Kosambi the Mathematician

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Apart from his more popular work on numismatics and genetics, D D Kosambi worked on path geometry, exploring the foundations of general relativity. He also worked on statistics in infinite dimensions, computing, and probabilistic number theory. His whole mathematical career appears as one long clash of values. A rejection of the value of specialisation saw him leave Harvard. The high value he placed on research saw his exit from Banaras Hindu University and Aligarh Muslim University. His attempt to impart real knowledge of mathematics saw him sacked from Fergusson College, Pune. His insistence on ethical and relevant research led to his exit from the Tata Institue of Fundamental Research where, too, the diversity of his interests was portrayed negatively, though he continued his mathematical research till the end of his life. His mathematical career raises a number of questions regarding science management in post-independence India. These questions are vital today when the state is again making huge investments in science and technology.

I would like to thank R P Nene for a variety of inputs.

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D D Kosambi successfully applied mathematics (or rather statistics) to history (through numismatics).[1] Why not the other way around? Why did he not do anything on the history of Indian mathematics? So asks Romila Thapar.[2]

Perhaps, Thapar might have better addressed a related question to herself. Why did not the leading historians in India provide any space for the history of science in the last 60 years? Capitalism breeds specialisation (which leads to higher profits): therefore a scientist's wider interests (in, for example, history) are viewed negatively, as a disqualification and may jeopardise his job – as happened with Kosambi. Consequently, neither historians nor scientists, in India, are willing to do the history of science, except as a post-retirement pursuit. (And there are no professional historians of science, for there is no university department for the history and philosophy of science in the country.)

Remarkably, even our Marxist historians, who keep holding up Kosambi as a model,[3] never bothered in these 40 years to study his main preoccupation (and means of sustenance), which was mathematics. I am no expert in the history of mathematics in that period. Moreover, Kosambi published also in French,[4] German,[5] Italian,[6] Japanese,[7] Chinese,[8] and Russian[9] journals, so I know only some of his publications in one language. If I have nevertheless written this article on Kosambi, it is only to set the ball rolling and because consideration of Kosambi's scientific career highlights some key current issues, which need discussion. Science has become the new holy cow in Indian tradition on which a critical historical perspective is badly needed at this juncture,[10] particularly when the state is again making enormous new investments in it.

Apart from ordinary poverty, capitalism also breeds information poverty – or scientific illiteracy. The capitalist welcomes new technology, as a source of profits, but is disinterested in educating the general populace. This is evident in the enormous disparity between well-funded high-end research institutions and ill-funded (or non-funded) ordinary schools and colleges. This disparity creates a huge knowledge gap between the high-end researcher and all others. Even the otherwise educated elite are scientifically illiterate – to the point that they cannot even discriminate between mathematics and statistics – and are not ashamed of it, as they would be if they wrote bad English. Widespread scientific illiteracy is especially true of India where people have simply transferred their *bhakti* (devotion) from things like astrology to science. Science is today the dominant form of epistemic authority which people blindly accept without understanding, and without question. In practice, this amounts to uncritical reliance on a few persons in positions of authority ("experts") in the scientific community, howsoever that authority was obtained.

This "expert raj" suits the capitalist, who uses it to exploit scientific illiteracy just as ordinary illiteracy was exploited earlier. Kosambi was almost a solitary critical voice who could see through this uncritical belief in science and ask: "Is it any less a superstition?".

For example, Kosambi mentions the case of an Indian solar cooker, which used a pot polished from the outside to improve its looks and make it more saleable.[11]

> We know that the cooker produced some years ago with such fanfare and self-congratulation is useless. Even a schoolboy should have known that the pot at the focus of the solar cooker, being nickelled and polished, would reflect away most of the heat. But our foremost physicists and research workers, who rushed to claim personal credit and publicity, did not realise this.

It is a common mistake to suppose that this sort of thing happens only in India. For example, Stephen Hawking's singularity theory is a sophisticated discourse on creationism.[12] But few people, even among relativists, understand singularity theory. So, to decide scientific truth, they weigh social authority. Thus, millions can be persuaded about creationism simply by marketing Hawking as the new scientific messiah.

Second, we must situate Kosambi in the context of a historical transformation in the understanding of science in India. In the 19th century, the Indian elite had concluded, rightly or wrongly, that Indians were colonised because they were inferior, and that they were inferior because they lacked science and were steeped in superstition. This led to the well-known reform movements during which the elite demanded education in "western science", not Sanskrit. Calcutta became a well-known centre for scientific activities, with key achievers such as J C Bose, S N Bose, K S Krishnan, and C V Raman. During the first part of the 20th century, science was pursued in mission-mode.

Post-independence, however, science became professionalised – a quest for funds, not truth. Post-independence scientists in India (think of any famous name) are people who control (or have controlled) vast funds and power. It is impossible to say how their scientific work, if any, has benefited the people in their everyday lives. There is a vast difference between J C Bose, inventor of the radio, and Raja Ramanna, Kosambi's junior colleague, and former chairman of the Atomic Energy Commission. On the other hand, scientific illiteracy ensures that scientific truth is decided purely by authority, making scientists even less accountable than bureaucrats, or businessmen, or the judiciary. Big funding with no accountability has led to the emergence of a veritable science mafia in the country, and Kosambi was one of the isolated few to resist it.

> But by research is not meant the writing of a few papers, sending favoured delegates to international conferences and pocketing of considerable research grants by those who can persuade complaisant politicians to sanction crores of the taxpayers' money. Our research has to be translated into use.[13]

Kosambi, therefore, needs to be celebrated as a symbol of dissent, and studied if only to understand what is wrong with Indian science today.

First, however, an account of Kosambi's career and achievements as a mathematician is in order. I will suppose that Kosambi's map function,[14] which replaced Haldane's, is well known, like his work on numismatics, and I will restrict myself to the neglected aspects of his work on path geometry, statistics in function space, and number theory. Part 1 deals with "pre-history": or his career and work on path geometry (which, along with tensor analysis, he describes as his "main field of work for 20 years"), and statistics in function space, which took place before the publication of his book on history. Part 2 concerns his later years, where his work was on number theory, and also considers the contemporary lessons that can be drawn.

1

Career

Kosambi was initially weak in mathematics[15] in his early days of school in Poona. But he worked on it, and soon turned mathematics into a strength, just as he transformed his initially frail body into an impressive physique while at Harvard Grammar School and Cambridge High and Latin School in the early 1920s. He went on to study mathematics at Harvard University, under the famous G D Birkhoff, to become the "strongest" mathematician, as he called himself (meaning physically strongest). Birkhoff spotted Kosambi's talent, and admitted him to a personalised student research course on the many-body problem. He also advised Kosambi to focus on mathematics. Kosambi sensed that specialisation was a disguised form of semi-literacy; he valued what he called the "Renaissance type of versatility: wide range of knowledge without sacrificing depth". He consulted his father who concurred that he should acquire knowledge as widely as possible. Thus encouraged, Kosambi used the freedom available in American universities to offer 18 courses in a year. Though he excelled in his studies, and graduated summa cum laude in 1929, together with a Phi Beta Kappa, his refusal to specialise went against him, even at the very beginning of his career, for, on the capitalist value of specialisation, non-specialists are taken non-seriously. In any event, Birkhoff did not offer Kosambi a fellowship for further research in mathematics at Harvard. The Great Depression had just started, and Kosambi returned to India. On the way, he stopped by in France, to try (unsuccessfully) and meet T Levi Civita (who later communicated some of his papers) and Eli Cartan (of Bourbaki fame,[16] who later commented on his work).

Shortly after his return, Kosambi took up a job at Banaras Hindu University. Despite a sympathetic vice chancellor (Madan Mohan Malviya, Kosambi senior's friend), Kosambi did not find the atmosphere there conducive to knowledge. He shifted within a year to Aligarh Muslim University where the mathematics department was headed by André Weil, also of Bourbaki fame. Weil could not cope with the politics at Aligarh, and was soon sacked, and Kosambi followed in 1933. He had got married in 1931 and settled in Poona, in a house built for him off Bhandarkar Road. Eventually, like his illustrious father, Kosambi settled for a lower social position and presumably greater peace of mind. Like his father, he took up a low-paid job in the nearby Fergusson College, where he taught mathematics for many years. As recognition for his research in path geometry, he received the Ramanujan prize in 1934. He described this situation as 12 years *vanvas* (banishment to a forest), for he lived in Erandavane, and had no one with whom he could discuss his mathematical research. His students were only interested in passing examinations to secure a job.

The few mathematicians he could interact with in Poona included a person with the improbable name of Wrangler Paranjape, who carried his undergraduate achievements in Cambridge as a sort of British-conferred title all his life (and even beyond in that at least one building was named after him) though he never did any research in mathematics. The anecdote goes that, one day, Paranjape arrived to meet Kosambi and was late by some 10 minutes. An annoyed Kosambi, who had imbibed the value of punctuality in the US, opened the door himself and said "Professor Kosambi is not at home" and then shut it. (There are other, milder, versions of this anecdote.) One must recall that, as principal of Fergusson College, Paranjape had given jobs, even if only ill-paid ones, to both Bapu and Baba Kosambi.

Eventually, Fergusson College sacked Kosambi, on the alleged grounds that students did not understand the mathematics he taught. Mathematics is a difficult subject to teach, especially to exam-oriented students in India. If one tries to teach the subject in its proper spirit, as Kosambi did, one loses most of one's students whose aim is a job or a degree, and not knowledge. The best mathematics teacher I had in my life, M S Huzurbazar, could also have been sacked for the same reason. Forced out of his vanvas, Kosambi met Homi Bhabha who was then expanding the Tata Institute of Fundamental Research (TIFR) in Bombay. Bhabha offered Kosambi a lucrative job which involved pure research, and which paid him more than five times what he had got as a teacher in Fergusson College. Kosambi accepted this in June 1947,[17] the same year he was given a special "Bhabha prize".

As is well known, Kosambi commuted from Poona on the Deccan Queen, switching from daily commute to weekend commute and back to daily commute. The two-way train journey itself was seven hours. Additionally, in the morning, he trudged a few kilometres from his house to the Poona station with a haversack full of books to read on the journey. The countryside, which Kosambi loved, had great natural beauty. As J D Bernal perceptively commented,[18] for Kosambi, "history was not only in the past but also in the present", so his thoughts on these journeys naturally turned to history. (His book on history was written on the Deccan Queen during these lengthy commutes. He is also known to have consumed three mystery novels in three hours, and donated hundreds of science fiction and mystery novels to the common room in TIFR.) Since he had a few kilometres to go on the Bombay side, first to Kenilworth, on Peddar Road, and then to the Old Yacht Club at Gateway of India, and finally to Holiday Camp (now Navy Nagar) beyond Afghan church, Kosambi naturally joked that TIFR was just a "third-class waiting room", which he used between train journeys.

First Paper

The first time is always of great interest. Kosambi's first paper was on "Precessions of an Elliptical Orbit".[19] As one would suspect, it links planetary orbits to atomic orbits: "The solar system is but a step away from the atomic model". But the sweep is breathtaking. The paper starts unexpectedly with hydrodynamics, giving the simplified equations of motion for an infinite cylinder. It then points out that these equations could be used to describe "an electron in a magnetic field, a Foucault pendulum, and even the restricted problem of three bodies". It then moves on, via observations of the motion of banjo strings, to general relativity, and points out that "relativity has still to account for the observed precession of planetary orbits". He then focuses on Venus and just the effect of its direction of rotation on the direction of precession (recognising the limitations of the simplified theory he is using). He then tries to link it to the Zeeman effect and the electron spin-orbit interaction in the Bohr model, and moves on to the Raman effect. He concludes with a passing mention that "electrodynamically unsound models have usually been employed for purposes of illustration" and refers to the notes of Birkhoff. The paper is dated "Harvard, September 1927; Banaras Hindu University, April 1930". It may not amount to much, but for a paper written by an undergraduate student just out of his teens, it is absolutely astounding.

Path Geometry

From the very beginning, Kosambi's mathematical research was where the action was: on the frontiers. Relativity was one of the most intellectually exciting and challenging developments of that period. One of the most dramatic scientific events of the time was the sensational newspaper reporting on the three empirical tests of general relativity in 1918, which made Albert Einstein into a celebrity. Arthur Eddington had then boasted that only three people understood relativity.[20] As a schoolboy in the US, during 1918-24, with a keen interest in science, Kosambi grew up with these stories, which must have had a deep impact on him. As he puts it, the theory "aroused theological passions". That is the theatre Kosambi's mathematics of path spaces[21] impinged upon, and on which he worked for 20 years.

The correct equations of general relativity theory (GRT) were first obtained by David Hilbert in 1916, as a sort of mathematical exercise during vacation, and accepted with great alacrity (within five days) by Einstein,[22] eager not to lose out on the credit. Possibly because of the haste in its formulation, GRT leaves many questions unanswered.

The broad agenda of GRT was to replace the Newtonian concept of force by geometry, and this idea had been around for some time. But which geometry? As currently formulated, GRT uses Riemannian geometry. (This is named after Bernhard Riemann, a famous 19th century mathematician, who speculated along the lines of a four-dimensional space-time continuum, but was unsuccessful, since he lacked the special theory of relativity, and was not clear how to handle the physical notion of time, so his time coordinate was no different from a space coordinate.)[23] But the reasons for this choice of Riemannian geometry in GRT are not entirely clear, and raise several questions. Should one use what are called Finsler spaces instead? (In Finsler geometry, the metric, or the measure of "distance", need not involve a quadratic form, as it does in Riemannian geometry.) Kosambi's path geometry included both Riemannian and Finsler geometry as special cases.[24]

Another key dispute of the times was whether geometry should be metric at all. While devising new foundations for mathematics, around 1900, Hilbert (and Bertrand Russell) interpreted Euclidean geometry as synthetic or non-metric. This was contested by Birkhoff who advocated a metric interpretation of Euclidean geometry. (The difficulty was that the metric interpretation trivialises the *Elements*, attributed to "Euclid", for the "Pythagorean" theorem can be proved in one step.)[25] A similar dispute applied a fortiori to non-Euclidean geometry. Length measurement requires lengths to be picked and carried. But, in curved space-time, picking and carrying lengths can distort them, just as a shadow moving over an uneven surface gets distorted. Geometry, it was thought, should be concerned with invariants. So, should one drop the metric, and do non-metric geometry (purely with the affine connection, or some notion of parallel transport) as suggested by Cartan? (From the "debauch of indices" perspective, use of only the affine connection amounts to working with the Christoffel symbols, $\Gamma^{\mu}_{\nu\sigma}$, without the metric tensor $g_{\mu\nu}$.) The absence of a metric, or the strange idea of defining area without defining length, however, makes geometry useless for mathematical physics, as Kosambi noted. Kosambi explored the circumstances under which his path spaces admitted a metric.

Yet another question relates to geodesics. A geodesic is an extremal path, either the shortest or the longest path between two points; in flat space, geodesics are straight lines, but on a curved surface they may be curved lines. For example, on the surface of the earth (regarded as a sphere) the geodesics are great circles. Geodesics may be regarded as characterising the surface or higher-dimensional geometry in question. A key idea of general relativity is that one can substitute the Newtonian force by a space-time geometry in which the geodesics are approximately the particle trajectories that would have been obtained with that force. Thus, instead of saying that the planets move in elliptical orbits around the sun because of the force exerted by the sun on the planets, one says that planets move in geodesics of the Schwarzschild geometry, due to the solar mass. The three famous tests of general relativity actually tested geodesics in Schwarzschild geometry. (The Schwarzschild geometry was obtained as the first solution of the equations of general relativity; it was obtained even before the equations were finalised. The solution corresponds to the "field" of a point mass. Planets being at large distances from the sun, the difficulty that a point mass must be surrounded by a black hole is not relevant for planetary orbits.) But is it meaningful to speak of geodesics in the absence of a metric? The calculus of variations studies extremal paths, and Kosambi considers an affine (or non-metric) calculus of variations.[26]

The replacement of force by geometry also brings in aesthetic considerations. To see this we need to consider the relation of force to geometry in another way. In Newtonian physics, masses are usually treated as geometric points, and geometric curves arise naturally as the trajectories of these points. For example, if we throw a ball, its path is approximately a parabola. The exact trajectories are obtained by solving Newton's equations of motion (Newton's second "law"), which are ordinary differential equations of the second order. In the Lagrangian reformulation of Newtonian physics, one can regard the very same curves as the extremal paths of the action (which is the integral of the Lagrangian with respect to time). These extremals are studied in the calculus of variations, where they are obtained as solutions of the Euler-Lagrange equations, which may be ordinary or partial differential equations. An interesting aspect of this Lagrangian reformulation is that the Lagrangian function can be decided largely or entirely by considerations of symmetry. Mathematically, these symmetries form a group, which helps characterise the geometry in question. From this perspective, the key difference between Newtonian gravitation and general relativity may be roughly stated thus: Newtonian physics is required to be the same for all observers moving with constant velocity (invariance under the Galilean group), while general relativity is required to be the same also for all accelerated observers (general covariance). In contrast, special relativity is restricted to the Lorentz group (observers moving with constant relative velocity, when time measurement depends upon relative velocity).

This awe-inspiring grand narrative that the nature of the cosmos is decided from purely aesthetic (and geometric) considerations was seen to have some flaws even in the 1930s when Kosambi started his research. In his theory of path geometry (a term which he coined himself),[27] Kosambi explored these ideas. He also took up an issue which seems not to have been much discussed in the literature. Newton's equations of motion are the same for all inertial observers (moving with constant velocity) because they are of the second order. (Since velocity is the first derivative, taking the second derivative kills any constant velocity term that one might add.) However, if one now demands general covariance (indifference to the acceleration of the observer as well), there is no longer any reason why the equations of motion should still remain of the second order – they might well be of higher order. In this situation, what sort of geometry would emerge? This was a question that Kosambi investigated in his theory of path spaces of higher order.[28]

Although Kosambi does not explicitly motivate the use of higher-order differential equations of motion (or perhaps he mentions it somewhere, for I have not seen all his papers), he could easily have stressed the following compelling point. When a charged particle accelerates, it radiates energy (the technology of the radio, television and microwave is based on harnessing electromagnetic radiation). Consequently, the motion of the charged particle is damped. However, the force of radiative damping involves the third order derivative in time. This upsets the stock scheme of Newtonian physics, and the issue of what to do with this third-order term has remained unclear practically to this day.[29] This issue is also closely related to the difficulty of the "unified field theory" which fruitlessly occupied Einstein for the final decades of his life: whether and how one could include electromagnetic and other forces, apart from gravitation, into geometry, possibly in a higher dimensional space. Kosambi had, in 1934, already applied path geometry to this question of the "unified electro-gravitational field theory", as he called it,[30] particularly what is today called the unified field theory of De Donder, Einstein and Mayer.

Apart from the very high level of abstraction involved (even by the standards of relativity) in the kind of non-Riemannian geometry involved in Kosambi's path geometry, these papers are remarkable for their breadth of vision and for their use of sophisticated mathematical concepts. Kosambi extensively uses the theory of Lie groups, then still in its infancy. He repeatedly cites

Richard Courant and Hilbert's *Methods of Mathematical Physics* (in the original German) which banks heavily on variational principles. There is also a noticeable amount of explicit computation with tensors, suggesting that Kosambi enjoyed playing with them.

There were other philosophical questions such as those raised by Edward Athur Milne: whether science is inductive (beginning with observations and moving to principles) or whether it is "hypothetico-deductive" (beginning with hypothesis and deducing consequences which are then compared with experimental data). These related to the "cosmological principle", which was later turned into the "perfect cosmological principle" by Hermann Bondi in the formulation of the steady-state theory in 1948 in association with Thomas Gold, and Fred Hoyle. Kosambi links his path geometry programme to Milne's cosmology.[31]

Through all this maze of mathematical sophistication, the persistent vision was to explore an alternative, less arbitrary and more general reformulation of the general theory of relativity. Kosambi eventually visited the Institute of Advanced Study, Princeton, as a guest, and discussed his work with Einstein in what he described as "several long and highly involved private technical discussions".[32]

It would be inappropriate for me to assess Kosambi's work, for my own point of view is somewhat different. I have opined that GRT is an incomplete theory since it lacks the notion of "particle". (The simplest solution due to Schwarzschild gives rise to a black hole, so it is not easy to speak of even the "test particles" needed to formulate the geodesic hypothesis. The geodesic hypothesis states that "test particles" naturally follow geodesics.) An actual notion of particle (as distinct from the geodesic hypothesis) would bring in the equations of particle motion, which get coupled with the equations of the field.[33] Therefore, my opinion is that a purely geometric approach (which was Hilbert's forte, and which presumably led to his profound neglect of matter while formulating general relativity) is not likely to be ultimately productive for a physical theory of gravitation.

However, there are other indicators. As already noted, Kosambi received the Ramanujan prize for this work. Another way to assess Kosambi's work on path geometry is to note the difficulty of the questions he tackled, and that can be judged by the fact that the answers are not clear yet, even after seven decades. Then, one can look at the impact among his fellow mathematicians. Some Japanese mathematicians took great interest in his work, and translated it into Japanese. On what I have called the "Indian scientific method", the value of science (especially mathematics) is judged solely by the reactions of prominent western scientists. From this perspective, Kosambi was very successful, for the foremost western mathematicians of the time (such as Levi Civita, Cartan and Weil) responded well to him. As already stated, Cartan wrote an extended response to Kosambi's paper.[34] Further, as described by M S Raghunathan, in conversation, Weil told him, "Kosambi is one of the finest mathematicians in your country".[35]

Computing

Everyone has heard of how Kurt Godel knocked down Hilbert's number-theoretic program in 1929. However, few seem to understand that Godel's metamathematics involved, first, a whole-hearted acceptance of Hilbert's formalisation of mathematics, which has had far-reaching consequences, extending far beyond his celebrated theorems. In the 1930s, there was extensive development of formal mathematics leading to the formalisation of set theory by John von Neumann, Paul Bernays, Godel, and others. This was followed by the reworking of all mathematics in formal terms by the Bourbaki group, whose books became the Bible of mathematicians. (Non-mathematicians can perhaps connect to this as the "new math" using set theory, which was introduced in schools and colleges in India in the late 1960s and early 1970s.)

One of the consequences of Hilbert's formal and mechanical vision of mathematics was its influence on the theory of computation, around which present-day digital computers are built. Like atomic energy, this technology emerged during the second world war. The Tatas were interested in it, and Kosambi was awarded a Unesco fellowship in 1948-49 to tour the UK and the US to study this new technology of electronic calculating machines. He was probably the first Indian to study it. Curiously, in a paper,[36] in connection with an infinite dimensional stochastic integral (later related by others to Feynman path integrals, but still hard to formalise today in any context other than Brownian motion – such as Lévy motion), Kosambi proposes an elaborate and detailed design for an analogue computer. Note that Kosambi proposes an analogue rather than a digital technique of computation. The key relevant difference is this: the theory of digital computation was (and still is) based on formal mathematics, whereas analog computation is obviously based on physics. So, this attempt to bring in physical and empirical methods of computation into mathematics at this high level of abstraction is very interesting,[37] although it is not clear that Kosambi intended this as a direct challenge to the dominant philosophy of formalism in mathematics.

About his study tour, Kosambi said,[38]

> One of my theoretical papers deals with probability and statistics in infinitely many dimensions. There has been no effective use, because the attempts at getting a special electronic calculating machine to translate this theory into practice failed.[39]

However, that trip was productive in other ways. Kosambi revisited Harvard and renewed his links. He saw the changes that the war had brought about in American science. He also visited Chicago and gave a whole course of 36 lectures on tensor analysis there. He visited Einstein at the Institute of Advanced Study in Princeton (also to discuss path geometry) for a couple of months. He met with A L Basham in London. As regards computers, TIFR did later try to build its own computer (Oldap) which I do not recall ever having seen working beyond printing out coy statements like "Oldap is a good boy". Today, of course, Tata Consultancy Services is a corporate giant in the field of information technology outsourcing, and is now trying to do cutting edge research on its own.

Statistics in Infinite Dimensions

Kosambi said he got into numismatics because he used coins to teach himself statistics, and his first paper on statistics[40] related to numismatics. However, he also combined statistics with his deep knowledge of differential geometry, and successfully applied the geometric approach[41] to statistics in (infinite-dimensional) function space.[42] The infinite-dimensional analogue of the multivariate normal distribution is today known, in the theory of stochastic processes, as the Wiener measure, and is related to the stochastic process known as Brownian motion. It should be recalled that Wiener senior was a colleague of Kosambi senior, also teaching languages at Harvard, and he taught Russian to Bapu Kosambi. He had some radical theories about the origin of the native American languages and their relation to African languages. Norbert Wiener, whom he reared into a child prodigy (based on his radical theories of parenting), was Kosambi's friend from an early age. Although Wiener was 12 years older, he had a similar temperament, and the two became good friends. When Kosambi revisited the Massachusetts Institute of Technology in 1949, Wiener greeted him, "Welcome, wise man from the East", and Kosambi responded, "No, a wise guy from Cambridge".[43]

In the geometric approach to statistics, a random variable is regarded as an infinite-dimensional vector which can be expanded in an infinite basis in function space, somewhat in the way a finite-dimensional vector can be so expanded in a finite basis. While the meaning of a finite sum is unambiguous, there are issues of convergence in the infinite-dimensional case. Kosambi studied this first in a paper of 1942. This leads to an orthogonal decomposition, similar to the expansion in Fourier series, and known as Proper Orthogonal Decomposition. Today this is also known as the Karhunen-Loeve expansion after Kari Karhunen who seems to have picked up the idea five years later, in a 1947 publication. One wonders why G Kallianpur[44] has attributed "independent rediscovery" to Kosambi, for it was Kosambi who discovered it first, and Karhunen who "independently rediscovered" it later.

Regarding this business of "independent rediscovery", people seem unaware that Western scientists have been rather liberally and comically claiming the privilege of "independent rediscovery" for centuries. This is not confined to isolated incidents such as Guglielmo Marconi "independently rediscovering" the work of J C Bose (which piece of historical nonsense it took a century to correct) but applies also to purported scientific revolutionaries such as Copernicus and Newton. As such, this is a systematic phenomenon of appropriation. During the religious intolerance of the Inquisition, the claims of "independent rediscovery" were understandable, for theological deviance spelt big trouble. (Copernicus, a priest, waited till he was on his deathbed, to avoid the fate of his colleague Scultetus who was arrested by the Inquisition. Acknowledging his Arabic-Islamic sources would not have helped Copernicus' case for theological correctness in the preface of his "revolutionary" book. Similarly, Mercator was arrested by the Inquisition, and did not reveal the non-Christian sources for the projection and table of secants needed to construct his map.) Later day colonial historians built on these false claims. Such unethical claims by influential westerners of having "independently rediscovered" the earlier published work of an Indian continue to this day, as happened more recently with this author.[45] Even if there is no dishonesty involved, there is, as Newton remarked, no reason why the "second inventor" of an idea should get the credit for it. Therefore, the "Karhunen-Loeve" expansion ought to be renamed Kosambi's expansion.

2

In 1954, Kosambi published his last paper on path geometry and tensor analysis.[46] The next year, his book on history was published, and he led the Indian peace delegation to Helsinki, and toured China and the USSR. Kosambi published nothing in mathematics or statistics during 1955 and 1956. Sumit Sarkar recalls that when he graduated in 1958, Kosambi was not so famous as a historian. Irfan Habib (whose father was Kosambi's colleague at Aligarh) concurs. However, Kosambi's fame had spread as a Marxist statistician. In May 1957, he gave a talk to the Mathematical Institute of the Chinese Academy of Sciences on probabilistic aspects of Tauberian theorems.[47] Later he published a paper in a Chinese journal on an extension of the least-squares method to abstract function spaces.[48] He visited China under an Indo-China exchange programme, and according to biographer C Deshmukh,[49] was invited by the Chinese government to apply statistics to agricultural production. (His genetic mapping function is still widely cited in Chinese publications.) In any case, he subsequently began publishing in the *Indian Journal of Agricultural Statistics* on the applications of probability to number theory.

Probability and Number Theory

By way of background, Norbert Wiener, Kosambi's childhood friend, also contributed to number theory. The prime number theorem says that if $\pi(n)$ is the number of primes less than or equal to n, then asymptotically (i e, for large n) $\pi(n)$ can be approximated by the function $\frac{n}{\log(n)}$, where lo g denotes the natural logarithm. (Another version uses instead the logarithmic integral $\mathrm{Li}(n) = \int_2^n \frac{dx}{\log(x)}$.) After his PhD at the young age of 18, Wiener went from Cambridge to Cambridge, and, on Russell's advice, studied number theory with G H Hardy. Wiener obtained the prime number theorem by means of his famous Tauberian theorem of 1932 which used only the methods of real analysis, surprisingly contrary to Hardy's belief that complex variables were a must for its proof.

As regards the terminology "Tauberian theorem", broadly speaking, there are two categories of theorems, Abelian and Tauberian. These theorems originally related to the notion of (Abel) summability of a divergent series, which Abel stated to be the work of the devil. Abel's theorem asserted that if a series converged in the ordinary sense, then it also converged in the sense of Abel sums. The converse of an Abelian theorem is known as a Tauberian theorem: the original theorem by Tauber asserted that if the Abel sums of a series converge, then, under certain conditions, the series itself converges. This general idea of deducing the behaviour of a series from the behaviour of its weighted averages, such as Abel sums (or of a function from its integral transforms, such as Fourier transform) has come to be known as a Tauberian theorem. Towards the end of his life, Kosambi got drawn into the application of probability to number theory through the class of Tauberian theorems.[50]

The prime number theorem is also closely related to the Riemann zeta function which is defined as (the analytic continuation of) $\zeta(z) = \sum_{n=1}^{\infty} n^{-z}$, where z is a complex number (z is not equal to 1). The famous Riemann hypothesis, not settled to this day, asserts that all the (non-trivial) zeros of the Riemann zeta function lie on the vertical line Re $(z) = \frac{1}{2}$. In terms of the above functions, regarded as functions of a real variable *x*, the Riemann hypothesis has been shown to be equivalent to the statement that $|\mathrm{Li}(x) - \pi(x)| \leq c\sqrt{x}\log(x)$. Taking off from his work on Tauberian theorems, Kosambi developed a statistical approach to the Riemann hypothesis. This was a bold and innovative step. However, what Kosambi did next was a bit strange: he published a purported proof of the Riemann hypothesis in the *Indian Journal of Agricultural Statistics*.[51] In his own annotated list of papers, however, he carefully noted that this only gave a conditional proof: he said that it "shows that if the primes in suitably defined covering intervals behave like an unbiased random sample, then the Riemann hypothesis follows".[52]

Possibly the unconditional statement was one of those jokes which needed to be explained,[53] as sometimes happens with even the simplest jokes.

Kosambi's possible joke about the Riemann hypothesis may be better understood if it is placed in the context of the dilemma faced by B Bagchi, who did his PhD thesis on the Riemann hypothesis. He, too, applied statistical techniques to number theory, and subjected me to his "endless ramblings" on it, as he acknowledges in that thesis.[54] The Riemann hypothesis is one of the most difficult unsolved problems in mathematics. Bagchi could neither prove nor disprove the Riemann hypothesis. But he obtained some results that he thought were of value. An issue that worried him then was this: would that research be judged adequate for a PhD? What was he to do with those results?

This dilemma brings out a central difficulty with formal mathematics: the object of research in formal mathematics is to prove theorems – a good (formal) mathematician is one who proves good theorems. While a mathematical proof (as enunciated by Hilbert) can be mechanically checked for *validity*, there is no objective way to ascertain *value,* for all theorems are tautologies, some less obvious, some more so. The empirical world does not intrude at all into formal mathematics, unlike the way it does even in a pseudoscience like astrology. Therefore, the value of a mathematical theorem, like the value of a piece of art, lies solely in the eyes of the beholder. Unlike a piece of art where the beholders can be many, only a few people may understand a piece of advanced formal mathematics well enough to assess it. Due to the combined effects of scientific illiteracy and semi-literacy (over-specialisation) this number may sometimes be five or six people in the world – if one is lucky. Therefore, value is typically assessed by a few "experts" who can be deeply influenced by various subjective considerations. Had the Riemann hypothesis been proved, the value of the theorem would have been demonstrated. But, for a partial result, no one can say for sure whether it is the right road or a blind alley; so, judging its value, like judging the value of most other research in formal mathematics, calls into play the subjective perceptions of those in positions of social authority. Since formal mathematics has no external empirical anchor, the dependence on authority is total. On the maxim that absolute power corrupts absolutely, those in positions of authority in mathematics tend to get corrupted, and to use this subjectivity in ways that are not above board. This is especially true of formal mathematicians in a dystopian society.

So what could Kosambi have done? He had some results that he thought were promising, but could not establish their value unambiguously, for the non-expert, since he could neither prove nor disprove the Riemann hypothesis. On the other hand, there were too few experts around. In the matter of Bhartrhari, Kosambi was concerned that his "heavy labours" should not go to waste, and should at least provide guidance to future workers. So, the natural thing in this case too was to create a record of what he had done, and also explain its value to others – most of whom might otherwise miss the connection, for connecting probability to number theory was such a very radical idea at that time.

But doing something so unsubtle would have been contrary to Kosambi's academic upbringing at Harvard. On the other hand, Kosambi was well known from childhood for playing practical jokes.[55] Therefore, it is possible that he combined subtlety with wit, and as yet another prank, published a "proof" of the Riemann hypothesis in the *Indian Journal of Agricultural Statistics*. The curious choice of journal in which so momentous a claim was made does suggest that he was perhaps not serious about the claim and well knew that the purported proof of the Riemann hypothesis was not unconditional, as he clearly recorded in his own list of papers.

Kosambi had played such a prank earlier in his second paper.[56] That was quite a historic prank, for it confirmed the christening of Bourbaki, the famous "mathematician" whose works are the Bible of formal mathematicians.

> In later years, Weil was at a meeting in India and told his friend Kosambi the story of the incident at the Ecole [where a student donned a false beard and strange accent and gave a much advertised talk which was balderdash from start to finish]. Kosambi then used the name "Bourbaki" in a parody that he passed off as a contribution to the proceedings of some provincial academy. Soon the still-nascent Bourbaki group determined that this would be its name. Weil's wife Eveline became Bourbaki's godmother and baptised him Nicolas.[57]

However, after Hilbert, a mathematical proof is required to be addressed to a machine or a moron, so present-day mathematicians often have neither a sense of humour nor of ethics, but only a sense of quibble.

A young man asked me why Kosambi played such a rash prank about the Riemann hypothesis when he had so many enemies. My only answer is that he probably did not expect his enemies to be so undignified as to hit below the belt – that too at a joke. Inevitably, however, this possible attempt at subtlety and humour had a tragic consequence. The Riemann hypothesis is notorious for the numerous wrong proofs of it that have been published. Kosambi's prank was misconstrued by the humourless mathematicians in TIFR to portray him as a crank and to sack him by not renewing his contract. Typical of the culture of back-room gossip,[58] which characterises the TIFR mathematics school built by K Chandrasekharan, Kosambi was not asked for his version nor given a chance to explain his side.

Kosambi remained active in mathematics, and continued his work on probability and number theory[59] even after his removal from TIFR. Characteristically, the same idiosyncratic sense of humour was again on display. Realising that the people at TIFR were petty enough to block his papers, he wrote papers under the pseudonym of S Ducray, who thanks Kosambi.[60] The name Ducray sounds vaguely French (*a la* Ducré), but *dukkar* means "pig" in Marathi (where it means rather more than "pig" in English – closer to "swine"). Sure enough the papers were published. Kosambi kept up his interest in this area. A few days after completing his first book on mathematics (the typescript of which has mysteriously disappeared[61]), Kosambi died in his sleep.

Contemporary Lessons

As the late Ravinder Kumar used to say: history is futuristic. What contemporary lessons can we learn from Kosambi's history? Why did India have no room for talented and knowledgeable people like him? Instead of being honoured for his lifelong devotion to research, he was repeatedly sacked, and left in bad financial circumstances. Can something be done today to prevent this history from repeating itself?

These questions are particularly acute today when we again stand at the threshold of large-scale changes in the education system. Post-independence India happily continued with the educational system it blames on Thomas Macaulay. Education was seen as a passport to a scarce white-collar job, so the bulk of the country was deliberately kept illiterate. Money was available for almost anything except education. However, recent years have seen a boom in outsourcing IT, business processes (BPO) and knowledge processes (KPO) to India. The fear, now, is that the comparative advantage of lower labour costs may evaporate. So, the large illiterate population in India is now being seen as a resource. Accordingly, a serious attempt is now on to educate the masses, since the perceived future of corporate profit depends upon that. A National Knowledge Commission has been constituted to this end. The changes that it recommends may have as substantial and long-term an impact as the changes made by Macaulay and Nehru earlier. Under these circumstances, repeating history by not learning from past mistakes could be an unmitigated disaster.

Let us examine Kosambi's case in more detail. Kosambi was, by all accounts, an extremely erudite and creative person, and such people are needed for research. So why was Kosambi sacked from TIFR? We have to understand Kosambi as a representative case: the systematic elimination of knowledgeable and creative persons has practically characterized science management in post-independence India – Kosambi was repeatedly eliminated by the system. The reasons are not hard to understand.

In the first place, like many knowledgeable and creative persons, Kosambi tended to be outspoken – what is the point in being knowledgeable if one cannot communicate this knowledge to others? According to a well-known anecdote, at a social gathering in TIFR, speaker after speaker got up to praise the Tatas for their benevolence in contributing their hard earned money for the noble causes of research and education in setting up TIFR. Kosambi, a Marxist, knew the real reasons why TIFR was set up. However, he disagreed only with the description of the Tata money as "hard earned". He felt compelled to explain that capital often starts off as rogue capital before turning respectable. And he illustrated this by pointing out that the Tatas had earned big during the opium wars. The truth, however, may not be spoken loudly in a hierarchical system of management – Kosambi's remarks naturally did not go down well with Bhabha who was a close relative of the reigning Tata.

Post-Independence Science Management

Those remarks might have been shrugged off, but there was a more fundamental reason: Bhabha's knowledge was insecure. Bhabha's initial training was in engineering, which was what was considered valuable for the family business. (We must remember that Bhabha was once seen as a potential successor to the Tata empire since his aunt was married to the reigning Dorab Tata, and the couple was childless.) It is another matter that the youthful Bhabha, smarting under the intellectual snobbery of Cambridge, later switched to physics, like Paul Dirac, the idol of the times. Though Bhabha subsequently took the mathematics Tripos, it did not teach enough physics either. So, he hired a displaced Jew, Heitler, to teach him physics. Bhabha is known in physics primarily for the paper that he wrote jointly with Heitler. It is a common tactic among our science managers to use their personal or institutional wealth to exploit those (such as research scholars) who are financially insecure. As Kosambi put it:

> The Byzantine emperor Nikephoras Phokas assured himself of ample notice from superficial observers, at some else's expense, by setting up in his own name, at a strategic site in the Roman Forum, a column stolen from some grandiose temple. Many of our eminent intellectuals have mastered this technique. There is little point in discussing the personal experiences of the scum that naturally floats to the top...[62]

While the written medium permits one to exaggerate one's knowledge and innovativeness, through joint or commissioned publications, it is difficult to hide one's ignorance during a live, oral discourse. For example, Bhabha's deputy and successor, Raja Ramanna, publicly asserted, at a seminar in Calcutta, in response to a query by this author, that "every matrix has a diagonal, even a rectangular matrix has a diagonal", and demonstrated this visually by drawing a rectangle and its diagonal on the blackboard!

A hierarchical knowledge-management system does not allow the worker to be more knowledgeable than the manager, for this risks exposing the ignorance of the manager. To put Kosambi in his place, Bhabha appointed Chandrasekharan, a junior and undistinguished mathematician, over his head as deputy director. It always seems a smart move for the manager to appoint a subservient deputy. To ensure constant dependence, sycophants are selected for incompetence. This can gravely damage the entire system through a chain reaction, leading to long lineages of sycophants of progressively degraded quality, institutionalising incompetence.

Chandrasekharan, naturally, was rather more insecure than Bhabha. As Kosambi put it:

> The greatest obstacles to research in any backward, underdeveloped country are often those needlessly created by the scientist's or scholar's fellow citizens...The meretricious ability to please the right people, a convincing pose, masterly charlatanism, and a clever press agent are indispensable for success.[63]

As pointed out earlier, widespread scientific illiteracy ensures that the scientific method, in practice, is reduced to mere reliance on scientific authority. On the "Indian scientific method" this scientific authority is taken to reside in the west – western social approval (such as publication in *Nature*) is thus regarded as the ultimate test of scientific truth. Consequently, it was natural for Bhabha to conflate development in science and technology with mimesis of the west, as many people still do today. Bhabha made a clear distinction between an east canteen in TIFR (where ordinary folk ate, and which served Indian food), and a west canteen (where the scientists ate, which served western food). He always wore western clothes, and reportedly even went so far as to demand that scientists in TIFR do the same.[64]

Bhabha's affinity for the west suited Chandrasekharan who built strong linkages with the west on one side, and with people from his caste and college in Chennai on the other. (TIFR was and remains exempt from the purview of reservations, so this freedom was bound to be misused. Andre Weil, in his autobiography[65] mentions the dominance of "brahmins of southern India" in the TIFR math school, but gives a hopelessly naïve sociological explanation for it.) Chandrasekharan's strategy fetched the mathematics school of TIFR a good reputation in the west.

However, the TIFR mathematics school contributed nothing of value either for the people of India, or for the atomic energy programme, despite more than half a century of intensive public funding, and despite access to the very best talent in the country. On the contrary, it exerted a strong negative influence across mathematics departments in India, and systematically (and often unethically) eliminated those who could do something of practical value, Kosambi being just the first. (When it was my job to implement various applications of national importance on the first Param supercomputer, despite offering high salaries, I could find no competent mathematicians in the country for the practical tasks at hand. Kosambi, incidentally, did link his work on statistics to diffusion in atomic reactors.)[66] The stranglehold of the TIFR math school has led to the systematic failure of mathematical creativity across the country. Admittedly, salaries in the west are far higher than in India, but if western social approval is what the country wants to spend money on, in the name of research in mathematics, that should not be done under the misleading heading of expenditure on atomic energy.

Nehruvian Vision

A little deeper enquiry would help to place the issue also in the surrounding economic and political circumstances. Briefly, the Tatas were then interested in diversifying in a big way into energy, especially in exploring the possibility of atomic energy, which could bring in super profits like any new technology. But they lacked the know-how. Moreover, as Kosambi pointed out, the whole project of atomic energy was "fantastically costly" if one took into account the costs of the research involved in generating the know-how. The investment in research made the project unprofitable, unless the state was persuaded to underwrite the costs of research, while privatising the profits. Though the plan was initiated by the Tatas, this policy suited Nehru.

Nehru had a vision of modern India where capitalists would replace the traditional rajas, and science would replace superstition. In this vision, both private capital and science needed to be built up in the country through the agency of the state. So he created large research enterprises: vast sums of money were pumped in by the state into the departments of atomic energy, space, and scientific and industrial research. Nehru appointed the scions of leading industrial families to the apex management of these enterprises: Bhabha for atomic energy, Vikram Sarabhai for space, and the Birlas for the Council of Scientific and Industrial Research (CSIR). The sole expected outcome from the vast investments in these research enterprises was knowledge – to which those in apex managerial positions naturally had unhindered access. Thus, these vast state investments in knowledge were intended also to act as an invisible subsidy to help accumulation of private capital in India.

It is routine to pass on the results of state-funded research to private capital, but Nehru built a unique system of science management in India, by going a step further and giving representatives of private capital managerial control over state-funded research. Therefore, he forced scientists to be subordinate to capital. Those whom Kosambi called "official Marxists" happily went along, for they clung to the antiquated idea of science as a "progressive" force regardless of how it is managed.

Nehru reinforced this policy of "science for the capitalist" with two further profoundly anti-democratic measures. The first was to take the Department of Atomic Energy outside the purview of even Parliament – Bhabha accounted directly to Nehru. (In retrospect that was excessive, for science and technology issues are rarely debated in Parliament, except for absurd things like the petrol-from-water case!) Second, the last vestiges of any resistance through the university system were crushed by sidelining the enterprising Meghnad Saha in favour of Bhabha.[67] Over the next 60 years, this led to a concentration of funds and powers with science managers, who have now virtually established a dictatorship, while the relatively more democratic universities were systematically starved of funds bringing them to their present plight.

In line with the "Indian scientific method" of relying on western social authority as the sole means of validating science, our science managers became mere brokers of western scientific knowledge. The huge funds available to science managers were mostly used to import outdated technology and assimilate scientific developments from abroad, establishing linkages with the west, and getting also their brokerage in this process. Since funds and power were ends in themselves (and not the means to an end) for key science managers in post-independence India, this created an oppressive atmosphere to escape which the really talented and knowledgeable people often emigrated to the west. Like the dependence on western authority to decide truth, or the dependence created by the import of outdated technology, the export of talent is a process most appropriate for a vassal state.

The importance of the last factor is not recognised: western dominance depends upon a lead in technology, which, in reality, is very fragile. Pumping large sums of money into research does not guarantee innovation; so, to maintain this lead, it is crucial to ensure that scientific and technological innovation does not take place outside of direct western control. Bad science management in India, by punishing innovators, and encouraging mediocres, willy-nilly plays a supportive role in maintaining western hegemony – just as crucial as the supportive role of the Indian civil service in maintaining the British empire. In a word, post-independence science has become an instrument to maintain dependency.

Kosambi, however, had grown up in the US, and had seen the post-war situation there and had no illusions that it offered greater freedom to the scientist.[68] Having rejected the option of emigrating, he had opted out of the larger western scheme of things. There was no possibility of his being a sycophant, so he had no place in India. Therefore, he had nowhere to go.

As is clear to any unbiased person with the slightest commonsense, mimesis of the west hardly suits the requirements of the people of this country. Contrary to the Nehruvian vision of mega projects, suited to transfer of capital, Kosambi argued in vain for technology better adapted to the needs of the Indian people, such as solar energy or small dams (even small atomic reactors) – and we are still having those debates.

Atomic Energy

The immediate cause of Kosambi's elimination has been described as his opposition to the atomic energy programme,[69] funding for which was the raison d'etre for the Tatas to have started the TIFR. It is quite true that Kosambi was an active campaigner for peace from the early 1950s. He participated in many public meetings in Bombay and led the mammoth 90-member Indian delegation to the World Peace Conference in Helsinki in 1955. It is also true that shortly before his exit from TIFR he gave a talk to the Rotary Club, in 1960, outlining the dangers of atomic energy.[70] (One of the arguments he advanced in this connection was that, since radioactivity brings about genetic changes, it might make everyone too stupid to be able to reverse the changes!)

However, championing peace or solar energy, or opposing large atomic energy plants in urban areas, could hardly have been the official cause of discontinuing Kosambi's contract, especially so soon after independence when democracy was linked to free speech and Tata's unwavering support for the British was still fresh in everyone's mind. I was unable to get hold of the papers at either the Bhabha or Nehru end when I first investigated the matter, long before the Right to Information Act. (This is an invitation to others to pursue the matter.) I regard it as more likely, as stated by Deshmukh,[71] that a case was made out on the basis that Kosambi had published a wrong proof of the Riemann hypothesis, and that this had damaged the reputation of the institute. If this is correct, it suggests a clear intention to manipulate: even supposing Kosambi had made a mistake, why should one mistake in a long and distinguished career invite such retribution? In a subsequent letter that Bhabha wrote to Kosambi,[72] he snidely commented that Kosambi would now have more time for his other interests (namely history), indirectly suggesting that Kosambi had ceased to be a mathematician because of his interest in history.[73]

Of course, we have seen that Kosambi kept doing research in mathematics till the very end of his life. However, in the logic of specialisation, a mathematician who did no serious research (and there were many such in TIFR) was preferable to one who did some mathematics but also did history, for by doing history he ceased to be a mathematician. Who says there is no identity politics in science! Also, Kosambi erred in being innovative in his mathematical research, for innovative ideas are very often initially socially disreputable, and what is required in the "Indian scientific method" is respectability.

All this is not to deny that the system of keeping people on contract has been instituted just to silence criticism. People are rarely removed for non-performance, but quickly removed for defiance. Kosambi, of course, had ample strength of character and would not be cowed down in his criticism of Bhabha or the atomic energy programme, though he was well aware that his contract could be discontinued.

> In one matter it was necessary to speak out though it meant considerable damage to finances, health and research. Atomic war and the testing of nuclear weapons must stop. A flimsy 'Indian Report' on the effects of atomic radiation shows our moral and scientific bankruptcy by ignoring the extensive data compiled since 1945.[74]

Just about the time Kosambi's contract was discontinued, there was the Panshet dam disaster. A dam burst in July 1961, inundating large parts of Pune, especially the heart of the old city on the banks of the Mutha river. Kosambi, known to be extremely generous in helping people, helped in the relief effort, giving succour to people and offering them food and money.[75] According to an unconfirmed report, Kosambi donated his entire savings for the relief effort; although this is denied by other sources.[76] Anyway, his dismissal from TIFR created difficult financial circumstances, as Kosambi himself noted in the above quote (apart from depriving him of a library and other facilities he needed for research). According to A Rahman, when he visited Kosambi in this period, instead of the usual hearty Goan meal of fish, all he got was tea and biscuits. Bhabha's influence with Nehru prevented any alternative appointment for Kosambi,[77] and he could get an Emeritus position in CSIR only after Nehru's death in 1964, and had some difficulty even getting affiliated to the Maharashtra Association for Cultivation of Science.

To summarise: Nehru's method of building modern India imposed a culture of managing science hierarchically, and the knowledge insecurity of the top managers generated a culture of sycophancy. The resulting situation suited western hegemony, but was fatal both to knowledge development and to knowledgeable people like Kosambi who refused to turn into sycophants, and also refused to emigrate to the direct control of the west. In an atmosphere of widespread scientific illiteracy, it is the subservient sycophants who later become top science managers, and are often conflated with top scientists. Through a process of the "survival of the unfittest", over time, these lineages of sycophants lead to a noticeable deterioration in quality. The only way to prevent perpetual recurrence of this state of affairs is to disband "expert raj" and ensure accountability, by subjecting our science managers to sustained public scrutiny, and by making them personally accountable for the grants they sanction, and the people they promote, compared to those they reject. Such accountability will help dismantle the huge empires they have built through lifelong science management.

Final Response

Finally, it is interesting to consider Kosambi's response, as a mathematician, to his dismissal. In 1964 he again published a proof of the Riemann hypothesis in the *Indian Journal of Agricultural*

Statistics.[78] Kosambi was not repeating the same joke a second time, so the viewpoint this time was different. As he explained it,

> The method of proof for deductions based upon probability differ radically from those of pure mathematics. Conclusions cannot be 'true or false' without qualification.[79]

He thought the "modern statistical method can be an excellent guide to action". So what he meant was that the truth of statements must be understood "only in the sense of unit probability", a point he had made in his earlier paper but not emphasised. Anyway, in the above quote he was clearly drawing a distinction between "statistical proof" (with probability one, which is "almost sure", and an excellent guide to action) and formal mathematical proof (which is purportedly certain).

For a proper understanding of these remarks about mathematical proof, we should place them in their appropriate historical context. At about this time, three influential Indian academics, P C Mahalanobis,[80] J B S Haldane,[81] and D S Kothari,[82] had taken up the issue of the Jain logic of *syadavada*. Haldane (a non-official Marxist, and Kosambi's friend) was willing to engage with Indian tradition, and interpreted Jain logic as a three-valued logic, while Mahalanobis sought a new foundation for probability theory based on this, and Kothari thought of explaining quantum physics on this basis.[83] Although Jains and Buddhists started off as contemporary (though rival) groups, the Jain ethic of non-violence has made them the richest community in India.[84] Buddhists were eliminated, and revived Buddhism still remains associated with dalits. Therefore, while Jain logic found a supporter in Kothari (a Jain and Chairman of the University Grants Commission), the Buddhist position on the matter was left unarticulated until recently, by this author.[85] Kosambi, however, should certainly have known of the Buddha's use of *catuskoti* (or the logic of four alternatives) in the *Brahmajala sutta* (Bapu Kosambi knew the *Tripitaka* by heart).

The other part of the historical context is the rise in the 1930s of a group of mathematicians known as the intuitionists, who challenged Hilbert's notion of formal mathematical proof. Though the intuitionists, led by L E J Brouwer and Arend Heyting, did not go so far as to suggest the use of a non-two-valued logic, they rejected proof by contradiction (or the "law" of the excluded middle), and this changed notion of proof radically altered the theorems of mathematics. (Proof by contradiction certainly fails with Buddhist *catuskoti*, which I have explained[86] as a quasi truth-functional system, though the situation is more complex with Haldane's interpretation of *syadavada,* as a three-valued logic.) It is irrelevant that the intuitionists lost the subsequent battle for social acceptance among mathematicians.

This historical context also enables us to counterfactually answer Thapar's question with which we started. Had Kosambi seriously applied himself to the history (and philosophy) of Indian mathematics, he could hardly have failed to notice that traditional Indian mathematics accepted the *pratyaksa* or empirical as a means of proof – as indeed did all systems of Indian philosophy,[87] without exception. This is in striking contrast to the western understanding of mathematics, which has deep religious roots,[88] and led to a formal mathematics that is divorced from the empirical, and admits only purely metaphysical means of proof. This metaphysics, being divorced from the empirical, is naturally a pure social construct. What has not been noticed is that this social construct has a religious bias, for it declares two-valued logic to be "universal", while we have seen that it is not (Buddhist and Jain logics are not two-valued, and the debates between Buddhists and the Naiyayikas dragged on for a thousand years for this reason). Therefore, mathematical proof offers no certainty to the non-believer.

As for the practical applications of mathematics, they all relate to calculation, where formal mathematical proof is more of a hindrance (which has made it difficult to teach mathematics). Therefore, if Kosambi had indeed taken up the study of traditional Indian mathematics, he would have noticed its focal concerns with practical calculations, which linked it to the key means of production in India (monsoon-driven agriculture and navigation needed for overseas trade).[89] He would also have uncovered and attacked the key western superstition underlying the taboo against empirical means of proof in mathematics, and the belief that certainty can be found only through metaphysics of a certain religious sort. So, had Kosambi taken up the study of traditional Indian mathematics, it might have significantly altered the entire mathematics scene in the country, and perhaps, the world by now.

NOTES

1 D D Kosambi, "A Statistical Study of the Weights of Old Indian Punch-Marked Coins", *Current Science*, 9 (1940), 312-14. There was a long series of subsequent papers. A popular summary is in "Scientific Numismatics", *Scientific American*, February 1966, 102-11. Also, *Indian Numismatics* (Hyderabad: Orient Longman), 1984.

2 R Thapar, "Early Indian History and the Legacy of D D Kosambi", *Economic & Political Weekly*, 43(30) 26 July 2006, 43-50, 47.

3 K N Panikkar, "Culture as a Site for Struggle", *The Hindu*, 27 January 2009, 11.

4 D D Kosambi, "Les metriques homogenes dans les espaces cosmogoniques", *Comptes rendus*, 206 (1938), 1086-88. "Les espaces des paths generalises qu'on peut associer avec un espace de Finsler", *Comptes rendus*, 206 (1938), 1538-41. "Sur la differentiation covariante", *Comptes rendus*, 222 (1946), 211-13.

5 D D Kosambi, "Affin-geometrische grundlagen der einheitlichen feld-theorie", *Sitzungsberichte der Preuss B Akademie der Wissenschaften, Physikalisch-mathematische klasse*, 28 (1932), 342-45.

6 D D Kosambi, "Geometric differentiale et calcul des variations", *Rendiconti della R Accademia Nazionale dei Lincei*, 16(6) (1932), 410-15.

7 D D Kosambi, *Tensor* (old series) No 2 (1939) 36-39.

8 D D Kosambi, "The Method of Least Squares", *Acta Sinica* (1957).

9 D D Kosambi, "Indo-Ariiskii Nosovoi Ukazatel", *Sovetskaya Einografia* (Ak Nauk USSR) 1 (1958), 39-57.

10 C K Raju, "Indian Science at the Crossroads", plenary talk at the 32nd Indian Social Science Congress, Delhi, December 2008. Also, "Swaraj in Science", International conference on the centenary of Hind Swaraj, Delhi, February 2009.

11 D D Kosambi, *Atomic Energy for India* (Poona: People's Book House), 1960, 10. Transcript of talk given to Rotary Club, Poona on 25 July 1960.

12 Hawking's serious book ends, "The actual point of *creation*, the singularity, is outside the presently known laws of physics". S W Hawking and G F R Ellis, *The Large Scale Structure of Space-Time*, Cambridge University Press, 364 (emphasis added). This linkage was made even more explicit by Tipler who uses Hawking's authority, and the authority of the journal *Nature* in which he has published on related issues, to begin his book by asserting that "Theology is a branch of physics". F J Tipler, *The Physics of Immortality* (London: MacMillan), 1995. A more detailed analysis is in C K Raju, *The Eleven Pictures of Time*, Sage, 2003.

13 D D Kosambi, *Atomic Energy for India*, cited above, 9.

14 D D Kosambi, "The Estimation of Map Distance from Recombination Values", *Annals of Eugenics*, 12(3) (1944), 172-75. This formula appears in the stamp issued in honour of Kosambi. What Kosambi himself said about it seems not widely known: "It seems to have given a new lease of life to genetical theories which I, personally, should like to see revised; so that I am accused at times of not appreciating my own formula". "Steps in Science", 198, in *Science and Human Progress* (D D Kosambi Commemoration Volume) (Bombay: Popular Prakashan), 1974.

15 Chintamani Deshmukh, *Damodar Dharmanand Kosambi: Life and Works* (trans title) (Mumbai: Granthghar), 1993. Anon Eng trans. from Marathi. Deshmukh mentions Kosambi's poor marks in mathematics in school.

16 For the uninitiated, Bourbaki is a romantic figure in the history of mathematics. This name was used by a group of the foremost French mathematicians, and Bourbaki's books remain the Bible of formal mathematicians today.

17 There is a discrepancy in my secondary sources on this point. Deshmukh's biography gives the date as 1945 (51), while the Kosambi commemoration volume gives the date as 1947 (310). R P Nene confirms that Kosambi left Fergusson College only in 1946, and joined TIFR in 1947. However, the date in Deshmukh's book is not a typo, for he again states on 84 that Kosambi spent 17 years in TIFR.

18 J D Bernal, "D D Kosambi" in *D D Kosambi Commemoration Volume*, cited above, 331.

19 D D Kosambi, "Precessions of an Elliptical Orbit. Notes on: Vibrating Strings; Planetary Orbits; The Raman Effect". *Indian J. Phys.* 5 (1930), 359-64.

20 One is not sure today whether Einstein was among the three. Einstein made a mistake regarding the general relativistic many-body problem, which exposed his lack of understanding of even special relativity vis-a-vis Poincaré. For an elucidation of the mistake (and its correction), see, C K Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994, 122.

21 D D Kosambi, "Geometric differentiale et calcul des variations", *Rend R Ac dei Lincei*, 16(6) (1932), 410-15; "Parallelism and Path-spaces", *Math Zeit*, 37 (1933), 608-18; "Systems of Differential Equations of the Second Order", *Quart J Math*, 6 (1935) 1-12; "Homogeneous Metrics", *Proc Ind Acad Sci*, 1 (1935), 952-54.

22 See, for example, Jagidish Mehra, *Einstein, Hilbert, and the Theory of Gravitation*, Dordrecht, D Reidel, 1974.

23 That is, his space-time was locally Euclidean rather than locally Lorentzian.

24 For example, D D Kosambi, "Path-Spaces of Higher Order", *Quart J Math*, 7 (1936), 97-104.

25 On the other hand, the difficulty with Hilbert's interpretation of the *Elements* was that it either fails after *Elements* 1.35 or else requires us to define area, while declaring the notion of length to be meaningless. See, C K Raju, "Euclid and Hilbert", chapter 1 in *Cultural Foundations of Mathematics*, Pearson Longman, 2007.

26 D D Kosambi, "An Affine Calculus of Variations", *Proc Indian Acad Sci*, 2 (1935), 333-35.

27 D D Kosambi, "Steps in Science", 195.

28 D D Kosambi, "Path-Spaces of Higher Order", *Quart J Math*, 7 (1936), 97-104.

29 See, for example, C K Raju, *Time: Towards a Consistent Theory*, cited above for a detailed list of references.

30 D D Kosambi, "Systems of Differential Equations of the Second Order", *Quart J Math*, 6 (1935) 1-12.

31 D D Kosambi, "Path-geometry and Cosmogony", *Quart J Math*, 7 (1936) 290-93.

32 D D Kosambi, "Steps in Science", cited earlier, 195.

33 This coupling leads to functional differential equations. This point, which has caused considerable confusion, is explained in detail in C K Raju, "The Electrodynamic 2-body Problem and the Origin of Quantum Mechanics", *Found Phys*, 34 (2004), 937-62.

34 Eli Cartan, *Math Zeit*, 37 (1933), 619-22.

35 M S Raghunathan, *Current Science*, 85(4) (2003), 526-36 (531), "Young man, I find that people who know nothing about Kosambi want to talk about him! Let me tell you this: he was one of the finest intellects to come out of your country". However, the word I recorded in conversation with Raghunathan, eight years before this publication, was "finest mathematician" and not "finest intellect", and that would also better fit the context of Raghunathan's question to Weil, which Raghunathan stated was Kosambi's publications on the Riemann hypothesis, considered later.

36 D D Kosambi, "Statistics in Function Space", *J Ind Math Soc*, 7 (1943), 76-88.

37 A key difference between the Indian and the European philosophy of mathematics is that Indian mathematics permitted empirical methods of proof, on which Western mathematics has placed a taboo. C K Raju, *Philosophy East and West*, 51(3) (2001), 325-62. Draft at http://ckraju.net/papers/Hawaii.pdf.

38 D D Kosambi, "Steps in Science", 198.

39 Today, of course, this is possible, and in 1994, this author implemented a package "Stochode", which solves stochastic differential equations driven by Brownian or Lévy motion, as his last piece of work in C-DAC. Some details were reported several years later in C K Raju, "Supercomputing in Finance", *Pranjana*, 3 (1&2) (2000), 11-36.

40 Ref 1.

41 D D Kosambi, "The Geometric Method in Mathematical Statistics", *Amer Math Monthly*, 51 (1944), 383-89.

42 D D Kosambi, "Zeros and Closure of Orthogonal Functions", *J Indian Math Soc*, 6 (1942), 16-24. "A Test of Significance for Multiple Observations", *Current Science*, 11 (1942), 271-74. "Statistics in Function Space", *J Ind Math Soc*, 7 (1943), 76-88.

43 L B Arguimbau, "'Baba' of Harvard Days" in *D D Kosambi Commemoration Volume*, cited above, 321.

44 G Kallianpur, *Bull Amer Math Soc*, 24(1) (1997), 46. "Incidentally, as long as we are mentioning authors, the original K-L theorem should be properly called the Karhunen-Kosambi-Loeve theorem. The result was independently discovered during World War II by the Indian mathematician and Marxist historian D D Kosambi."

45 M E Walker, "Retarded Differential Equations and Quantum Mechanics", *Notices of the American Mathematical Society*, 54(4) 2007, 473, belatedly acknowledges my earlier work. G W Johnson and Walker had in a 2006 article in the *Notices*, reported on Michael Atiyah's 2005 Einstein lecture, and renamed my ideas as "Atiyah's hypothesis". My theory correcting Einstein's error was already published in ref 14, and the Johnson and Walker article was published well after Atiyah was informed of my prior works – within a few days of his 2005 Einstein lecture.

46 Why did Kosambi suddenly lose interest in path-geometry? There could have been many reasons,

and one can only speculate. The atom bomb surely was not the only reason, for Kosambi did publish on pathgeometry even after Hiroshima and Nagasaki. Perhaps Kosambi's demotivation had something to do with Edmund Whittaker's 1952 book which accused Einstein of having plagiarised the special theory of relativity from Poincaré and Lorentz. E T Whittaker, *A History of the Theories of Aether and Electricity*, Vol II: *The Modern Theories*, [1951-53] (New York: American Institute of Physics), 1987.

47 D D Kosambi, "Classical Tauberian Theorems", *J Ind Soc Agricultural Stats,* 10 (1958), 141-49.

48 Bibliography in the Commemoration volume, entry at 105, says the Chinese text was published in *Ac Sinica* 1957, of unknown date, and later published in English as D D Kosambi, *J Ind Soc Agricultural Stats,* 11 (1959), 49-57.

49 Deshmukh, *D D Kosambi: Life and Works*, cited above, 72.

50 D D Kosambi, "Classical Tauberian Theorems", *J Ind Soc Agricultural Stats,* 10 (1958), 141-49.

51 D D Kosambi, "An Application of Stochastic Convergence", *J Ind Soc Agricultural Stats,* 11 (1959), 58-72.

52 (D D Kosambi), "Bibliography of Kosambi's Writings", compiled from Kosambi's notes by R P Nene. In *Science and Human Progress* (Prof D D Kosambi Commemoration Volume) Popular Prakashan, Bombay, 1974, entry number 119, 373.

53 Martin Gardner, in *The Annotated Alice*, explains various jokes in *Alice in Wonderland.* That many people would otherwise miss the joke is clear from the fate of Swift's satire against formal mathematics in *Gulliver's Travels*, which has sunk into oblivion.

54 B Bagchi, "The Statistical Behaviour and Universality Properties of the Riemann Zeta-Function and Other Allied Dirichlet Series", PhD thesis (Calcutta: Indian Statistical Institute), 1981.

55 Deshmukh, cited above, 17, points to the doggerel which described Baba Kosambi as a school student at Cambridge High: "The rest to some faint meaning make pretense/but Baba never deviates to sense".

56 D D Kosambi, "On a Generalization of the Second Theorem of Bourbaki", *Proc Acad Sciences UP*, 1 (1931) (3 pages).

57 S G Krantz, *Mathematical Apocrypha*, Mathematical Association of America, 2002, 144.

58 The systematic use of malicious gossip by the TIFR math school to mislead the unwary is clear from the way R S Anderson, who researched in TIFR for his PhD thesis, was planted with the information that Kosambi was "just a college teacher from a Bombay [Poona] college", while Chandrasekharan was from Princeton. This information is in Anderson's PhD thesis, though not in the subsequent book. RS Anderson, "Life of Science in India: Comparative Ethnography of Two Research Institutes", PhD thesis, University of Chicago, 1971, Ann Arbor, Michigan, and *Building Scientific Institutions in India: Saha and Bhabha*, McGill University, Centre for Developing Area Studies, 1975.

59 D D Kosambi, "The Sampling Distribution of Primes". *Proc Nat Acad Sci* (USA), 49 (1963), 20-23; "Statistical Methods in Number Theory". *J Ind Soc Agricultural Stats*, 16 (1964), 126-35.

60 S Ducray (D D Kosambi), "Normal Sequences", *J Univ Bombay*, 31 (1962), 1-4; "Probability and Prime Numbers", *Proc Ind Acad Soc*, A 60 (1964), 159-64; "The Sequence of Primes", *Proc Nat Acad Sci*, A 62 (1965), 145-39.

61 R P Nene recalls that he posted a copy of the typescript of the book to the publisher. Kosambi had apparently discussed number theory with Paul Erdos, who visited him in Pune. Nene also recalls that John Irwin, the curator of a London museum, was interested in the book, and was trying to get it published by Routledge (who do not normally publish mathematics). But Routledge seems to have lost the typescript. Nene also recalls that Kosambi was very meticulous about keeping copies of all his papers. But copies of the typescript of this book no longer seem to be available.

62 D D Kosambi, "Steps in Science" in *Science and Human Progress* (D D Kosambi Commemoration Volume) (Bombay: Popular Prakashan), 1974, 201.

63 Ibid.

64 Bhabha's sartorial proclivities were reportedly the cause of M L Mehta leaving TIFR. I interviewed some people about this in 1995. Yashpal, former Chairperson of the University Grants Commission, whose kurta is his sartorial trademark, denies that there was any such demand. According to N R Puthran, Bhabha's registrar, Bhabha said: "The answer to a shirt is a tie". "Those were his exact words", Puthran added. A contemporary of Chandrasekharan was K G Ramanathan; the only occasion I saw him without a tie, he offered an explanation for it.

65 André Weil, *The Apprenticeship of a Mathematician*, trans Jennifer Gage, Birkhauser Verlag, Basel, 1992, 80.

66 D D Kosambi, *J Ind Soc Agricultural Stats,* 11 (1959), 49-57. The paper mentions that the solutions it obtains are "of considerable importance for diffusion theory and the integral equations for atomic energy piles". This is repeated in his annotated bibliography in the Commemoration volume in the entry at 105 which notes that this paper "solves abstract equations, including integral equations for atomic piles (diffusion)".

67 Some details are in R S Anderson, *Building Scientific Institutions in India: Saha and Bhabha*, McGill University, Centre for Developing Area Studies, 1975.

68 D D Kosambi, "Science and Freedom", *Monthly Review*, 4 (1952), 45-49.

69 Dhirendra Sharma, *India's Nuclear Estate* (New Delhi: Lancers Publishers), 1983, 120. I discussed this with Dhirendra Sharma in Hawaii, January 2000, but he had no recollection as to the source of this information.

70 D D Kosambi, *Atomic Energy for India* (Poona: People's Book House), 1960, cited above at 11.

71 C Deshmukh, *Damodar Dharmanand Kosambi: Life and Works*, cited above, 82. Deshmukh did not put down his sources, and died before he could do so.

72 My information about this letter comes from R P Nene, a friend of Kosambi, who saw the letter. His account is corroborated by that of Deshmukh, in the place cited above.

73 Deshmukh, in the place cited above. Interestingly, M S Raghunathan also describes Kosambi as a "mathematician turned historian" thereby suggesting that Kosambi had ceased to be a mathematician. Now, it is a simple empirical fact that Kosambi remained active in mathematics right until his death, although, of course, empirical facts are of no concern to formal mathematicians or postmodernists. M S Raghunathan, *Current Science*, 85(4) (2003), 526-36 (531).

74 D D Kosambi, "Steps in Science", cited above, 265.

75 Deshmukh, *D D Kosambi: Life and Works*, 86.

76 The source is A Rahman. R P Nene, who was close to Kosambi, in his last years, denies this. He however points out that Kosambi did donate the royalties from his book on the *Culture and Civilisation of Ancient India* to Cuba for hurricane relief. Deshmukh, *D D Kosambi: Life and Works*, 85-86 cites some of Kosambi's petty extravagances on chocolates, travel, and the like, which would be "profligacy" only from a strict Gandhian perspective, which Kosambi, a Marxist, obviously did not accept. On the other hand, Deshmukh also points out that Kosambi was excessively generous, and always ready to help others, and hence had little savings.

77 Deshmukh, *D D Kosambi: Life and Works*, 96-97 goes into elaborate detail. This was corroborated by A Rahman.

78 D D Kosambi, "Statistical Methods in Number Theory". *J Ind Soc Agricultural Stats*, 16 (1964), 126-35. This is not mentioned in the bibliography in the commemoration volume.

79 D D Kosambi, "Steps in Science", cited above, 197.

80 P C Mahalanobis, "The Foundations of Statistics (A Study in Jaina Logic)", *Dialectica,* 8 (1954), 95-111; reproduced in *Sankhya*, *Indian Journal of Statistics*, 18, 1957, 183-94; reproduced as Appendix IV B in *Formation of the Theoretical Fundamentals of Natural Science,* vol 2 of *History of Science and Technology in Ancient India*, by D P Chattopadhyaya (Calcutta: Firma KLM), 1991, 417-32.

81 J B S Haldane, "The *Syadavada* system of Predication", *Sankhya*, *Indian Journal of Statistics*, 18 (1957), 195-200; reproduced as Appendix IV C in *Theoretical Fundamentals of Natural Science*, by D P Chattopadhyaya, cited above, 433-40. This account, together with Bhadrabahu's syllogism, is discussed at the level of the layperson in C K Raju, *The Eleven Pictures of Time*, Sage, 2003, chapter 10.

82 D S Kothari, "Modern Physics and Syadavada", Appendix IV D in *Theoretical Fundamentals of Natural Science*, by D P Chattopadhyaya, cited above, 441-48.

83 Unfortunately, Kothari (or perhaps his student adviser, Kishan Lal) overlooked the similar unsuccessful explanation of quantum mechanics in terms of three-valued logic advanced two decades earlier by Reichenbach.

84 There is a simple causal nexus between wealth and the ethic of extreme non-violence: the Jain ethic compelled them to become traders and moneylenders, which most of them still are. (According to the *Census of India 2001*, only 3.2% of Jains were into agriculture, compared to an all-India figure of 31.65%, and similarly for cultivators. The reason is this activity may involve the incidental killing of living beings while ploughing, etc). For more details, see *The Eleven Pictures of Time*, cited above.

85 C K Raju, *Philosophy East and West*, 51(3) (2001), 325-62; draft available from http://ckraju.net/papers/Hawaii.pdf.

86 For an exposition of the different logics, see my article "Logic" *Springer Encyclopedia of Nonwestern Science, Technology, and Medicine* 2008, a draft of which is available from http://ckraju.net/papers/Non_western_logic.pdf.

87 C K Raju, "Proof vs Pramana", chapter 2 in *Cultural Foundations of Mathematics,* Pearson Longman, 2007. This is an elaboration of the argument in my paper in *Philosophy East and West*, 51(3) (2001), 325-62, a draft of which is available from http://ckraju.net/papers/Hawaii.pdf.

88 This point perhaps needs to be explained. The word "mathematics" derives from *mathesis*, meaning learning, and Plato opined, in *Meno*, that all learning is recollection of knowledge of past lives by the soul. Proclus explains that mathematics, "the science of learning", is particularly suited for this purpose since mathematics concerns eternal truths which sympathetically move the eternal soul. Accordingly, Plato, in his *Republic*, recommended teaching mathematics for the good of the soul, and expressly not for any practical purpose. This theme was picked up by the Neoplatonists who used mathematics to contest changes to Christian theology in the 4th century, and then got incorporated into Islamic rational theology (*aql-i-kalam*) and from that (with some key changes) into post-Crusade Christian rational theology, the beliefs of which are reflected in present-day formal mathematics. C K Raju, "The Religious Roots of Mathematics", *Theory, Culture, and Society*, 23(1-2), 2006, 95-97. For the current *political* uses of rational theology, see C K Raju, "Benedict's Maledicts", *Indian Journal of Secularism*, 10(3) (2006), 79-90.

89 *Ghadar Jari Hai*, 2(1) (2007), 26-29. http://ckraju.net/IndianCalculus/Cultural-Foundations-Mathematics-review-from-GJH.pdf. C K Raju, *Cultural Foundations of Mathematics*, Pearson Longman, 2007.