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Zeroism

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Zeroism is an alternative philosophy of mathematics,[1] based on *śūnyavāda*, a realistic philosophy often ascribed to the Buddhist teacher Nagarjuna (2nd c. CE).[2] It is now called zeroism to emphasize that the concern is with the practical and contemporary benefits of that *śūnyavāda* philosophy, as distinct from fidelity to this or that interpretation of the textual sources of *śūnyavāda*, which have often been misunderstood and mangled by scholars unfamiliar with the idiom. Indeed, the whole idea of relying on the *authority* of textual sources is a practice of scriptural traditions, and *contrary* to *śūnyavāda*, which denies the validity of proof by authority.

In a nutshell, zeroism is a realistic philosophy, which accepts universal practical procedures in mathematics, and rejects as *erroneous*, and culturally biased, the formalist (or idealist) attempt to understand mathematics as metaphysics. (Formalism is the philosophy underlying mathematics as taught in schools and colleges today.)

To understand how the West came to the peculiar conclusion that mathematics is metaphysics, we need to go back all the way to Plato. In Plato's story of Socrates and the slave boy,[3] Socrates first elicits the boy's innate knowledge of mathematics, and then declares that he has proved the existence of the soul! (His argument is that since the slave boy did not learn mathematics in this life, his innate knowledge of mathematics proves that he learnt it in a previous life.) Indeed, "mathematics" derives from mathesis. Though mathesis just means "learning", Plato had a special understanding of "learning", and stated that "all learning is recollection"—of the knowledge the soul had acquired in its previous lives but has forgotten since its

[1] C. K. Raju, *Cultural Foundations of Mathematics*, Pearson Longman, 2007.

[2] Nagarjuna, author of *Mūlamādhyamakakārikā*.

[3] Plato, *Meno*, in *Dialogues of Plato*, trans. B. Jowett, Encyclopaedia Britannica, Chicago, 1996, pp. 179–180.

birth.[4] Proclus[5] believed that mathematics (and not geography, for instance) contains eternal truths, and is *hence* best suited to arouse the eternal soul.)

This notion of the soul and its previous lives ran afoul of the church, which cursed it in the 6th c., when it also banned mathematics and philosophy from Christendom.[6] The Western understanding of mathematics changed further in the 12th c., during the Crusades, when the church accepted back mathematics, but reinterpreted it as concerned not with arousing the soul, but with rational persuasion or metaphysical proofs. Nonetheless, the belief that mathematics contains eternal truths lingered on. Indeed, Aquinas claimed that God ruled the world with eternal laws of nature, and it came to be believed that those laws were hence written in the language of mathematics (since mathematics was believed to contain eternal truths).

This belief in eternal truths, led to the related belief in the "perfection" of mathematics: any imperfection was bound to be exposed during eternity! As Berkeley[7] put it "It is said, that the minutest Errors are not to be neglected in Mathematics". The belief in "perfection" led to the rejection of the empirical in Western mathematics. Indeed, Plato deprecated the empirical world as inferior, and Westerners came to believe that this "perfection" of mathematics could only be achieved through metaphysics, which gave a higher form of truth than empirical truth.[8] Consequently, the present-day philosophy of formalism still supposes that mathematics (being a higher form of truth) must be 100% metaphysics. The number 2 is today defined solely with reference to Peano's axioms, and *not* ostensively by pointing to 2 oranges, 2 dogs etc.

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[4]This belief in the previous lives of the soul was based on the belief in quasi-cyclic time. (See article on TIME.) With a quasi-recurrent cosmos, rebirth takes place in successive cycles of the cosmos. Not only are humans reborn, but all events repeat approximately, so it is indeed natural to suppose that memories too commence afresh. Mathesis then becomes a mysterious way to arouse and bring back those lost memories of previous lives in a previous cycle of the cosmos.

[5]Proclus, *Commentary*, trans. Glenn R. Morrow, Princeton University Press, Princeton, New Jersey, 1970, Prologue part 1.

[6]C. K. Raju, "The curse on 'cyclic' time", *The Eleven Pictures of Time*, Sage, chp. 2.

[7]George Berkeley, *The Analyst or a Discourse Addressed to an Infidel Mathematician*, ed. D. R. Wilkins, 1734, IV, available online at `http://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/Analyst.html`

[8]It is still believed, on Tarski-Wittgenstein semantics, that empirical truths are contingent truths (true in some possible worlds), weaker than mathematical truths, which are necessary truths (true in all possible worlds), relative to the axioms.

All these beliefs are in sharp contrast to zeroism. To begin with, how do we know that there are any eternal truths? Like science, zeroism admits only two means of knowledge: *pratyakṣa* or the empirically manifest, and *anumāna* or inference. Totally contrary to the belief in eternal truths, it is manifest that nothing persists unchanged for even two instants. This is often wrongly called the Buddhist doctrine of flux. It is, however, not a doctrine but a simple observation, repeated thousands of times each day. Those who claim the existence of something (such as the soul) which stays constant and unchanged across time need to prove its existence. This has never been done.[9]

Denying the continuation of identity may seem counter-intuitive, since our thinking involves language, and the *belief* in persistence of identity is part of natural language. Thus, the simple statement "When I was a boy", suggests the persistence of some unchanging identity, the "I" which applies to both me and the boy. This clearly neglects the manifest differences, such as my gray hair, or reduced physical activity, compared to the boy. On *śūnyavāda*, I and the boy are really two distinct individuals with some memories in common. For practical purposes, we may neglect the difference as somehow "small". Zeroism accepts this practical process of neglecting or zeroing things regarded as unimportant in the context, on the grounds of the paucity of names: it would be cumbersome to speak of Ashok1, Ashok2, Ashok3, etc., with one Ashok for each instant. Treating slightly different multiple entities, as if they were one, is a necessity, because on realistic zeroism, or *śūnyavāda*, there is no valid basis for the idealistic doctrine of eternal (or persistent) entities or truths, which doctrine is rejected as *erroneous*. This corresponds to the exact anti-thesis of the formalist doctrine that only metaphysics gives "perfect" mathematics, and that all practical representations (which ignore minute differences) are erroneous.

Let us illustrate this with an elementary mathematical example. Consider the iterative procedure to calculate square roots, the earliest record of which comes to us from Āryabhaṭa.[10] If we try to extract the square root of 2, this

[9]Thus, this *śūnyavāda* denial of the soul (*anātmavāda*) is fundamentally different from the church curse on 'cyclic' time which sought to dictate the nature of the physical cosmos on doctrinaire and political grounds. All that is denied in *śūnyavāda* is the continuation of an unchanged identity from one instant to the next, and this is done solely on grounds of observation, and absence of any proof for the existence of something unchanged.

[10]*Āryabhaṭīya*, *Gaṇita*, 4. *Āryabhaṭīya* of Āryabhaṭa ed. K. S. Shukla, and K. V. Sarma, Indian National Science Academy, 1976, p. 36.

algorithm will not terminate. If it terminates, that will give us a finite decimal expansion or a fraction, formally called a rational number, and $\sqrt{2}$ cannot be represented by a fraction. The pattern of digits in the infinite decimal expansion for $\sqrt{2}$ will not recur either, for a recurrent decimal expansion too corresponds to a fraction by the formula for the sum of an infinite geometric series (see CALCULUS).

This (non-terminating, non-recurrent) decimal expansion corresponds to an infinite sum: $\sqrt{2} = 1.414\cdots = 1 + \frac{4}{10} + \frac{1}{100} + \frac{4}{1000} + \ldots$. Carrying out the infinite sum by physically adding successive terms is impossible, a supertask (or an infinite task which will take an eternity of time). Therefore, the universal practical procedure is to stop at the level of accuracy needed for the practical application at hand, and neglect or zero the remaining digits as unimportant to the practical context. For example, $\sqrt{2} = 1.414$. Or, for example, a calculator might give $\sqrt{2} = 1.4142135623730950488016887242097$ and zero the remaining digits. The exact number of decimal places after which we stop is not important. The point is we have no option but to stop, after a finite number of steps, no matter how large.

This universal procedure is good for *any* practical application of mathematics. Indeed, a practical task must be accomplished in a limited period of time, therefore, for practical application of mathematics, this is the only procedure available. Since each zeroed digit is at most 9, we can easily estimate the maximum possible inaccuracy by using the formula for the infinite geometric series (which formula too involves zeroing the insignificant). We can improve the accuracy to any desired level, but can *never* achieve "perfection": contrary to Berkeley, some "minute Error" would have to be neglected.

However, in search of "perfection", formalism today insists that the only correct procedure is to sum the entire infinite series. Since that cannot be done physically, it is done metaphysically. Since the sum cannot be a fraction, it is assigned a meaning through a special metaphysics of infinity which is used to construct a number system misleadingly called the "real" number system. In fact, the usual construction of this number system requires set theory which brings in further metaphysics of infinity. Unlike practical procedures, this metaphysics is not universal, but the universal practical procedures are declared as "erroneous". For example, most practical calculations today involve the use of computers which cannot handle the Western metaphysics of infinity implicit in formal "real numbers", and computers use a different arithmetic (of floating point numbers) which is declared erroneous.

Indeed, on zeroism, this "discarding of insignificant quantities" is not an error; it involves a fundamentally different understanding of mathematics, not as eternal truth, not as something perfect, but as concerning practical calculations. This was the understanding with which mathematics developed in India since the days of the *śulba sūtra*-s, through Āryabhaṭa down to the 15th c. Nīlakanṭha. Thus, *śulba sūtra*-s used the term *sa-viśeṣa* (with something left out),[11] or *sānitya* (impermanent)[12] while Āryabhaṭa used the term *āsanna*[13] (near value), and Nīlakanṭha explained why the "real value" cannot be given.[14] There was never any religious doctrine of mathematics as eternal truth, nor its corollary that mathematics must be "perfect", and that this "perfection" can only be attained through a particular metaphysics of infinity. On zeroism, the representation $\sqrt{2}$ is acceptable only due to a paucity of names, and functions exactly like the single name "Ashok" which refers to a multiplicity of slightly different entities, 1.4, 1.41, 1.414 etc.

Though we have taken $\sqrt{2}$ as an example, the practical use of numbers is inevitably tied to neglecting or zeroing of some tiny differences. Thus, when we speak of 2 oranges, we neglect as unimportant the fact that no two oranges are identical any more than any two persons are identical. What we mean is that the differences between the two oranges are irrelevant to the context. Hence, even the attempt to understand integers formally, through Peano's axioms, involves the metaphysics of infinity in a subtle way. This is clear from the fact that computers can never do supposedly "perfect" Peano arithmetic.[15]

This contrast between the two different philosophies of mathematics, the religious and the practical, created enormous difficulties in understanding the infinite series of the Indian calculus when it was first imported into Europe, and Europeans conflated the two distinct streams of mathematics, and tried to understand the practical in religious terms.

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[11] Baudhāyana *śulba sūtra*, 2.12. *The śulba sūtras*, ed. S. N. Sen and A. K. Bag, Indian National Science Academy, 1983.

[12] Apastamba *śulba sūtra* 3.2, *śulba sūtra*-s, cited above

[13] *Āryabhaṭīya*, Gaṇita, 10.

[14] *Āryabhaṭīya*, part 1 with commentary of Nīlakanṭha, ed. K. Sambaśiva Śastry, Kerala University, Trivandrum, 1930 (Trivandrum Sanskrit Series, 101), reprint 1977, p. 56, commentary on Gaṇita 10.

[15] For ints on a computer, see C. K. Raju, "Computers, mathematics education, and the alternative epistemology of the calculus in the Yuktibhāṣā", *Philosophy East and West*, **51**(3), 2001, pp. 325–62. A more detailed account of why this happens can be found in `http://ckraju.net/hps2-aiu/ints.pdf`.

Thus, Descartes went so far as to declare[16] that the ratios of curved and straight lines were beyond the capacity of the human mind! A charitable interpretation of his statement is that he was alluding to the ratio of the circumference to the diameter of a circle. That is a number today called π and represented by the infinite series $\pi = 3.1415\ldots$, and especially the series today wrongly called the Leibniz series (see CALCULUS). Descartes thought this involved an infinite sum or a supertask which was beyond the human mind. In fact, this comparison is very easily done using a flexible string to measure the length of the curved line, and then straightening it to measure the straight line. This was how it was taught to children in India, since the days of the *śulba sūtra* (*śulba* means string) and can still be taught in preference to the ritualistic compass box which has no instrument to measure curved lines.[17]

Of course, even on this charitable interpretation, Descartes was completely wrong in imagining that the issue specifically concerned curved lines. Thus, exactly the same problem arises with $\sqrt{2}$, which corresponds to the diagonal of a unit square: hence concerns a ratio only of two straight lines. The real issue is zeroism versus idealism. Historically speaking, however, Western mathematicians accepted Descartes' objections, and Newton thought he had resolved them and made calculus "perfect" by making time metaphysical.[18] Ironically, Newton's physics failed for the very reason that he made the conceptual mistake of making time metaphysical.[19]

This historical legacy of Western problems with the "perfect" way to understand infinity/eternity persists to this day in calculus as taught in schools and colleges today. Though Newton's fluxions have been abandoned, all university calculus texts state the belief that metaphysical, formal "real"

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[16] R. Descartes, *The Geometry*, trans. D. Eugene and M. L. Latham, Encyclopaedia Britannica, Chiacago 1996, Book 2, p. 544.

[17] C. K. Raju, "Towards Equity in Mathematics Education. 2: The Indian Rope Trick", *Bharatiya Samajik Chintan*, New Series, 7(4), 2009, pp. 265–269. `http://ckraju.net/papers/MathEducation2RopeTrick.pdf`.

[18] C. K. Raju, "Time: What is it that it can be measured", *Science&Education*, *15*(6) (2006) pp. 537–551, `http://ckraju.net/papers/ckr_pendu_1_paper.pdf`; "Retarded gravitation theory", in: Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro (ed.), *Sixth International School on Field Theory and Gravitation*, American Institute of Physics, New York, 2012, pp. 260–276. `http://ckraju.net/papers/retarded_gravitation_theory-rio.pdf`.

[19] C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, 1994. Fundamental Theories of Physics, vol. 65.

numbers are essential for the calculus. This belief is false.[20] Recall that (see article on CALCULUS) the Indian calculus developed with a different number system ("unexpressed fractions", formally the field of rational functions, a field larger than formal real numbers, which is hence "non-Archimedean") in which limits are not unique, and, formally speaking, infinitesimals must be discarded. Today, differential equations are numerically solved on a computer using a smaller number system of floating point numbers, and this too involves discarding or zeroing small numbers regarded as insignificant for the practical purpose at hand.

In the practical application of calculus to physics, there are many current problems with infinity, such as the renormalization problem of quantum field theory,[21] the runaway solutions of Maxwellian electrodynamics,[22] or the problem of differentiating discontinuous functions, as in shock waves in general relativity.[23]. These problems all arise just because the Indian calculus was not properly understood in Europe, just because of the belief in the "perfection" of mathematics which led to the metaphysics of infinity on which present-day formalism is based.[24] The huge metaphysical structure of set theory needed even to define simple numbers such as 2 creates enormous learning difficulties in mathematics. Zeroism is of contemporary importance since it helps to resolve both pedagogical and scientific problems.

Western philosophers and mathematicians did not notice a key problem with their beliefs. On the one hand they believed mathematics is universal, on the other hand they believed perfection could only be found in metaphysics. They neglected to ask: which metaphysics? Unlike the universal procedure of practical calculation, Western metaphysics is obviously *not* universal. For example, empirical proofs are universally accepted by all Indian

[20]See "Retarded gravitation theory" cited above.

[21]C. K. Raju, "On the Square of x^{-n}", *J. Phys. A: Math. Gen.* **16** (1983) pp. 3739–53

[22]Suvrat Raju and C. K. Raju, "Functional Differential Equations and radiation damping", *Mod. Phys. Lett. A*, **26**(35) (2011) pp. 2627–2638. arXiv:0802:3390.

[23]*Cultural Foundations of Mathematics*, cited above, appendix on "Renormalization and Shocks".

[24]More fundamentally, for example, the metaphysics of infinity in present-day mathematics is incorporated into (formal) set theory. Though many of the earlier problems (such as Russell's paradox, or problems with transfinite induction are deemed to have been resolved) we still have, for example, the Banach-Tarski paradox which provides us with a source of unlimited wealth: it says that a ball of gold can be cut into a finite numbers of pieces which can then be reassembled into two balls of gold each identical to the original one!

systems of philosophy, without any exception. They all accept the *pratyakṣa* or the empirically manifest as a valid means of knowledge or proof as does Islam (*tajurba*), and science (experiment). Further, empirical proofs are the only means of proof accepted by all Indian systems, since the Lokayata (or common people's philosophy) rejects *anumāna* (inference or deduction) as unreliable (and Buddhists reject as unreliable any means of proof other than *pratyakṣa* and *anumāna*). Thus, the elevation of metaphysical proofs above empirical proofs is a cultural idiosyncrasy of the West which is decidedly not universal, and its use in the philosophy of mathematics, makes that Western mathematics non-universal. (It is another matter, that the colonial education process makes Western mathematics normatively universal.)

Further, Western thought has dogmatically maintained, since the Crusades that inference (or deduction) is the only reliable means of proof. Now, inference or reasoning is based on logic, and the West naively assumed that "reason" (meaning two-valued logic) is universal. This belief in the universality of "Aristotelian" logic (see article on LOGIC) was tied to the dogmas of the Christian rational theology of Aquinas and schoolmen, which developed during the Crusades, and maintained that even God was bound by logic in the sense that he could not create an illogical world. Since they, however, believed that God was free to create facts of his choice (while creating the world) therefore they thought that logical proofs are "stronger" than (or superior to) empirical proofs (since God is bound by logic, but not by facts). This belief is reflected in Tarski-Wittgenstein semantic of possible worlds: in contrast to facts which are contingent truths (true in some possible worlds) mathematical theorems are believed to be necessary truths (true in all possible worlds). The only difference is that instead of possible worlds which God could create, one speaks of possible logical worlds in the sense of Wittgenstein! In actual fact, there is nothing divine or unique about logic except in the Western imagination: there are an infinity of logics to choose from.

Logic is not even culturally unique (e.g. Buddhist *catuṣkoṭi*, or Jain *syādavāda* are not 2-valued or even truth-functional, see article on LOGIC). Accordingly, if logic is decided on cultural grounds, then that obviously involves a cultural bias. If, on the other hand, logic itself is determined empirically, from facts, logical proofs cannot be stronger than (or superior to) empirical proofs. (That is, in principle. In practice, of course, deductive proofs are far more fallible than empirical proofs, as is clear from the most elementary first proposition of the Elements, which uses an empirical proof, but this was not noticed for 700 years,[25] during which it was regarded as a model of deductive proofs.) Changing logic would naturally change the theorems that can formally deduced from a given set of axioms; therefore, contrary to the Western belief, mathematical theorems are not necessary truths relative to axioms, but are at best truths relative to both axioms and logic.

To summarise, zeroism denies the existence of persistent truths and, consequently, treats the belief in eternal truths as an error. The West, because of its earlier belief that mathematics contains eternal truths, believed that mathematics must be "perfect" and this "perfection" could be achieved only through metaphysics, especially the metaphysics of infinity (closely related to the theology of eternity). However, Western metaphysics is not universal as has falsely been claimed. For example, all Indian schools of philosophy accept empirical proof as applicable to mathematics, and superior to metaphysical proof. Further, even logic is not universal, as has wrongly been claimed in the West, since Buddhist logic and Jain logic are fundamentally different from the two-valued logic declared universal in the West. (The Western belief in the universality of logic is based on the wrong theological argument that logic supposedly binds the Christian God, and is hence universal.) Choosing logic on cultural grounds would naturally result in a cultural bias, but choosing it on empirical grounds means the belief that metaphysical proofs are "stronger" than empirical proofs must be abandoned. (Also, as quantum logic shows, 2-valued logic is still not the automatic choice.) In either case, there is no basis left for formalism.

Zeroism is ideally suited to the practical applications of mathematics. Indeed, zeroism enhances the practical reach of mathematics, since there are many situations where calculations are possible, but metaphysical proof is not: for example, stochastic differential equations driven by Levy motion, where solutions can be calculated, but cannot be formally proven to exist.[26] The advantages of zeroism are particularly evident in physics where the classical calculus with limits fails, as does its extension, the Schwartz theory of distributions. Limits also fail in statistics, for relative frequency converges to probability only in a probabilistic sense.[27] Zeroism works very well in this situation.

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[25] C. K. Raju, *Euclid and Jesus*, Multiversity, 2012.

[26] C. K. Raju, in *Philosophy East and West*, cited above.

[27] C. K. Raju, "Probability in ancient India", *Handbook of the Philosophy of Science*, vol. 7. *Philosophy of Statistics*, ed. Prasanta S. Bandyopadhyay and Malcolm R. Forster.

The use of zeroism has the further advantage that it greatly simplifies mathematics in situations such as the calculus. The derivative no longer represents the slope of the tangent (the best linear approximation to a curve), but only the slope of a chord (a good linear approximation to the curve). As usual, one name serves to represent a multiplicity of slightly different entities, the differences being irrelevant for practical applications. All that is given up is the oxymoronic demand for the "best approximation". This not only suits computational mathematics, it simplifies pedagogy. Making math easy means that students can tackle much harder problems.

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