose only encounter with numerical methods is in numerical analysis. In fact, numerical analysis is concerned with efficient numerical methods, whereas calculus without limits is concerned with core concepts, to remedy the confusion arising from the days of Newton and Leibniz. Note that, in calculus without limits, these concepts are based on non-Archimedean arithmetic, not the Archimedean arithmetic of real numbers used not only in calculus *with* limits but also in current numerical analysis.

Again, some people wonder where are the symbols for the derivative and integral in the Indian calculus. Their naïve expectation is that, since this was the way in which *they* learnt the calculus, so this is the only way to do calculus. This is as wrong as pope Sylvester’s 10^{th} c. laughable reaction of building a special abacus for Indian arithmetic (“Arabic numerals”) on the wrong assumption that an abacus is the only way to do arithmetic. In fact, since both the derivative and integral in the current teaching of the calculus are defined as limits, these symbols are not used in the calculus without limits.

They are not needed. All practical applications of the calculus, from the simple pendulum to rocket trajectories, are still obtained by the method of numerically solving differential equations initiated by the $5^{th.}$ c. Aryabhata. The khanda-jya (or finite difference) was used for interpolation/extrapolation, as the “unit rate of change”, on the very elementary rule of three, which one learns in primary school. While finite differences are conceptually quite clear, as even Karl Marx opined, since they do not require the use of limits, there is the issue of non-uniqueness. This issue is handled by the philosophy of zeroism.

The derivative is not needed except as a compact notation for writing down differential equations. There is no need for the integral sign either, since the integral as anti-derivative is just the solution of a differential equation. This immediately frees the integral from the limitation to elementary functions, in the usual fat college calculus texts. Thus, the whole junk-part of calculus, memorizing formulae for derivatives, and learning tricks for symbolically evaluating integrals (solely of elementary functions) is rightly thrown out of the window. Obviously, this way of doing things (by throwing out of the ballast) enhances the practical value of what students learn, for this is the way calculus is used in actual applications (such as calculating rocket trajectories), together with all the attendant complexities, of air resistance etc., for one no longer has the luxury of being limited to the elementary functions. (For those who still insist on the ritualistic knowledge of symbolic integrals and derivatives, of elementary functions, the use of a free and open source computer program such as MACYSMA/MAXIMA should be reassuring, for there is no merit in memorizing those formulae or tricks.)

Then, there is the question of the sum of infinite series. How is that obtained without limits? How did Indians manipulate and handle such infinite series? The simple answer is that, with non-Archimedean arithmetic, though limits are non-unique, one can arrive at a useful answer through zeroism by discarding infinitesimals, exactly as the 15th Nilakantha did when he first summed the infinite geometric series in his *Aryabhatiyabhashya*. This is the natural and simpler method, which is what makes calculus without limits pedagogically so useful, because the difficulties of the calculus with limits are a major stumbling block for most students, for they never learn limits properly even from the perspective of formal mathematics, and forever remain conceptually confused.

However, the issues are *not* limited to pedagogy. There is, for example, the question of the limitation of calculus with limits, that it does not allow discontinuous functions to be differentiated. Such discontinuities naturally arise in applications of calculus to the physics of, say, shock waves, where one must understand a differential equation at or across a discontinuity.

The Schwartz theory allows a function with a simple discontinuity to be infinitely differentiated. But, it is an inadequate substitute for the college calculus since it is linear, so that the non-linear differential equations of physics (whether of fluid dynamics or of general relativity) still do not make sense at a discontinuity. The *ab initio* use of non-Archimedean arithmetic, as in the Indian calculus, means that there is no need to go first to the Schwartz theory then to non-standard analysis, to handle products of Schwartz distributions, as I once did. The same mathematical problem, of course, arises as the famous renormalization problem of quantum field theory. As such, calculus without limits is able to naturally resolve many of the severe mathematical problems arising from the limitations of calculus with limits.

Though calculus is basic to Newtonian mechanics, and Newton is regarded as the “discoverer” of the calculus, on the obnoxious (and genocidal and epistemicidal) doctrine of Christian discovery, the fact is Newton did not properly understand the calculus, and on my epistemic test, that is proof that the calculus was stolen (from India). Newton’s failure to understand calculus is clear from his conceptually confused notion of the derivative as a fluxion. This "fluxion" was based on the incoherent notion that time itself flows, a notion which the 8th c. Sriharsa long ago made much fun of (and is today known as McTaggart's paradox after McTaggart "discovered" it, on the same doctrine of Christian discovery). On the same incoherent notion, Newton did not define equal intervals of time, imagining that time flows with an "even tenor" (as known to God). This internal incoherence directly led to the fall of Newtonian mechanics, and its replacement by special relativity, by Poincare, in 1904 (*not* Einstein in 1905).

Since Newton's "laws" of motion come as a package deal, together with his "law" of gravitation, the fall of Newtonian mechanics forces a correction of Newtonian gravitation. Instead of general relativity, which could not solve the many-body problem in a century, my retarded gravitation theory suggested a simple correction to Newtonian gravitation to make it consistent with special relativity, in a way which enables solution of the many-body problem. This would enable one to at least explore the possibility that the failure to explain galactic rotation curves, without accumulating hypotheses about dark-matter halos, is due to the failure of Newtonian gravitation beyond the solar system, for Newtonian physics was back-calculated from ancient observations of planetary motion.

Apart from collecting together earlier articulations, some aspects are entirely new to this book. For example, while my retarded gravitation theory has existed for a decade, a completely new set of formulae (RGT 2.0) is derived and used here for the first time, by applying it afresh to hyperbolic orbits such as the flyby anomaly, and possibly to Oumuamua etc. The potential advantage of RGT2.0 is that this works in ways quantitatively impossible in general relativity.

There are other aspects of my writings which, though stated earlier, were not explained in detail, on the same mistaken assumption of the knowledgeability of the other. For example, the point that most present-day school mathematics is historically of Indian origin, and Europeans imported it from India between the 10th and 16th c., before giving back a modified version through coloniual education. This *history* is important to help understand the *philosophical* difficulty that Europeans had with imported Indian ganita, starting from the notion of zero, in arithmetic, and going on to imported trigonometry, calculus and probability, and the resulting consequences for formal mathematics, as taught in universities today.

The crux of the matter is, of course, that, contrary to the superstition of the colonised, mathematics is not universal, but varies with culture. Hence, even when it is appropriated by another culture, it changes, usually for the worse, as happened when Europeans appropriated Indian ganita. Indian ganita was practical. In contrast, mathematics in the West was always tied to religious beliefs since Plato. Subsequently, the post-Crusade church appropriated "Neoplatonic" (Egyptian mystery) mathematics, attributed it to an unknown Greek "Euclid", and brazenly "reinterpreted" it for its own political purposes of using "reason" to convert Muslims. Therefore, it insisted that the key element of mathematics is proof (which the church needed for persuasion). People failed to notice the church doublespeak about "reason": that this "proof" involves metaphysical reasoning MINUS facts. The prohibition of facts (as in formal mathematics) suited church dogma, which is frequently contrary to facts, but not the practical applications of mathematics to science and engineering.

The church turned mathematics into metaphysics, to suit its theology of reason. Though the falsehood of the *myth* about formal proofs in "Euclid" was exposed by the end of the 19th c. the church *superstitions* about the purported "superiority" of axiomatic proof are deeply ingrained in the Western psyche, and a key part of the rant of civilizational superiority which accompanied colonialism, and still carries on. The persistence of this church superstition resulted in the invention of formal mathematics in the 20th c, despite the exposure of the absence of formal proofs in "Euclid".

However, while Indian ganita was imported for its practical value by Europeans, its philosophy did not fit into Western religious myths and superstitions about math (for example the myth that math is eternal truth hence exact). To make it compatible with its indigenous myths and superstitions, the West added a layer of metaphysics to the imported Indian ganita, in the form of a metaphysics of infinity (aligned to church dogma of eternity). Together with customarily false Western history of science, this package was returned through colonial education, and declared as "superior" in exactly the same utterly irrational way in which racists under apartheid declared whites as superior. Since colonial education still teaches uncritical imitation, the colonised imitate it today. And, because colonial education manifestly made most of the colonized ignorant of mathematics, they are superstitiously terrified of any change, long after colonialism supposedly ended.

The Western metaphysics, added on to Indian ganita, culturally suited the West, but it did not enhance the *practical* or *epistemic* value of mathematics in any way, though it added enormously to the difficulty of mathematics, and to its *political* value for the coloniser. Therefore, eliminating that metaphysics of formal math and reverting to the original normal math, or ganita, makes math easy. The validity of the metaphysics (of infinity), added by the West to math, cannot be decided empirically, for the validity of any metaphysics can only be decided by authority. For example, calculus is taught with real numbers whose axioms (and the axioms of the underlying set theory), not the non Archimedean arithmetic with which it originated, purely on account of Western authority; only Western authority is *de facto* permitted to lay down the axioms. Consequently, it is no surprise that this Western metaphysics of infinity is colored by Western prejudices about eternity, shaped by church political dogmas of iniquity.

Since math is used for science, this allows a religious bias to be injected into science. For example, the use of real numbers for calculus already forces a view of time as a linear continuum, which has a pro-Christian, but anti-"pagan", anti-Hindu, anti-Buddhist, and anti-Islamic bias. But colonial education has ensured that the colonized are too ignorant of mathematics, especially real numbers, and too alienated from their own traditions, even to understand this bias. From that position of ignorance, they are dead certain that "science is at war with all religion", based usually on the single, unverified *story* of Galileo. It is hard to shake such superstitions of ignorant people, but that is what makes this "science" such a great weapon for religious propaganda, for the colonized simply cannot imagine or believe that science can be manipulated, though mathematics, in support of a particular religion. They may swear by secularism, but cannot ensure the religious neutrality of the kind of math taught as a compulsory subject in school. They may swear by 'scientific temper', but "science" for them only means superstitious faith in Western authority, and implicit trust in scientific journals, which decide truth by the age-old church method of secretive refereeing, having rejected the Indian tradition of public debate as "inferior".

Specifically, a singularity is a point where the college calculus fails, so the differential equations of physics *prima facie* fail to make sense at a singularity. From before my book the *Eleven Pictures of Time*, I have long tried to explain how the celebrated scientist Stephen Hawking was sold for his interpretation of a singularity as a unique moment of creation in accord with church theology. But the West knows how to play on the ignorance of the colonized, and their foolish trust in the authority of the colonizer, who so manifestly exploited them for centuries. In a recent article on the award of the Nobel Prize to Roger Penrose, I tried to bring out some of these issues. In contrast to formal mathematics, ganita is religiously neutral. Therefore, rejecting the metaphysics of formal math and reverting to religiously neutral ganita results in a better science, and secular education.

Over the years, the telling of various part of this counter-story has evoked a variety of objections. A common objection used to support formal math is to say "it works". Noticeably, this objection is often raised by people who don't understand mathematics (and formal mathematicians who, when pressed, will admit they do not understand its practical applications). Therefore, neither knows exactly *what* works.

Formal math does NOT work. What actually works is ganita (normal math), NOT formal math, for everything from bridge building to calculating rocket trajectories. The slightest thought should make it obvious: metaphysics cannot add scientific value, though it can easily add political value. My scattered responses to such arguments have not only been collected in one place in this book, they have been sharpened. For example, that something being globally accepted (as in the globalisation of colonial education) does not make it superior, or universal, as in the case of the inferior Western calendar which is globally accepted, but is manifestly inferior, and also does not suit Indian purposes. The time has come the write the obituary of formal mathematics, for future generations, even if so many in the present generation imagine it is still alive.

Of course, in its present form, this is still a draft of the book, because of the serious health issues I have had during the Fellowship. Despite these challenges, the important thing is that the difficult tasks including those of planning and organizing the book and drafting it have been completed. Also completed is the very difficult task of establishing the relevance to science in a simple and comprehensible way by working out the new formulae for RGT2.0. Pedagogically, school teaching requires a text book, and the task of producing a school text for Rajju Ganita has also been completed, though the text book for teaching calculus without limits is yet to be completed. The remaining tasks, for completing the present book, are only the relatively easy ones of elaborating, summarising, eliminating repetition (where not needed), uniformity in typesetting, formatting, etc.

For the purposes of my final seminar at the IIAS, because the book may seem very complex and radical for many people, detailed synopses of the book and of each chapter are provided in three layers. The first layer is for the Twitterati with limited attention span: in 21 words, or less than 140 characters. The second layer provides an overview for those who would like to know a little more about the radical assertions in this book such as the church invasion of mathematics: both of its history and philosophy. The third layer is for those who want to peruse the detailed arguments, and follow the references to my earlier articulations. The footnote to each chapter heading provide a reference to the *online* material actually written up, though in slightly different contexts, which most closely corresponds to the contents of that chapter.

Since I am physically able to type only with my left hand, at the time of finalising this preface, there may by typos and even howlers due to the use of dictation software.

Ganita vs formal mathematics

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Outline: 1. Ganita (गणित) differs from (formal) math, 2. it makes math easy, and 3. makes science better. 4. This is an obituary of formal math.

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Preface

Part 1: Introduction

1. **Colonial education as church education, and the propaganda of civilizational superiority**
   Colonial education (including university education) was church education designed to create missionaries with a missionary mindset. It was imposed to prevent revolts by making the educated (colonised elite) mentally submissive (and amazingly receptive to Western propaganda). This goal was explicitly clarified (for the case of the British poor) by the little-known Macaulay 2.0. The trick was to sell this church education to the colonised by declaring it as "superior", using the dogma of Western civilizational superiority (as in the well-known Macaulay v1.0). This dogma *mutated* from the earlier dogmas of racist and religious superiority. While the dogma of racist superiority has been recently hotly contested, the colonized, as a result of colonial education, have accepted and internalized the dogma of Western civilizational superiority, especially in science. All three dogmas of "superiority" were based on the same false and concocted Christian chauvinist history of science (with minor variations). The politics of Christian chauvinistic history dates back to the 4^{th} c., to Eusebius and Orosius. But the Christian chauvinist history of science went ballistic during the Crusades, when all scientific knowledge in Arabic texts was appropriated by attributing it to early "Greeks", then regarded as the sole "friends of Christians". Hence that knowledge was deemed a Christian inheritance which enabled its translation. During the "Age of discovery", the genocidal "doctrine of Christian discovery" was used to morally justify the appropriation of vast lands and labour (slavery). But it is little known that the same doctrine was used also to appropriate a great variety of indigenous scientific knowledge, such as heliocentricity today attributed to Copernicus, or calculus today attributed to Newton and Leibniz. The colonized never checked that false history, and colonial education has made them too ignorant of science to be able to do so: there is not a single department of the history and philosophy of science in any of the thousands of Indian universities even 75 years after independence. Instead, the colonized learnt to worship these false gods of science, from Newton to Stephen Hawking. Hence, they strongly *resist* doing any checks on their false gods today, given their created mindset of unflinching loyalty to the master.

2. **Ganita versus formal mathematics: an outline**

   Part 1: History. Most present-day school mathematics (arithmetic, algebra, trigonometry, calculus, probability and statistics) was actually imported by Europe from India, as ganita, for its practical value, between the 10th and 17th c. Europeans struggled to understand it; they made a long series of hilarious blunders, across centuries while trying to understand this imported knowledge (starting from elementary arithmetic algorithms and zero). By the 20^{th} c. they assimilated the ganita of calculus etc. into the Western religious understanding of math, by adding junk metaphysics (of "real" numbers, measure theory etc.) of nil practical value. Combined with systematically false history ("Euclid did some special geometry", "Newton invented calculus", etc.), the whole package was returned as "superior" through colonial education, and that is how math is currently taught in schools and universities today.

   Part 2: Philosophy. Misrepresentation is a common technique of propaganda. Utterly false claims are made, even today in Indian school texts, that Indian ganita lacked reasoning or lacked proof. This involves a church doublespeak about “reason”. During its Crusades against Muslims, the church developed the Christian theology of reason, to compete with the Islamic theology of reason. This Christian theology of reason (of Aquinas and schoolmen) involved metaphysical reasoning, *prohibiting* facts or empirical proofs, because facts are so often baldly contrary to church dogmas such as virgin birth. The church declared this politically convenient method of reasoning *without* facts as “superior”, and falsely attributed its origin to the “Greeks”, such as “Euclid”. Due to the then church hegemony over the European mind, this method of axiomatic reasoning, or metaphysical reasoning MINUS facts, was blindly accepted as “superior”, by the best European minds, and is still a key aspect of formal math today. Since the church then was interested solely in persuasion, not in scientific applications of math, (metaphysical) proof was declared also as the key objective of formal math, as it is today. This was incoherently combined with persistent earlier superstitions about math, from Plato’s exposition of Egyptian mystery geometry, which declared mathematics arouses the eternal soul since it has eternal truths, hence must be exact, and is of spiritual or aesthetic importance. Formal math is still just declared “superior”, and taught worldwide today. In contrast, ganita explicitly accepts empirical proofs as also deductive proofs, as does science, but focuses on the practical value of *inexact* calculations, as does science. This is superior to proving formal mathematical theorems which are *not* even valid knowledge in the real world. For example, the Pythagorean theorem is *not* valid knowledge on the curved surface of the earth (or anywhere in the cosmos). Nor can it even be regarded as valid *approximate* knowledge. To be useful, an “approximation” needs an error estimate, but the theorem purports to be *exact* truth, hence comes without any error estimate, which is available only from the two “Pythagorean” *calculations* known to Indian ganita, since the sulba sutra, but improperly known to Europeans until the 18th century.

Part 2: The false history and superstitions of Western (formal) mathematics

3. False history of math-1: Pythagoras, Euclid, and geometry

The claim of Western civilizational superiority in mathematics is anchored on the myth of Pythagoras and his theorem, for which there is nil evidence, *and* much *counter* evidence. Western historians make up for the lack of evidence by “myth jumping”: they provide “evidence” for the myth of Pythagoras by jumping to the myth of Euclid, for whom, too, there is nil evidence and much counter evidence. Finally, in desperation, they jump to the myth of the book purportedly written by Euclid, as some kind of solid proof. However, the “Euclid” book was just brazenly “reinterpreted” by the church in support of “church (metaphysical) reasoning”. In actual fact, it is a book on Platonic mystery geometry (it has diagrams), but does NOT have a single axiomatic proof. After centuries of gullibility, and an amazing collective failure to read the text, the absence of axiomatic proofs in the book was finally publicly admitted towards the end of the 19th century, by Dedekind, Russell and Hilbert. Laughably, though they denied the myth, all three still accepted the church *superstition* of the “superiority” of axiomatic or metaphysical “church reasoning”, which dodges facts. Even more laughably, the myth of axiomatic proofs in the “Euclid” book still persists among the best Western historians, despite its public exposure, and is used by Greediotic historians like Gillings, Needham, Clagget etc., and in Indian government’s school math texts. All of them use the Western *myth* to dodge facts about the “Euclid” book, *and* simultaneously press the claim of Western civilizational (and racist) superiority in math. The written rule of the current Indian government is that Indian students should do likewise. Because the Pythagorean *theorem* is invalid knowledge, even as an approximation, and inferior to the two “Pythagorean” *calculations*, known to Indian ganita, this belief in the “superiority” of theorems led to huge European navigational disasters from the 16th to the 18th centuries.

4. False history of math-2: Newton, Leibniz, and the calculus

The calculus originated in India with the fifth century Aryabhata, as a numerical technique to solve differential equations, to calculate precise sine values (precise to the first sexagesimal minute). These values were all-important then for navigation, to determine the radius of the earth, latitude, longitude, and, in the case of backward Europeans, loxodromes. The Indian calculus technique was further enhanced by the non-Archimedean arithmetic (avyakt ganita, polynomial arithmetic) of the 7th c. Brahmagupta, who also initiated the use of higher order polynomials for interpolation/extrapolation (“Stirling’s formula” etc.). This later led to the infinite series used by the Aryabhata school in Kerala, culminating in “trigonometric” values precise to the third sexadecimal minute (about nine decimal places). European governments fully recognized their own backwardness in navigation, and repeatedly offered huge prizes for a solution of their navigational problem, culminating in the British Parliament's legislated prize of 1711. Consequently, in the 16th c., Cochin-based Jesuits stole this valuable knowledge available in Indian math texts, for its use in navigation and astronomy. This stolen knowledge was later shamelessly attributed to various Europeans (such as Tycho Brahe, Clavius, Newton, Leibniz) on the genocidal "doctrine of Christian discovery". On my epistemic test, those who steal knowledge, like students who cheat in an exam, often fail to understand it. Clavius stole precise Indian trigonometric values but failed to use then to measure the earth. Newton and Leibniz failed to understand the imported Indian calculus which they supposedly "discovered", and Newton advanced an incredibly incoherent notion of the derivative as "fluxion". This is reflected in its consistent criticism from the 17th c. Descartes to 18th c. Bishop Berkeley to 19th c. Karl Marx, and the "fluxion" had to be eventually abandoned, as hopelessly confused. Dedekind's "correction" of "Euclid", by inventing metaphysical (unreal) "real" numbers, supposedly also solved the conceptual problem of the calculus for Westerners. However, even within inferior formalism, that happened only *after* the axiomatic set theory of the 20th century. Though "real" numbers are deemed essential to calculus today, all practical value of calculus for Newtonian physics was obtained *before* their invention, and they still cannot be used for practical purposes today, whether for rocket technology or for machine learning.

5. Superstitions of formal math-1: Politics of reason and the fallibility of deduction

The "reason" offered by the church for the "superiority" of metaphysical reasoning (prohibiting empirical proofs) was the claim that metaphysical deductive proofs are infallible. This gross church superstition is still the basis of formal mathematics today. The fallibility of deduction is manifest: students frequently turn in faulty proofs, hence flunk in mathematics tests. Authorities too have published or claimed faulty proofs of, say the Riemann hypothesis. Given that manifest errors are possible in pure deductive proofs, the only way to check the validity of a deductive proof is by repeated re-checks (induction) or by asking an "authority"; in either case, deductive proofs are *more* fallible than inductive and empirical proofs. In fact, in a complicated task of deduction, like a game of chess, the human mind *almost invariably* errs, so that deduction is far *more frequently* fallible than empirical proof. However, formal math totally depends on authority; even the axioms of formal mathematics are laid down solely by (Western) authority (e.g. "real" numbers for calculus, not even formal non-Archimedean arithmetic). This makes the theorems or "truths" of formal mathematics totally dependent on authority. This dependence of mathematical truth on (Western) authority greatly suited the church and colonialism, for math is used for science, which is a universal requirement, for which the world is made dependent on Western authority. Even the logic underlying mathematical proofs is decided by authority. Because of its ignorance and cultural parochialism, the church knew only about two-valued logic as the basis of reasoning: the church theology of reason said "logic binds God". And Europeans, as usual, blindly accepted that. Hence, that is the logic "authoritatively" accepted in formal math today. However, the church never asked *which* logic binds God? Logic is neither culturally universal (e.g., Buddhist catuskoti, or Jain syadavada) nor empirically certain (e.g. quantum logic). Hence, mathematical theorems are (at best) either mere cultural truths, or depend *essentially* on the empirical. Hence, formal mathematical proofs are always weaker than empirical proofs, to which they are hence inferior.

6. Superstitions of formal math-2: Politics of eternity and the metaphysics of infinity

Because formal mathematics is totally dependent on authority, it can be manipulated to suit prejudices, and these (religious) prejudices then creep into science through formal math. For example, "real" numbers are deemed essential for the calculus, used to formulate the equations of physics. "Real" numbers involve a *metaphysics of infinity*, closely tied to political church dogmas of *eternity*. Consequently, time in physics, is forced to be like a line, irrespective of any physical inputs. That suits the church, which, for political reasons of of opposing equity, cursed "cyclic" time, in the 6th c. However, this notion of time, as a linear continuum, is anti-Hindu, anti-Buddhist, and anti-Islamic. People believe Stephen Hawking was a great scientist, but his major book was on singularity theory, as applied to the cosmos. That involved imitating, in detail, the fiat**s** and defective arguments of Augustine, transferred to general relativity. Hawking similarly concluded, like Augustine, in favor of biblical creationism but now justified by science (general relativity). Last year, Roger Penrose was awarded the Nobel prize in physics for his work on singularity theory. However, a singularity is merely a failure of the Western understanding of calculus (chp. 12), not really a moment of biblical creation or apocalypse, or even a black hole, as has been so strongly peddled.

Part 3: The alternative (pedagogy)

7. **Zeroism**
   In reality, nothing stays unchanged for two instants. Nevertheless, we commonly speak of an individual across time, in much the same way as we use the abstraction "dog", though no two dogs are identical. We do this by "zeroing" differences unimportant to the context. This common-sense way of handling non-uniqueness works equally well with non-unique "limits" in a non-Archimedean field, by discarding or zeroing infinitesimals. One can equally well handle the non-uniqueness of finite differences. This is preferable to setting up a complicated and biased metaphysics of infinity, which few understand, just with a view to arrive at some arbitrary metaphysical exactitude in the notion of derivative. The point has already been illustrated using the Pythagorean theorem versus the Pythagorean calculations: a real inexactitude is preferable to unreal (or metaphysical) exactitude.

8. **"Euclidean" geometry vs Rajju ganita or string geometry (of the sulba sutra)**
   The current teaching of geometry in schools incorporates the most rotten features of colonial/church education, and the colonised are completely impotent. To begin with, it mandatorily recites the propagandist myths of Euclid and Pythagoras, and shows images of both as white skinned males, to emphasize the claim of racist superiority hidden under the claim of civilizational superiority. (Actually, the evidence strongly suggests the author of the "Euclid" book was a black woman, in the 5^{th} c., lynched by a church mob.) NCERT admits it has no primary evidence for Euclid, but says primary evidence is not needed for Western history, since blind trust in the "superior" master is the first lesson of colonial education. Its math text states that geometry done by the Greeks was civilizational superior to the geometry done by all other cultures. The text then goes on to incoherently mix up four distinct and incompatible geometries: (i) "Euclidean" geometry, (ii)_Hilbert's synthetic geometry which sought to correct the "Euclid" book by providing axiomatic proofs missing in it, (iii) Birkhoff's axiomatic metric geometry which sought to correct Hilbert and introduced metric geometry axiomatically, and (iv) empirical metric geometry of the compass box, a ritual feature of colonial education. The authors of the NCERT text, Jayant Narlikar et al. while extolling "reason" seem unaware that these four distinct geometries are mutually contradictory. It is well known that a contradiction can be used to prove anything whatsoever as a theorem; hence this technique of contradictory assumptions was a favourite of church rational theology. Even for the purpose of empirical metric geometry, the compass box is inferior since no instrument in it can measure a curved line. As such, this geometry is completely impractical and cannot be used to determine, say, the area of a farm with non-straight boundaries. For that, it is necessary to fall back on the string geometry (rajju ganit) traditionally used in India since the days of the sulba sutra. In fact, the eco-friendly and local string can replace all the instruments of the compass box, many of which are purely ritualistic and never used. Empirical string geometry is superior and makes geometry easy. It enables students to solve harder problems, such as determination of pi not taught in "Euclidean" geometry, or measurement of the earth, or determination of latitude and longitude. Pedagogical experiments with school teachers and students were conducted across four districts in India which confirm this. A school textbook on Rajju Ganita for class 9 has been prepared, together with a teacher's manual.

9. **Calculus without limits-1: Critique of calculus with limits**
   The stock way to teach calculus follows its historical trajectory in Europe. In short, it is a complete disaster which leaves people stuck in the "confusion stage" of European attempts to trying to understand Indian calculus. Most students only memorize a few formulae, and learn tricks to integrate elementary functions. Among the hundreds of students I tested, who had typically gone through the fat calculus texts of some 1300 pages ("Thomas calculus"), none could define basic notions such the derivative or the integral, or even the exponential function, since they are indoctrinated into limits but never learn the definition of limit. Apart from such pedagogical issues, calculus with limits is limited since one cannot differentiate a discontinuous function; but the need to do so arises in physics. Differentiating a discontinuous function is enabled by the Schwartz theory, which, despite its complexity, is also inadequate since it is a linear theory. Limits require real numbers which require axiomatic set theory, which results in absurd conclusions such as the Banach-Tarski theorem (that one ball of gold can be subdivided into a *finite* number of pieces which can be reassembled, without stretching or tearing, into two balls identical to the original, and so on, to repeatedly double your money using set theory).

10. **Calculus without limits-2: The pedagogical experiments**
    Calculus originated with the 5th c. Aryabhata's numerical method of solving differential equations ("Euler's" method), and numerical solution of differential equations is still all that is required for all key practical applications of calculus, such as calculation of rocket trajectories. Today it is believed that metaphysical or unreal "real" numbers and limits are essential to calculus, but it is *impossible* to use real numbers in practice, on a computer, which uses floating point numbers instead. Formally speaking, limits fail to exist not only in

ordered fields which are smaller than the reals (such as rationals), they also fail to exist in ordered fields which are *larger*, hence non-Archimedean. Brahmagupta's avyakt ganita (polynomial arithmetic) is an example of the latter. (Of course, this has nothing whatsoever to do with Archimedes, and this corrupt terminology is yet another example of how false Western history of math is used to confuse people and propagate the false claim of Western civilizational superiority.) The philosophy of zeroism permits infinite series to be summed in the absence of limits, as was first done for the infinite geometric series by Nilakanth in the 15th c. The complexity and inadequacy of the present day calculus arises from the collective Western failure to understand the last two features of the Indian calculus, though they understood the first feature (of numerical solution of differential equations) and used it to derive practical value from the calculus for Newtonian physics. Both the complexity and the inadequacy of calculus with limits can be avoided by teaching calculus without limits. In the last decade, this course has been successfully taught to various levels of students, from undergraduate to postgraduate, in pedagogical experiments carried out with 10 groups of students in 6 universities in 3 countries. These pedagogical experiments have demonstrated that this way of teaching calculus makes calculus easy, even for students of social science and humanities. It enables students to solve problems too hard to be included in the 1300 page "Thomas" calculus courses. Related decolonized courses on statistics have also been created so that those who write machine learning programs can have basic conceptual knowledge of the techniques they use, and how those could fail. The major problem, however, is to overcome the persistent colonial mindset of extreme gullibility in and fear of the colonial master.

Part 4: The alternative (science)

11. Retarded gravitation theory v. 2.0

For Newton's "laws" of motion to be even meaningful, they must be supplemented by a definition of equal intervals of time. Newton provided no such definition, and instead retracted Barrow's earlier one. Because of Newton's confusion about the Indian calculus, he attempted to pass off the derivative as a nonsense "fluxion", coupled with his incoherent belief that time itself flows. On the same incoherent belief, he thought that God understood what it meant for time to flow with an "even tenor", and had revealed the "laws of nature" personally to him. Newton's physics failed just *because* of these delusions, and was replaced by special relativity (of Poincare, not Einstein) which provided a new definition of equal intervals of time by declaring the speed of light to be constant (nothing to do with any experiment such as Michelson-Morley). However, it is only the combination of Newton's "laws of motion" and his "law of gravitation" which is falsifiable, not either "law" individually. Therefore, special relativity forces a reformulation also of gravitation, such as that achieved by general relativity theory (GRT). In contrast, my retarded gravitation theory (RGT) seeks a simpler corrected theory of gravitation by first making gravitation compatible with special relativity. In comparison with the earlier version of RGT (1.0), the present version (RGT 2.0) surely has quantitative departures at the *v/c* level, impossible with GRT. These departures apply mainly to hyperbolic orbits, since they cancel out for elliptic orbits. It is possible to empirically test RGT using the velocity dependent effect of Earth's rotation on spacecraft flybys (as has been reported with the flyby anomaly). If RGT(1.0) is correct then RGT is just an improvement on Newtonian gravity similar to but far simpler than GRT. If, however, RGT (2.0) is correct, as seems likely, then GRT must be wrong. Of course, it is no part of the aim of this book to prove general relativity is wrong; the aim is restricted to point out how Newton's errors in understanding calculus impacted his physics, and how those can be corrected.

12. The general (relativistic) theory of shocks in general continua

College calculus with limits ("Thomas" calculus) cannot be used to differentiate discontinuous functions which arise in physics in the case of shocks and singularities. The very complicated Schwartz theory can be so used, but it is a linear theory, hence still inadequate to make sense of the non-linear differential equations of physics at or across a discontinuity. To be able to do so, I had earlier used an additional layer of complexity, in the form of non-standard analysis applied to the Schwartz theory. However, empirical inputs are necessarily required, as illustrated by Riemann's error with shocks. Hence, also, mere authority won't do to pick a product of Schwartz distributions. Further, the key feature of non-standard analysis which is required is non-Archimedean arithmetic, which is already present *ab initio* in the Indian calculus. My resulting junction conditions for (relativistic or non-relativistic) shocks, subsume all previous conditions, including those for shocks in general relativity. They go beyond all previous conditions, in applying also to shocks in real fluids (in the presence of viscosity and thermal conductivity), and may be empirically tested. This demonstrates the power of the present approach. Many singularities too can be handled, showing that a singularity need not to be end of the world or its beginning. In particular, shocks, which might naturally arise during gravitational collapse, could easily be confounded with Penrose "singularities", so that a singularity is not even a robust indicator of a black hole as recently wrongly declared by the Nobel prize committee.

Ganita vs formal mathematics

Re-examining the philosophy of mathematics, its pedagogy, and the implications for science

Outline: 1. Ganita (गणित) differs from (formal) math, 2. it makes math easy, and 3. makes science better. 4. This is also an obituary of formal math.

Synoptic Contents

Preface

Part 1: Introduction

1. **Colonial education as church education, and the propaganda of civilizational superiority**
   (a) Macaulay[1] v. 2.0 (1847) in a speech to British parliament proposed free and compulsory education as the best and cheapest means to tame the British poor, and exorcise the spectre of revolt then haunting Europe.[2] Why? Because for centuries, Western education was church education. Specifically, the church had a 100% monopoly over Western education, including all top Western universities, such as Paris, Oxford and Cambridge, which the church set up during the Crusades,[3] and fully controlled until the end of the 19th century. As church education, intended to create missionaries, it was designed to instil submissiveness and respect for authority (hence blind acceptance of church superstitions) among those it educated. Church educated (submissive) people don't revolt. (b) The same Macaulay v 1.0 (1835) is wrongly blamed for colonial education in India. This is wrong, since colonial education was not limited to India alone, but spread globally, including to all non-British colonies, and everywhere it went, in whatever language, English, French, Spanish, Portuguese, or Dutch, it was the same church education. The submissiveness it taught, helped to offset colonial military weakness and prevent revolts in the colonies, such as the Indian revolt of 1857, after which Western university education came to India in a big way. (c) Macaulay v 1 (1835) simply used stock church propaganda based on a brazenly false history of science to argue that Indians needed Western education for science, since the West was civilizationally "superior" in science. (d) This propaganda of *civilizational superiority* everywhere accompanied colonialism[4] exactly the way the earlier propaganda of *racist superiority* accompanied racism, slavery, segregation, and apartheid, and the still earlier propaganda of *religious superiority* accompanied genocide and extermination of indigenous populations in three continents (North America, South America, Australia), and slavery on a fourth (Africa). (e) These three claims of superiority, though seemingly distinct, are minor

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1 T. B. Macaulay, *Speech to the House of Commons, 18 April 1847*, vol. IV, Speeches of Lord Macaulay, 1847, http://www.gutenberg.org/files/2170/2170-h/2170-h.htm#2H_4_0031.

2 'Education and counter-revolution', http://ckraju.net/papers/Education-and-counter-revolution.pdf. C. K. Raju, 'Decolonising the Hard Sciences', Frontier Weekly, 23 August 2013, https://www.frontierweekly.com/archive/vol-number/vol/vol-46-2013-14/46-7/46-7-Decolonising%20Hard%20Sciences.html.

3 trans Dana C. Munro, *Translations and Reprints from the Original Sources of European History, No. 3, The Medieval Student*, vol. II: No. 3 (Philadelphia: University of Pennsylvania Press, 1897).

4 For more on the doctrine of civilizational superiority, see 'Euclid must fall: The "Pythagorean" "theorem" and the rant of racist and civilizational superiority", Distinguished Academic Lecture Series, No. 5, African School of Conversational Philosophy, http://ckraju.net/papers/Pretoria-talk.pdf.

mutations of each other. E.g. after the transatlantic slave trade many blacks converted to Christianity, so that the earlier claim of religious superiority became questionable as the moral justification to enslave blacks. Likewise, the profits of slavery clashed with the European attempt to export its poor to the New World, and ultimately yielded to the superior profits of colonialism. However, the bogus Aryan race conjecture (1786) suggested that the colonized were of the same race as the colonizer, resulting in the claim of racist superiority mutating into one of civilizational superiority, as the moral justification for the brutalities of colonialism. While religious and racist superiority have been hotly contested, civilizational superiority was not similarly contested, and is still openly taught with mother's milk by the colonial education system (e.g., Indian class IX government math text teaches about the superiority of "Greek" math). Instead of contesting the claim of civilizational superiority, the colonized internalized it, and all they ever wanted was membership in the master's club: knighthoods, Western prizes, and degrees from Western universities, or other certificates of Western approval.

(f) All three claims of superiority—religious, racist, and civilizational—are organically linked, since all are based on essentially the same brazenly false history of science (also used by Macaulay). That false history of science was first concocted as Christian chauvinist history during the Crusades,[5] and the "Age of Discovery". First, the origin of all mathematics and science, from around the world, in Arabic texts which came to Europe during the Crusades, was attributed wholesale to *early* Greeks, regarded as the sole "friends of Christians" (ever since Eusebius), hence deemed a "Christian inheritance" fit to be translated. The early Greeks were, in fact, superstitious, and too mathematically backward to do science. Indeed, they lacked even efficient arithmetic, as clear from the non-textual evidence of their laughably primitive calendar. The use of the obnoxious and genocidal doctrine of Christian discovery[6] to "legally" appropriate land is barely understood ("Columbus discovered America", hence on current US law, "Red" Indians etc. legally lost ownership of their traditional lands). But its use to appropriate all indigenous scientific knowledge to Christians ("Copernicus discovered heliocentricity", "Newton discovered calculus", etc.) has hardly been understood. This results in the stock ("Wikipedia") history of science where all credit goes to early Greeks or to Europeans after the "renaissance". Just as this false history was earlier used to promote the claim of racist superiority, by Hume, Kant, Hegel etc., Macaulay v1.0, used this false history to claim civilizational superiority, and argue that Indians needed "superior" Western education for the sake of science. (g) Indians never cross-checked that false history in two centuries, though they changed the education system on that belief. Today, because they have learnt to worship various false gods of science, such as Copernicus, and because of their great desire to join the master's club, they fight bitterly *against* its being checked, and there still isn't a single university department or institution for the history of science in the country, decades after independence. The colonized never understood how easily they were fooled, and continue to be fooled to this day. (h) The colonized should check that false history at least now,

5 C. K. Raju, *Is Science Western in Origin?*, Dissenting Knowledges Pamphlet Series (Multiversity, 2009).

6 C. K. Raju, 'The Meaning of Christian "Discovery"', *Frontier Weekly* 47, no. 29 (2015): 25–31, http://www.frontierweekly.com/archive/vol-number/vol/vol-47-2014-15/47-29/47-29-The%20Meaning%20of%20Christian%20Discovery.html. More details in C. K. Raju, *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of Calculus from India to Europe in the 16th c, CE* (Pearson Longman, 2007).

especially because of the little-understood trick of linking false history to a bad (church) philosophy of mathematics, globalised by colonialism. (i) The sweeping claim of “superiority”, as in the purported superiority of Western ethno-mathematics, also needs to be checked. For example, because the colonised failed to check what is truly superior, they have uncritically accepted the inferior Christian calendar, which, apart from spreading superstitions convenient to the church, damages both culture and agriculture in India.[7]

2. **Ganita versus formal mathematics: an outline**

Part I: History[8]

(a) The story that mathematics is universal is false, as ought to be manifest even from the false claim of the “superiority” of Western ethno-math, for math can hardly be universal if there is a superior and inferior math. Mathematics varies with culture, even at the most elementary level of the notion of “angle”, or 1+1=2.[9] (b) Colonial math, or Western ethno-mathematics, replaced previously global normal math with formal or axiomatic math. Again, formal math is just declared “superior” on the colonial propaganda of civilizational superiority, which the colonized implicitly believe and refuse to check—most cannot check because colonial education actually made them ignorant, and formal mathematicians are co-opted. This “superiority” has nothing to do with practical applications of math, as many ignorant people believe. Formal math (which prohibits the empirical) adds nil practical value to any application of math from the grocery shop to calculation of rocket trajectories to artificial intelligence; naturally not, adding metaphysics does not add practical value for science and engineering. All practical value comes from normal math (ganita, which accepts the empirical), but the formal math of even 1+1=2 is enormously complex, makes math very difficult, and is demonstratedly beyond most people, including all professors in JNU,[10] and most professors of mathematics in IITs,[11] who just accept it uncritically. (c) Most normal math (arithmetic, algebra, trigonometry, calculus, probability and statistics) was initially imported by Europe from India, between the 10th and 17th c, for its practical value, and Europeans made a series of hilarious blunders in trying and failing to understand this imported knowledge of ganita, even at the level of elementary arithmetic algorithms and zero (e.g. pope Sylvester foolishly made a special abacus for “Arabic” numerals, Florence passed a law against zero). (d) Eventually (by the 20th c.), Europeans absorbed the Indian calculus into their own inferior (religious) understanding of formal mathematics, and globalised it through colonial education just by declaring it “superior”. They were right in believing that colonial education would so indoctrinate the colonized that they would never actually debate its purported superiority, any more than they ever checked its false history, but would bitterly oppose both out of their loyalty to the master.

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7 C. K. Raju, ‘A Tale of Two Calendars’, in *Multicultural Knowledge and the University*, ed. Claude Alvares (Penang: Multiversity, 2014), 112–19; *A Tale of Two Calendars - Dr C K Raju - India Inspires Talks* (New Delh, 2015), https://www.youtube.com/watch?v=MvpuC7Dg4e0&feature=youtu.be.

8 'Pre-colonial appropriations of Indian ganita: epistemic lessons' http://ckraju.net/papers/ckr-indology-abstract.pdf

9 For a popular-level account of 1+1=2 changes with culture, watch ‘How Colonial Education Changed Our Math Teaching: and what we can do about it today’, https://www.youtube.com/watch?v=Rm6d-bUmmGg. For more details about “angle”, see, 'गणित बनाम मैथमेटिक्स', http://ckraju.net/papers/Hindi-article-for-IIAS-journal.pdf.

10 ‘Statistics for Social Science and Humanities: Should we Teach it Using Normal Math or Formal Math? , lecture at JNU, Sep, 2020, https://www.youtube.com/watch?v=A9Og1k-Z5O4.

11 Neither meaning nor truth (nor practical value) in formal mathematics.

Part II. Philosophy[12]

(a) On an absurd Western myth, math is about reasoning which is unique to the West. However, the fact is that all Indian schools of philosophy (except Lokayata) accepted reasoning/inference as a means of proof, and this goes back to long before Aristotle of Toledo, and even Aristotle of Stagira. Indian ganita followed Indian philosophy, and accepted inference as a means of proof, as in inferring the radius of the earth, something Europeans were unable to do until the 17th c. (b) On another propagandist Western myth, proof is absent in ganita. This is just false, there are well-known proofs of the Pythagorean proposition in Indian tradition, but Westerners routinely go by myth not facts. (c) The deep trick is in the doublespeak about proofs based on "reason", which "reason" could mean normal reason (reason + facts), as used in science, or in India, or formal reason (reason minus facts), as initially used exclusively by the church. These Indian proofs in geometry accept empirical means of proof, as does all traditional Indian philosophy. In the West, the church prohibited empirical proofs (chp. 3) in reasoning just because empirical proofs would immediately derail numerous church dogmas. (However, the church hid this innovation of prohibiting facts by false attributing it to Greeks, or "Euclid" (chp. 3).) Due to church hegemony over the Western mind, formal mathematics still obeys this church prohibition: it is all metaphysical reasoning minus facts. This prohibition of the empirical was declared to make mathematics infallible and "superior", which claim of infallibility or superiority is a bogus church superstition (chp. 5).

(d) Ganita is about calculation, which is superior to *formally* proving mathematical theorems which theorems may not have any practical applicability. E.g. the "Pythagorean" theorem does NOT apply *exactly* anywhere on the curved surface of the earth (or anywhere in the real world). It is no use saying it is "approximately" true, for an approximation is worthless without an estimate of the error. Such an error estimate is no part of the theorem, which pretends to exact knowledge. An error estimate can only be provided using the two "Pythagorean" *calculations* known to Indian ganita, but little known in Europe until the 17th century,[13] an ignorance which resulted in huge navigational disasters, and thousands of lives lost (chp. 3). (e) Once empirical proofs are accepted, there is bound to be inexactitude, and the common-sense philosophy of zeroism (chp. 7), used in ganita, is needed to handle inexactitude. (f) The Indian calculus developed with the novel numerical methods of Aryabhata (a dalit from Bihar), combined with the non-Archimedean arithmetic of Brahmagupta and his avyakt ganit (polynomial arithmetic), and zeroism (chp. 9-10). It was used to sum infinite series (such as "Newton's" sine series, or the "Leibniz" series for pi), by the Aryabhata school in Kerala, but without appeal to limits.

12 'Ganit vs formal math', abstract for IIAS seminar, http://ckraju.net/papers/abstract-2-iias.pdf.

13 C. K. Raju, 'Black Thoughts Matter: Decolonized Math, Academic Censorship, and the "Pythagorean" Proposition', *Journal of Black Studies* 48, no. 3 (2017): 256–78, https://doi.org/10.1177%2F0021934716688311 http://ckraju.net/papers/Manuscript-Black-thoughts-matter-accepted-version.pdf.

Part 2: The false history and superstitions of Western (formal) mathematics

3. **False history of math-1: Pythagoras, Euclid, and geometry**[14]
On the myth of civilizational superiority, it is commonly asserted, for example in the current class IX math school text in India, that the Greeks had a "superior" understanding of geometry, compared to all others (Indians, Egyptians, Maya etc.) because the Greeks proved the Pythagorean theorem. (a) However there is no evidence either for the existence of Pythagoras or that he proved any theorem. This is not mere absence of evidence: there is ample counter evidence that Pythagoreans were disinterested in proving theorems, and interested solely in the religious connection of geometry, or the Egyptian mystery geometry elucidated by Plato, with a view to arouse the soul. They deprecated the empirical (compared to the spiritual) but did not prohibit it.

(b) All "evidence" for such Western "history" of "Greeks" comes from myth jumping: jumping to a second myth as if "evidence" to support the first. To support the myth of Pythagoras the myth of Euclid is invoked. But there is no primary evidence for "Euclid" either. But, of course (chp. 1), "Euclid" was concocted during the Crusades, so there is no evidence even that he was the author of the text Elements attributed to him, or that the text originates anywhere near the date attributed to "Euclid", or that it originates from Greece, or that he was a white male as offensively and graphically depicted in Indian math school texts (and Wikipedia) used to pollute the minds of children. As in the case of Pythagoras, there is ample counter evidence against each one of these mythical claims about "Euclid", so dear to Western "historians". Specifically, the real author "Euclid" was probably a black woman from Africa, raped in a church, and brutally lynched by a Christian mob in the 5th c. because that geometry promoted a notion of soul proscribed by the church. (c) To save the myth of Euclid, the Western apologist (or the colonised mind) now jumps to a third myth, the myth of the book "Euclid" purportedly wrote. However, once again, the facts are contrary to the myth about the book. The received text (the earliest from about 10th c. Turkey) does NOT have a single formal proof in it from its first proposition to its last, as has been falsely maintained for centuries by Western historians. There are no known social conditions among the early Greeks which would have motivated either the desire for "irrefragable" proofs, or the use of metaphysical proofs divorced from the empirical.

(d) However, when this text first came to Europe, during the Crusades, as an Arabic text in 1125, and was translated into Latin, the church had a great political need for a text on metaphysical reasoning, and persuasive proofs, in its desire to catch up with the Islamic theology of reason (aql-i-kalam), to be able to convert Muslims it failed to convert by conquest. Therefore, the church brazenly "reinterpreted" the "Euclid" text to claim that it had axiomatic or metaphysical proofs in it. Because of total church hegemony over the European mind during this period, all Western scholars, without any exception, were gullible enough to believe for 750 years that the book actually had axiomatic proofs. As typical of the church, it glorified everything which served its political purposes, and Western scholars still believe metaphysical proofs are superior. (e) It was only towards the end of the 19th century, when church hegemony slightly waned, that Dedekind, Russell, and Hilbert pointed out the absence of even a single formal proof in "Euclid's" Elements. Hilbert even wrote a whole book to provide the formal proofs missing in the book. Russell noticed the numerous diagrams in the book which have no connection to formal proofs, but failed to relate the geometry in the book to Plato and Egyptian mystery geometry, which does use diagrams, as

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14 '"Euclid" must fall: The "Pythagorean" "theorem" and the rant of racist and civilizational superiority', http://ckraju.net/papers/Pretoria-talk.pdf. Also, C. K. Raju, *Euclid and Jesus: How and Why the Church Changed Mathematics and Christianity across Two Religious Wars* (Penang: Multiversity and Citizens International, 2012).

Proclus had so explicitly pointed out in his Commentary. (f) Because this myth of "superior" Greek mathematics was so politically important to racism, colonialism, etc. even the most respected Western historians such as Gillings, Needham, and Clagget, laughably keep repeating the myth of "superior" formal proofs in "Euclid's" Elements, even a century AFTER the absence of formal proofs in the Elements was publicly exposed. This shows that for Western historians myth is all that matters, and facts are irrelevant. (g) The Indian government has recently taken the written stand that whatever nonsense "history" (myth) is stated in Western texts must be accepted as true on blind trust without demanding primary evidence: this is the duty of the colonised. This blind trust in the master is exactly what colonial/church education taught to the colonised, and it still persists, though colonialism is deemed to have ended.

(h) A formal mathematical theorem is usuallty NOT valid knowledge in the real world. For example, the Pythagorean theorem is not true on the curved surface of the earth (or anywhere in the real world). (i) It is no use saying a mathematical theorem is "approximately true", because approximate knowledge without a precise error estimate is worthless. Such an error estimate can only come from the two Pythagorean calculations,[15] each superior to the Pythagorean theorem. Both Pythagorean calculations were well known in Indian ganita, but were not properly known to Europeans. (j) Because Europeans, under church hegemony, wrongly regarded inferior mathematical theorems as superior to calculations, they were unfamiliar with the second Pythagorean calculation, and had a huge navigational problem from the 16th to the 18th c. (chp. 4)

4. **False history of math-2: Newton, Leibniz, and the calculus**[16]

(a) As I have explained for the past two decades,[17] the calculus originated in India with the 5th c. Aryabhata's strikingly innovative use of numerical methods ("Euler's" method) to solve differential equations, to obtain precise sine values, precise to the first sexagesimal minute, or about 5 decimal places. Over the next thousand years, this developed, across India, into a sophisticated technique of calculating very precise "trigonometric" values (or, rather circle-metric values), precise to the third sexagesimal minute, about nine decimal places. (b) These precise trigonometric values were all-important for navigation, to determine the radius of the earth, latitude, longitude, and, in the case of backward Europeans, loxodromes. (c) European governments recognized their backwardness in navigation, on which their dreams of wealth depended, and from the 16th to the 18th century many European governments offered large prizes for the solution of the navigational problem, starting with loxodromes, and ending with the longitude problem (British prize of 1711). (d) Naturally, the Jesuits who had established a college in Cochin in the 16th century, gathered Indian mathematical knowledge relevant to navigation with the help of the local Syrian Christians. They appropriated this indigenous knowledge using their infamous

15 Raju, 'Black Thoughts Matter: Decolonized Math, Academic Censorship, and the "Pythagorean" Proposition' http://www.ckraju.net/papers/Manuscript-Black-thoughts-matter-accepted-version.pdf.

16 C. K. Raju, 'Marx and Mathematics-1: Marx and the Calculus', Froniter Weekly, 28 August 2020, https://www.frontierweekly.com/views/aug-20/28-8-20-Marx%20and%20mathematics-1.html; C. K. Raju, 'Marx and Mathematics. 2: "Discovery" of Calculus', Froniter Weekly, 31 August 2020, https://www.frontierweekly.com/views/aug-20/31-8-20-Marx%20and%20mathematics-2.html; Raju, C. K., 'Marx and Mathematics. 3: The European Navigational Problem and the Dissemination of the Indian Calculus in Europe', Frontier Weekly, 4 September 2020, https://www.frontierweekly.com/views/sep-20/4-9-20-Marx%20and%20mathematics-3.html; Raju, C. K., 'Marx and Mathematics. 4: The Epistemic Test', Frontier Weekly, 8 September 2020, https://www.frontierweekly.com/views/sep-20/8-9-20-Marx%20and%20mathematics-4.html.

17 C. K. Raju, 'Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the Yuktibhāṣā', Philosophy East and West 51, no. 3 (2001): 325–62, http://ckraju.net/papers/Hawaii.pdf; C. K. Raju, Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of Calculus from India to Europe in the 16th c, CE (Pearson Longman, 2007).

doctrine of Christian discovery. Later, the "discovery" of calculus was entirely misattributed to Christians, particularly, Newton and Leibniz,[18] in exactly the same sense in which India was "discovered" by Vasco da Gama. On the doctrine of Christian discovery, the prior knowledge or prior occupancy of the land by non-Christians did not legally matter, as ruled by the US Supreme Court (judgment is currently valid), on the strength of customary British law.

(e) However, neither understood calculus. Newton rightly called Leibniz the "second discoverer," and ridiculed him for needing help to understand the infinite series named after him today. But Newton, himself, did not understand calculus, and advanced the most ludicrous doctrine of derivatives as "fluxions", based on his utterly incoherent notion that time itself flows.[19] (f) Consequently, even until the late 19th century, most Westerners remained confused about the calculus. The use of higher-order polynomial interpolations in India, since Brahmagupta's critique of Aryabhata, had eventually resulted in infinite series which were summed using Brahmagupta's non-Archimedean arithmetic of polynomials (and zeroism). No European understood this process. (g) Hence, the Europeans critics of calculus persisted from the 17th c. Descartes, to the 18th Bishop Berkeley to the 19th c. Karl Marx, who rightly referred to the calculus of Newton and Leibniz as "mystical".[20] (h) On my epistemic test, this claim of "discovery without understanding" is conclusive proof of theft of knowledge, as when a students cheats in an exam by copying from another, without fully understanding what he copies.[21] On the same doctrine of "Christian discovery", which is still British law, my very thesis, that the calculus was stolen, was itself stolen, by UK-based serial plagiarists, whose acts of plagiarism were also nailed by their extreme mathematical blunders.[22]

(i) Though Dedekind's "real numbers", attempting to provide an axiomatic proof of the first proposition of "Euclid's" *Elements*, are today regarded as the solution, to the problem of how to sum infinite series of calculus, there are two issues. First, all practical value of calculus for science from Newton to Laplace was already obtained *before* the advent of real numbers, proving that real numbers are NOT needed for practical applications of calculus. Today they are deemed essential to the calculus, on university calculus texts, but are still *impossibl*e to use in practical applications, from calculating rocket trajectories to AI. Second, the construction of real numbers required axiomatic set theory (as distinct from naive Cantorian set theory), which came into existence only in the 1930's, and is a vast metaphysics of infinity with all kinds of unreal conclusions, such as the Banach-Tarski theorem.[23] In any case, the underlying complexity makes it impossibly hard, even for formal mathematicians, to fully understand calculus. Hence, also, they avoid public debate.

5. **Superstitions of formal math-1: The fallibility of deduction**[24]

18 C. K. Raju, 'Marx and Mathematics. 2: "Discovery" of Calculus'.
19 Raju, C. K., 'Marx and Mathematics. 4: The Epistemic Test'.
20 C. K. Raju, 'Marx and Mathematics-1: Marx and the Calculus'.
21 Raju, C. K., 'Marx and Matheamtics. 4: The Epistemic Test'.
22 "Prof. Raju's charge of plagiarism found correct: UK university warns lecturer", *Hindustan Times*, Bhopal, 8 Nov. 2004, Headline. Archived at http://ckraju.net/Joseph/HT_report_8_Nov_04.pdf, For some of the mathematical blunders the plagiarists made see Raju, *Cultural Foundations of Mathematics*, "Transmission of the transmission thesis". For mathematical blunders specifically made by Joseph, see 'George Joseph: serial plagiarist", http://ckraju.net/blog/?p=132, and "George Gheverghese Joseph serial plagiarist and mathematical ignoramus, invited for conference on math education by Hyderabad University, http://ckraju.net/blog/?p=166, and related blog entries."
23 http://ckraju.net/sgt/technical-presentations-faculty/ckr-sgt-tech-presentation-2.pdf.
24 C. K. Raju, 'Decolonising Mathematics', *AlterNation* 25, no. 2 (2018): 12–43b, https://doi.org/10.29086/2519-5476/2018/v25n2a2.

(a) As already pointed out (chapter 3), contrary to the long-selling myth, there are no formal or axiomatic proofs in "Euclid's" *Elements*, or in any actual Greek tradition. Though, finally, Dedekind, Russell, and Hilbert denied the *myth* and pointed out that there were no such axiomatic proofs in the book, they nevertheless continued to accept the church *superstition* that axiomatic proofs are superior. (b) This belief in superiority of formal (church) mathematics is central to the colonial claim of civilizational superiority, and is the reason why formal mathematics is globally taught in schools and universities. (c) While everyone accepts that empirical proofs are fallible, mathematics does not become superior or infallible by prohibiting empirical proofs or by eliminating facts from it, any more than science becomes "superior" by eliminating fallible experiment. Indeed, the Lokayata in India regarded deduction as inferior and rejected it as leading to invalid knowledge. (d) But, a debate on the purported superiority of deductive proofs has proved very difficult to carry out, because a public debate on this long cherished Western superstition threatens to leave centuries of Western scholars with egg on their face, and exposure of their long-term gullibility. Formal mathematicians, whom the colonised are taught to regard as "experts", have persistently refused to debate, for they understand the weakness of their position, and they fear losing their jobs, and all credit for any past efforts.

(e) The most superficial observation shows that formal mathematical proofs are fallible: as any mathematics teacher knows, students make numerous mistakes in deductive proofs, and hence are flunked in mathematics. Numerous authoritative mathematicians, too, have made mistakes in publishing or claiming proofs of, say, the Riemann hypothesis. No self-respecting journal in formal mathematics will accept a purported proof of the Riemann hypothesis as valid, merely on the absurd ground that "deduction is infallible". (f) Given that there are possible errors in a deductive proof, the only ways to ascertain its validity are (i) to repeatedly re-check the proof oneself (an inductive process), or (ii) to uncritically rely on authority, as most people do. Reliance on authority is obviously MORE error-prone than empirical proofs, hence sabda pramana was rejected by all nastik schools of thought in India. It is a pure superstition that any socially respected authority, such as the pope, or an expert mathematician, such as Riemann, is infallible. In either case, deduction cannot be "superior" to either induction or to an empirical process, as shown particularly by Riemann's error about shock conditions. (g) It is no use saying that a "valid" deductive proof is infallible, for that is an irrefutable tautology, which applies equally to "valid" empirical proofs, or "valid" anything: the whole problem is to ascertain validity. The prohibition of the empirical turns formal mathematics into pure metaphysics, which is irrefutable, and in which validity (of both axioms and theorems) can only be decided by authority. Hence, this method of "reasoning" (without facts) greatly suited the church, and it suits the colonizer that the colonized have no way to decide mathematical (hence scientific) truth except by reference to the colonizer. Amazingly, many of the colonized are very happy with this situation of slavish dependence on Western authority—they are glad to be mental slaves so long as they have a place at the table!

(h) The game of chess, based wholly on deduction, makes it abundantly clear that deduction is MORE fallible than empirical proofs, because the human mind is more easily deceived than the human senses. Thus, an error-free chess game must end in a draw, but even the topmost Grandmaster makes an error *almost every time*, hence almost invariably loses to a computer. This excessively frequent occurrence of error is true for any complex task of deduction, and part of every mathematician's experience. (i) In contrast, the classical example in Indian philosophy of the fallibility of an empirical proof is that one may mistake a snake for a rope or vice versa, in dim light. This has happened with me only once in my life. That is, deductive proofs are not only not infallible, they are *more frequently* erroneous then empirical proofs.

(j) Formal mathematics assumes two-valued logic must be used in mathematical proofs,. Why exactly? (Not for mathematician to reason why/Theirs but to prove and die!) Because of the church superstition (of rational theology) that this kind of logic binds God. Historically speaking, the crusading church got its methods of reasoning (including the “Aristotelian” syllogism) from Arabic texts, particularly those of Ibn Rushd (Averroes). Hence, quite possibly, the “Aristotelian” syllogism is derived from the Indian Nyaya syllogism.[25] At any rate, there are many other types of logic known in Indian culture, such as Buddhist catuskoti, or Jain syadavada. The theorems of formal mathematics will vary with logic (for example proofs by contradiction will fail with Buddhist logic, so one cannot prove the existence of a Lebesgue non-measurable set, for example). So, the core assumption of formal mathematics and its theorems—logic—is decided by cultural dictatorship. In this case, the theorems of mathematics are, *at best*, mere cultural truths. (k) The other way to decide logic is to do so empirically, but the logic of the real world at the micro physical level is quantum logic which is NOT two-valued, or even truth-functional.[26] Therefore, logic is *not* empirically certain. In any case, if logic is decided empirically, deductive proofs are necessarily *more* fallible than empirical proofs, contrary to what the West has wrongly believed for so long.

6. **Superstitions of formal math-2: Politics of eternity and the metaphysics of infinity**[27]

(a) Because of its divorce from facts, formal math is pure metaphysics. There is no empirical way to validate metaphysical truth, which must be accepted on authority. In particular, the axioms of formal math, are metaphysics (about infinity) not facts, whose truth must be decided on authority. For example, the current Indian class VI text declares a geometric point to be infinitely small (hence invisible), and children have no way to know about invisible points based on any experience: their sole “knowledge” of the nature of invisible points comes from the school text. (b) This reliance of formal math on authority enables a variety of prejudices to creep into axioms, hence theorems. The church hegemony over the Western mind, at least until the end 19th century, ensured that the related prejudices about infinity were, in fact, church superstitions, about eternity. (c) I have earlier explained (in *Eleven Pictures of Time*) how church dogmas about eternity are tied to the church politics of iniquity, after the church married the state in the fourth century. In particular, in the 6th c., the church cursed “cyclic” time, as part of its politics against the “pagan” notion of equity and soul, which notion was also part of early Christianity. This church curse is also directly contrary to beliefs about time in Hinduism and Buddhism, though few have understood the effects on science. The belief in time as a continuum is contrary to both Buddhism and Islam.[28] (d) However, under the influence of church dogma, Newton (contrary to his mentor Barrow) declared time to be like a line,[29] and this belief persists in science today because of the falsehood that Newton “discovered” the calculus, though his confused doctrine of fluxions was abandoned. Consequently, calculus is today done using metaphysical “real” numbers, and children in class IX are taught about the real line. Since calculus is used to formulate the equations of physics, this forces time in physics to be like a line, without any

25 C. K. Raju, ‘Logic’, in *Encyclopedia of Non-Western Science, Technology and Medicine* (Springer, 2016 2008). http://ckraju.net/papers/Nonwestern-logic.pdf.

26 C. K. Raju, *Time: Towards a Consistent Theory* (Springer, 1994) chp. 6b, ‘Quantum mechanical time’.

27 C. K. Raju, ‘Eternity and Infinity: The Western Misunderstanding of Indian Mathematics and Its Consequences for Science Today’, *American Philosophical Association Newsletter on Asian and Asian American Philosophers and Philosophies* 14, no. 2 (2015): 27-33.; C. K. Raju, ‘A Singular Nobel?’, *Mainstream* 59, no. 7 (30 January 2021), http://www.mainstreamweekly.net/article10406.html.

28 C. K. Raju, “Decolonising mathematics: Eliminating the anti-Islamic biases in mathematics and science”, Decolonial International Network, https://www.facebook.com/1203315329731108/videos/904342360333449. Presentation: http://ckraju.net/papers/presentations/din.html.

29 C. K. Raju, ‘Time: What Is It That It Can Be Measured?’, *Science & Education* 15, no. 6 (2006): 537–51.

reference to any physics or the empirical world. (e) Since mathematics is used for science, and the whole world needs science, the propaganda of civilizational superiority (together with direct Western control over university academics[30]) ensures that the rest of the world imitates what the West did in math. This globalizes the superstitions about time and eternity, a key superstition by which the church rules. This victory over science is a tremendous victory for the church (and colonialism) a victory which has gone unsung, and unnoticed.

(f) A classic example of how current science based on formal mathematics can be manipulated in support of religious prejudice is the singularity theory of Stephen Hawking and Roger Penrose. Hawking and Tipler etc. used singularity theory, together with the fiats of Augustine, to brazenly promote creationism, and, indeed, all Judeo-Christian theology. This is of great importance today, since Penrose got the Nobel prize for singularity theory last year, and the colonised who understand not science but only the *authority* of science and its symbols, such as the Nobel prize, believe the truth of one-time creation according to the Bible has been scientifically established. (g) A simple way to avoid such manipulation in science is to switch back to calculus as normal mathematics (ganita). (h) How this is possible is discussed in the next part.

Part 3: The alternative (pedagogy)

7. **Zeroism[31]**
(a) In my 2007 book *Cultural Foundations of Mathematics*, this was called sunyavada. Subsequently, the name was changed to zeroism to emphasize that the key issue was practical value, not a textual exegesis of the Buddhist philosophy of Nagarjuna, which exegesis can obviously be done in various ways. (b) In reality, nothing stays the same for two instants of time, and no two objects (apples, oranges, or a person at two instants of time) are exactly alike. The common sense way of dealing with this non-uniqueness is to neglect or “zero” differences regarded as unimportant to the context. A similar attitude is used in dealing with any abstraction with an ostensive empirical referent, such as “dog”. (c) Infinite series cannot be *physically* summed by adding successive terms, since that would take an eternity of time. One method is to work with a finite sum which may or may not be a valid thing in the context. (d) With non-Archimedean arithmetic, the presence of infinitesimals means there are no exact (or unique) limits. However, the sum of an infinite geometric series can very well be obtained in the manner of the 15th c. Nilakantha, by discarding infinitesimals, which makes no practical difference. (e) The key point is that mere non-uniqueness is not fundamentally problematic, until such time as it makes a practical difference. Therefore, also, it is possible to work with non-unique finite differences instead of trying to arrive at metaphysical uniqueness through limits and the notion of derivative.

8. **“Euclidean” geometry vs Rajju ganita or string geometry (of the sulba sutra)[32]**
(a) The first key difference is that Rajju Ganita uses the sulba or string, which is flexible and can be used to *empirically* measure both straight and curved lines. This is different from current teaching of geometry based on the compass box (or geometry box) no instrument of

30 C. K. Raju, ‘Benchmarking Science: A Critique of the ISI (Thomson-Reuters) Index’, in *USM-Prince Songkla Univ. Conference in Hat Yai*, 2011, http://ckraju.net/papers/Benchmarking-science-paper.pdf.

31 C. K. Raju, ‘Zeroism’, in *Encyclopedia of Non-Western Science, Technology and Medicine* (Springer, 2016), http://ckraju.net/papers/Springer/zeroism-springer-f.pdf.

32 C. K. Raju, ‘Towards Equity in Math Education 2. The Indian Rope Trick’, *Bharatiya Samajik Chintan (New Series)* 7, no. 4 (2009): 265–69 http://www.ckraju.net/papers/MathEducation2RopeTrick.pdf; *Sulba Sutra Geometry: Can We Teach It in Schools Today? | C.K. Raju*, 2020, https://www.youtube.com/watch?v=rLI_UU6dfnE. Also, http://ckraju.net/geometry/Rajju-ganit-draft-teacher-manual.pdf.

which can be used to measure curved lines. (b) Current teaching of geometry piles on the confusion by incoherently mixing up five distinct kinds of geometry: the (i) the myth of Euclid, and the associated geometry (ii) religious mystery geometry, elucidated by Plato (iii) the axiomatic synthetic geometry of Hilbert arising from his misguided attempt to repair the failed myth of axiomatic proofs in "Euclid", (iv) the axiomatic metric geometry of Birkhoff, an attempt to repair Hilbert and provide axiomatic metric geometry, with (v) empirical compass box geometry (which is what most people actually learn). School education since the Sputnik crisis, pretends they are all one and the same, when they are, in fact, mutually incompatible: for example Hilbert synthetic geometry is incompatible with Birkhoff's metric geometry. Neither allows empirical superposition, which is the empirical way to measure length, hence both are incompatible with compass box geometry, etc. This confusion of teaching five incompatible geometries as if they were the same is the result of blindly imitating the West, thus imitating both the mistakes made by the West, and imitating also *all* the successive ad hoc "corrections"!

(c) With any kind of geometry, based on the straight-line, the length of a curved line is a difficult concept. For non-empirical geometry, this requires calculus and limits (rectifiable arcs), as clear from the navigational problem of loxodromes. (d) Starting with the curved lines, in the manner of Rajju Ganita, also makes "trigonometry" (or rather circle-metry) very easy, along with all its important applications. In particular, it makes the transition to the Indian calculus easy. (e) The use of empirical methods makes the proofs of all theorems of "Euclid's" geometry very easy.[33] (f) The string is also an eco-friendly, local resource unlike the compass box made of steel and plastic. (g) By using the two-scale principle, or a pair of strings, as used in traditional Indo-Arabic navigational instruments, a much high degree of accuracy is possible, in principle, than available with the geometry box. (h) The related pedagogical experiments with school students and teachers are briefly described.[34]

9. **Calculus without limits-1** [35]

(a) The present day teaching of calculus proceeds on the same principle that phylogeny is ontogeny, and repeats the European historical experience with calculus in fast-forward mode in the classroom. Because of the prolonged European historical failure to conceptually understand the Indian calculus and its infinite series, this pedagogical method is a disaster which leaves most students utterly confused about the calculus. (b) All that the typical student learns is to memorise a bunch of formulae involving derivatives and integrals, and tricks of integration, which is worthless knowledge which can be done more efficiently by free and open-source symbolic computation programs. (c) Most students are indoctrinated into limits, from an early age, but cannot formally define limits, needed to formally define derivatives and integrals, or transcendental functions, or even define the real numbers, needed for limits.

(d) Students learn that the calculus with limits cannot be used to differentiate discontinuous functions, but they never learn that this is a failure of the Western calculus, not some natural "law". Such discontinuous functions arise in physics in the case of shocks, and singularities, prima facie invalidating the differential equations of physics. Very few students learn about even the formal "remedy" of the Schwartz theory, and the Dirac delta function. Learning this within formalism, involves advanced, and complex formal mathematics such as topological vector spaces, known to only a few specialist mathematicians. (e) In fact, it is long known that even that formal "remedy" fails because the differential equations of physics are

33 C. K. Raju, *Rajju Ganita, a textbook on string geometry for class IX*.

34 'Rajju Ganit workshop from today' http://ckraju.net/blog/?p=155. 'Alternative math: media reports', http://ckraju.net/blog/?p=156.

35 http://ckraju.net/sgt/technical-presentations-faculty/ckr-sgt-tech-presentation-1.pdf, http://ckraju.net/sgt/technical-presentations-faculty/ckr-sgt-tech-presentation-2.pdf.

nonlinear while the Schwartz theory is linear. Shocks in real continua and the renormalization problem of quantum field theory, both, result in the problem of defining products of Schwartz distributions. (f) While there are many definitions of the product,[36] the only one which has been successfully applied to both problems, of classical and quantum physics, is my product, which uses non-standard analysis. (g) However, the key feature of non-standard analysis that is needed is that the non-standard extension of real numbers is non-Archimedean, since any ordered field larger than the reals must be non-Archimedean (i.e., must admit formal infinities and infinitesimals). (g) Students are not taught that even elementary polynomial arithmetic is also non-Archimedean, that (unique) limits do not exist in a non-Archimedean field, and that it is perfectly possible to do calculus without limits using non-Archimedean arithmetic. (h) However, to handle the problem of non-uniqueness, a new philosophy (zeroism) is required. (i) Of course, this philosophical way of handling non-uniqueness can also be used with finite differences, which already makes irrelevant the need for derivatives defined as limits.

10. **Calculus without limits-2**[37]

(a) As already explained (chp. 4 (i)) real numbers and limits are not used for any practical application of the calculus, whether to calculate rocket trajectories or for artificial intelligence. In fact, it is impossible to use real numbers on a computer, which uses floating-point numbers instead. Specifically, all practical applications of the calculus involve numerical computation, never limits wrongly believed in the West to be essential to calculus. (b) As I pointed out long ago, Aryabhata made a striking innovation of calculating sine values by using a numerical method of solving differential equations, today called the Euler method on the chauvinist doctrine of Christian discovery, after Euler who came more than a thousand years later, and there is ample documentation of his access to Indian mathematical texts. This aspect, of the practical value of the calculus, was readily understood by Europeans.

(c) Europeans did also separately understood Brahmagupta's avyakt ganita, or polynomial arithmetic, but only as part of algebra, not calculus. Specifically, they did not understand the possible use of the non-Archimedean arithmetic for calculus until the advent of non-standard analysis, in the 1960's. But, of course, the use of non-Archimedean arithmetic via non-standard analysis introduces an enormous layer of complexity, which is absent in the ab initio use of non-Archimedean arithmetic, in the Indian calculus. (d) From Plato to the church, Western ethnomathematics has always been tied up with religious beliefs. Plato's idea of mathematics as a way to arouse the *eternal* soul, by sympathetic magic, led to the superstition that mathematics incorporates *eternal* truths, and hence must be exact. This belief persisted into church dogmas about eternity: specifically the post-Crusade dogma and superstition that God rules the world with eternal "laws of nature", which are written in the "language of eternal truth", namely mathematics. The metaphysics of real numbers and limits is required not for any practical value but only on account of this superstitious belief.

(f) The results of various pedagogical experiments to teach calculus without limits across countries and universities are described.[38] These demonstrate that it makes calculus so very

36 C. K. Raju, 'Distributional Matter Tensors in Relativity', in *Proceedings of the 5th Marcel Grossman Meeting*, ed. D. Blair and M. J. Buckingham (World Scientific, 1989), 421–23, arXiv:0804.1998.

37 C. K. Raju, 'Teaching Mathematics with a Different Philosophy. 1: Formal Mathematics as Biased Metaphysics', *Science and Culture* 77, no. 7–8 (2011): 274–79 arXiv:1312.2099; C. K. Raju, 'Teaching Mathematics with a Different Philosophy. 2: Calculus without Limits', *Science and Culture* 7, no. 7–8 (2011): 280–851312.2100; C. K. Raju, 'Calculus without Limits: Report of an Experiment', in *2nd People's Education Congress* (TIFR, Mumbai: Homi Bhabha Centre for Science Education, 2009) http://ckraju.net/papers/calculus-without-limits-paper-2pce.pdf .

38 C. K. Raju, 'Calculus without Limits: Report of an Experiment', in 2nd People's Education Congress (TIFR, Mumbai: Homi Bhabha Centre for Science Education, 2009); http://ckraju.net/papers/calculus-without-limits-

easy that it could be taught in five days,[39] to students of social science,[40] science and engineering,[41] and, of course, various categories of math students, from undergraduate to post-graduate level. (g) It is also explained, how contrary to the typical church superstition of the colonized that any change would precipitate an apocalypse, the fact is that calculus without limits makes calculus easier; hence it enables students to solve harder problems, such as ballistics with air resistance, omitted in the typical calculus courses in fat calculus texts.[42] (h) However, it is not sure if anything can really be done for/by the colonized who are today totally ignorant of mathematics, due to colonial education, and have been taught to rely solely on the authority of the West, and mistrust the non-West, so they are too afraid to revolt against the West. The West itself will not easily agree to such a change, which makes Western mathematicians such as Hilbert and Russell (and Bourbaki) look like fools. Formal mathematicians, whom the colonized have learnt to regard as "experts", will also not permit such a change, or even discuss it due to their vested interests..

Part 4: The alternative (science)

11. **Retarded gravitation theory v. 2.0**[43]
(a) Newton's misunderstanding of derivatives as "fluxions" was based on the incoherent belief that time itself flows. It was on this incoherent belief that he based his metaphysical even tenor hypothesis, quite distinct from Barrow's earlier physical even tenor hypothesis,[44] which he wrongly thought was a substitute for a definition of equal intervals of time required for Newton's "laws" to be even meaningful. (b) This intrusion of metaphysics into physics was due to Newton's religious predilections, or his superstitious belief that God had written eternal laws of nature in the perfect language of mathematics. (c) The consequent failure to provide a physical definition of equal intervals of time directly led to the downfall of Newtonian mechanics, due to its *internal* deficiencies. Newtonian mechanics was replaced by special theory of relativity, by Poincare in 1904 (not Einstein, in 1905).[45] (d) As I explained long ago, Newton's "laws" of motion, and Newton's "law" of gravitation come as a package deal: neither is individually refutable (falsifiable), only the two together are. Therefore, the failure of Newtonian mechanics means that Newtonian gravitation too has to be modified.

(e) While Newtonian gravitation was modified long ago by the general theory of relativity (GRT), this modification is limited to v^2/c^2 level effects, as in the anomalous perihelion advance of Mercury. This modification cannot explain the long-term failure of Newtonian gravitation to explain the rotation curves of stars in the galaxy, without accumulating hypotheses such as dark matter halos. Nor can the general theory of relativity explain the more recently observed possible anomalies with hyperbolic orbits such as the flyby

paper-2pce.pdf, C. K. Raju, 'Teaching Mathematics with a Different Philosophy. 1: Formal Mathematics as Biased Metaphysics', Science and Culture 77, no. 7–8 (2011): 274–79; arXiv:1312.2099, C. K. Raju, 'Teaching Mathematics with a Different Philosophy. 2: Calculus without Limits', Science and Culture 7, no. 7–8 (2011): 280–85, arXiv:1312,2100..

39 "The5-day course on calculus without limits". http://ckraju.net/blog/?p=34.,

40 "Calculus for social science", http://ckraju.net/blog/?p=83,

41 See poster, http://ckraju.net/sgt/poster-calculus-without-limits.pdf, facebook post https://www.facebook.com/sgtuniversitygurgaon/posts/835428686620798, and youtube video: https://www.youtube.com/watch?v=0sdimbGwUCA,

42 E.g, see a stock tutorial sheet posted at http://ckraju.net/sgt/Tutorial-sgt.pdf.

43 C. K. Raju, 'Retarded Gravitation Theory', in *Sixth International School on Field Theory and Gravitation*, ed. Rodrigues, Waldyr Jr et al. (New York: American Institute of Physics, 2012), 260–76; C. K. Raju, 'Functional Differential Equations. 4: Retarded Gravitation', *Physics Education (India)* 31, no. 2 (June 2015), http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-(2).pdf.

44 Raju, 'Time: What Is It That It Can Be Measured?' http://www.ckraju.net/papers/ckr_pendu_1_paper.pdf.

45 Raju, *Time: Towards a Consistent Theory*.

anomaly, or Oumuamua. (f) My earlier proposed retarded gravitation theory (RGT 1.0) was a simple modification of Newtonian gravitation to make it consistent with special relativity. The simplicity has the advantage that it enables a solution of the many body problem which could not be solved in GRT for a century. However, it was later found that this modification is limited to v^2/c^2 effects, similar to those in GRT. (g) The more recent version of retarded gravitation theory (RGT 2.0), first proposed here, decidedly allows effects at the v/c level, required to explain the flyby anomaly, but impossible in GRT. (h) However, for closed circular/elliptic orbits the v/c effects cancel out due to symmetry, so that the net effect is limited to the v^2/c^2 level required for the perihelion advance of Mercury. Thus, the v/c level effects (due to the rotation of the earth) will manifest themselves only for hyperbolic orbits (such as the flyby anomaly). (i) This can be experimentally tested, in principle, using spacecraft or laser-trackable satellites. The important conclusion is, if RGT 2.0 is right then general relativity (GRT) must be wrong. However, the purpose of this book is limited to Newton's errors about calculus, how they reflect in errors in his physics, and how that could be most simply corrected

12. **The general (relativistic) theory of shocks in general continua**[46]
(a) As already pointed out, the classical Western calculus with limits cannot handle shocks (discontinuities) and singularities. (b) The existing shock conditions cannot handle shocks in real fluids, such as air or water, with viscosity and thermal conductivity. It is simply deemed that no actual shocks in real fluids, although the design of rockets and supersonic aircraft requires a study of temperature increase across a shock in air. (c) The use of calculus based on non-Archimedean arithmetic is able to handle such cases, as the also some singularities. (d) However, empirical inputs are required because different forms of partial differential equations, with identical smooth solutions, may have inequivalent discontinuous solutions, as in Riemann's blunder of assuming that entropy is conserved across a shock. (e) Our resulting shock conditions can be empirically tested (for supersonic wing design) in a wind tunnel, etc. (f) They generalize to the case of general relativistic shocks and surface layers, generalizing all previous general relativistic junction conditions, such as those of Papapetrou, Israel, Kuchar, and Taub. (g) Singularities are now seen as merely the failure of the Western understanding of calculus, not as a moment of creation, or as something deserving the Nobel prize. In particular, shocks may arise during gravitational collapse, and this theory shows that a surface layer must accompany a shock in real continua. Since this surface layer would have a non-unique extrinsic geometry, geodesics would be inextensible across the shock, which could easily be confounded with a Penrose "singularity". That is, a mere "singularity" is *not* a robust indicator of a black hole, as wrongly stated by the Nobel prize committee.[47]

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46 C. K. Raju, 'Renormalization and Shocks', in Cultural Foundations of Mathematics (Pearson Education, 2007); Raju, 'Distributional Matter Tensors in Relativity', cited earlier.
47 C. K. Raju, 'A Singular Nobel?', cited earlier.