The Religious Roots of Mathematics

C.K. Raju

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Present-day science relies upon mathematics. And mathematics has always been closely connected with religious beliefs in the West. Thus, Plato, in his *Republic*, advocates the teaching of mathematics for its beneficial effects on the soul, and expressly not for its practical applications. (For Plato's thought that 'geometry will draw the soul towards truth', see *Republic* VII, 527, in Plato, 1996.) Likewise, Socrates' demonstration of the slave boy's intrinsic knowledge of geometry was explicitly intended by Socrates to prove the existence of the soul, and how its recollections could be elicited by a philosopher acting as a midwife (Plato, *Meno* 179–80, in Plato, 1996). Proclus, the alleged source of the single vague remark that gave rise to the strong rumours about the existence of 'Euclid', explicitly asserts in his commentary on the *Elements* that the real function of mathematics is to turn the attention of the student inward, towards his soul, and thus make him realize universal oneness – hence most of the propositions in the *Elements* are about equality of apparently dissimilar things. (Proclus' explanation of the etymology of the word 'mathematics' as the science of learning, or the science of the soul, is at the end of his Prologue, part 1, in Proclus, 1970.)

These beliefs about mathematics were so strong a force in Islamic rational theology that even staunch opponents like al Ghazali allowed that even God was bound by the laws of logic. This concession was seized upon not only by al Ghazali's opponent Ibn Rushd (Averröes) but also by the school-men who inherited his legacy, and whose trivial curriculum did not include practical mathematics (until Clavius introduced it in the Jesuit mathematical syllabus *c.* 1570). Accordingly, Christian rational theologians regarded mathematics not as a means of practical calculation but as teaching a 'universal' means of argument and proof aimed at those who did not accept an appeal to their scriptures as a means of proof. They also accepted al Ghazali's contention that the empirical world had to be contingent to allow for the creation of the world by God – although they denied his belief in continuous creation, held also by the Dunsmen, whom they labelled dunces, and subscribed instead to the belief in one-time creation.

It is against this theological background that western thought reached the peculiar conclusion that only metaphysical procedures (like logical deduction) can incorporate necessary truth (truth valid in all possible worlds, truth that binds God), though any empirically based truth must remain forever contingent (true in only some possible worlds, truth not binding on God).

In present-day (formal) mathematics, the locus of this belief in necessary truth has been shifted from theorems to mathematical proof, regarded as completely divorced from the empirical. That is, according to formal mathematics, even though neither the axioms nor the theorems of a formal theory incorporate any necessary truths, the *connection* between axioms and theorems does: that is, for a formal theory, there may be worlds in which its axioms are false, and worlds in which its theorems are false, but any world in which the axioms are true is a world in which the theorems are true.

This is also the belief underlying Popper's criterion of refutability (falsifiability), which supposed that any number of experiments could not verify a theory (since this process was inductive, and probabilities are not ampliative), while a single experiment could refute a theory. The point here is not merely that the process of refutation is also inductive (as I have pointed out earlier), but that refutation is believed to refute the physical hypotheses underlying the physical theory, rather than the mathematics that connects the physical hypothesis to the empirical conclusion. If mathematical proof does not represent certain or necessary truth, and if it were to be accepted that mathematical proof is also fallible (even when correct), there is no basis for this belief, for our actual world may happen to be the world in which the physical hypotheses of the theory are true, but its conclusions are false.

The completely cultural nature of the belief that metaphysics is somehow superior to physics, that deduction incorporates certain or necessary truth, becomes crystal clear the moment one turns towards Buddhism or Jainism. Both use logics that are different from the logic culturally assumed in western thought. Therefore the inferences drawn using these logics will be different from the inferences drawn using two-valued logic, e.g. many proofs by contradiction would fail in Buddhist or Jaina logic. (The Buddha's use of the logic of four alternatives is in the *Brahmajāla Sutta* of the *Dīgha Nikāya*. The very readable English translation by Maurice Walshe is not so clear on this point, and neither is the older English translation by T.W. Rhys Davids.) An interesting (but not necessarily correct) interpretation of the Jain logic of *Syādavada* is provided by J.B.S. Haldane's 'The Syadavada System of Predication' (1957; for more detail see Raju, 2003).

But what decides the nature of logic to use? If decisions about logic are purely cultural, why should one use one logic rather than another? In any case, it is hard to understand how cultural decisions can be regarded as infallible! Both Buddhism and Jainism, like *all* other Indian schools of thought, incidentally, accept the empirically manifest (*pratyaks*a) as the first means of proof, while also accepting it as fallible. Therefore, if, on the other hand, decisions regarding logic are empirical (and based on beliefs about the nature of time), they are bound to be inductive, and fallible. In either case, deduction turns out to be more fallible than induction.

The incorrect belief that deductive inference, divorced from the empirical, represents certain truth, provides an important example of how deep-seated cultural assumptions are woven into the *content* of present-day mathematics, scientific theories, and also the philosophy of mathematics and science. Indian mathematics, by contrast, permitted the use of empirical procedures, and aimed towards practical calculation rather than claims of a universal and necessary truth; however, it is, on that ground, deemed not to be mathematics at all. (In fact, curiously, it is deemed to be neither mathematics nor physics!)

These cultural assumptions also underlie present-day technology. For example, the above assumptions about the nature of logic are also built into the present-day theory of computation, and are incorporated in common computer chips. As shown by quantum computers, using the structured-time interpretation of quantum mechanics, an alternative technology of computation based on alternative logics, like Buddhist logic, is feasible. (For the structured-time interpretation of quantum mechanics, see Raju, 1993; for further amplification and formal proofs see Raju, 1994: ch. 6b.)

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C.K. Raju is based at the Centre for Studies in Civilizations, New Delhi & Centre for Computer Science, MCRP University Bhopal.