Talk at University of Auckland, New Zealand, 18 Oct 2005

The Indian Origins of the Calculus and its Transmission to Europe Prior to Newton and Leibniz. Part II: Lessons for Mathematics Education*

C. K. Raju
Centre for Studies in Civilizations, New Delhi
&
Centre for Computer Science, MCRP University Bhopal

`c_k_raju@hotmail.com`

Extended Abstract: I will only summarise the evidence for the transmission of the calculus from India to Europe in the 16th c. CE, since it has been considered elsewhere.[1] Briefly, unlike the wild claims of transmission from "Greece", typically advanced without serious evidence by Western historians in the last three centuries, the standard of evidence I use is a legal standard of evidence which examines motivation, opportunity, documentary, circumstantial, and epistemological evidence. The **motivation** was provided by the requirements of the specifically European navigational problem: precise trigonometric values were required for the calculation of the three "ells's"—latitude, longitude, and loxodromes—and these were the focus of European navigational theorists like Pedro Nunes, Gerhard Mercator, Christoph Clavius, and Simon Stevin during the 16th and early 17th c. CE. (Navigation was critical to the production of wealth in Europe from the 16th c. onward.) The **opportunity** was provided by the presence of Roman Catholic missionaries in Cochin since 1500, and their spread into the interior of Kerala with the help of the indigenous Syrian Christians. By 1550, Jesuits took over the Cochin college, and not only taught Malayalam to the locals, but also learnt Sanskrit, along with mathe-

*Draft: please do not quote without the author's permission.

[1] C. K. Raju, *Cultural Foundations of Mathematics: The nature of mathematical proof and the transmission of the calculus from India to Europe in the 16th c. CE.*, PHISPC, Vol X. (4), New Delhi 2005 (to appear). C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the *Yuktibhāṣā*", 8th East-West Conference, Hawai'i, Jan 2000, in *Philosophy East and West*, **51**(3) (2001) 325–62. Draft: `www.IndianCalculus.info/Hawaii.pdf`. C. K. Raju, "How and Why the Calculus was Imported into Europe". International Conference on Knowledge and East-West Transitions, NIAS, Bangalore, Dec 2000. Extended abstract: `www.IndianCalculus.info/Bangalore.pdf`

matics and astronomy.[2] The clearest exposition of the calculus in India, in the *Yuktidīpikā*, was written by Śankara Vāriyar (1500-1560) and his brother, both of whom were patronised by the king of Cochin who simultaneously patronised the Portuguese, ever since they fled Calicut. Various other books on Indian astronomy and mathematics were readily available in and around Cochin. The **documentary evidence** is provided by e.g. Vasco da Gama's diary recording that he carried back the *kamāl* or *rāpalagai*, by Matteo Ricci's letter, or by de Nobili's ca. 1608 CE polemic against an astronomical text from −1350 CE, long rejected as obsolete by Indian tradition. The **circumstantial evidence** is provided by the appearance in Europe of imported mathematical knowledge, from 1500 onwards—celestial methods of latitude determination, the Mercator projection (and, of course, the trigonometric values he needed for loxodromes), the "Tychonic" model, "Julian" day numbers, "Pascal's" triangle, and down to the very numbers used in say Fermat's challenge problem, exactly which problem is found as a solved example in Bhaskara II. Then there are, of course, Cavalieri's indivisibles, the "Gregory-Leibniz" series, and the sine series for which Newton himself claimed credit, or "Stirling's" formula for interpolation. Unlike India, where the series expansions developed over a thousand-year period 499-1501 CE, they appear suddenly in fully developed form in a Europe still adjusting to grasp arithmetic and decimal fractions. Despite the historical and epistemological discontinuity, the "pagan" sources of this new knowledge went unacknowledged because of the then prevalence in Europe of various sharply repressive religious institutions and practices (like the Inquisition or the Doctrine of Christian Discovery) which brutally suppressed the slightest sign of theological deviance—even in a Newton (whose key writings remain suppressed to this very day).

This talk, however, will focus on the **epistemological evidence**: transmission of knowledge is established by lack of understanding, as when one student copies from another, but displays evidence of his ignorance of what he claims to have originated. It is well known that Indian arithmetic and numeral notation went to Europe via Arabic "algorismus" texts, valued especially by Florentine merchants, as useful for commerce. Neither calculus nor arithmetic ("algorismus" texts) were immediately understood upon receipt in Europe. The lack of understanding originated due the different epistemologies of mathematics in India and Europe. The curious way in which Indian numerals were misunderstood by Gerbert (Pope Sylvester II), accustomed to numbers on the abacus, provides a very simple example of how contrasting epistemologies cloud understanding. In the case of the calculus, the process is a little harder to understand, especially for formal mathematicians.

By way of background I point out that since the theorems of formal mathematics vary also with logic, and since logic varies with culture (e.g. neither Buddhist nor Jaina logic is 2-valued), and is also not empirically certain (e.g. quantum logic), formal proof (or deduction) can never aspire to be either certain

[2] After Clavius' reform of the Jesuit mathematical syllabus to include practical mathematics, ca. 1570.

or universal[3] (or even more certain than induction[4]), contrary to the popular philosophical and cultural beliefs in the West. The origin of these popular Western beliefs about mathematical proof can be readily understood through the historical evolution of the Egyptian/Neoplatonic ("Euclidean") notion of proof, as modified to suit the requirements first of Islamic rational theology, and then of Christian rational theology, and later incorporated into formal mathematics by Hilbert, Russell etc. by modifying the *Elements*.[5] The Indian notion of *pramāṇa*, in contrast, rooted in an empirical and practical understanding of the world,[6], did not view mathematics through a cultural understanding in which metaphysics ("mathematical truth") was placed on a higher plane than physics.

These two views of mathematical proof were brought into confrontation with each other when Indian calculus travelled to Europe. The European difficulty with zero did not concern merely the *numeral* zero, but related also to the process of discarding or zeroing a "non-representable" during the course of a calculation—similar to the process of rounding. Though the Indian method of summing the infinite series constituted valid *pramāṇa* it was not understood in Europe; the earlier difficulty with non-representables zeroed during a calculation reappeared in a new form. This was now seen as a new difficulty—the problem of discarding infinitesimals. Berkeley's criticism makes it obvious that up to the time of their death neither Newton nor Leibniz had reconciled their methods of dealing with fluxions/infinitesimals with the clarity expected of them by their contemporaries. (Retrospective disambiguation of Newton and Leibniz is as irrelevant as retrospective disambiguation of the prophecies of, say, the Oracle of Delphi or Nostradamus.) In both cases of algorismus and calculus, Europeans were unable to reject the new mathematical techniques because of the tremendous practical value for calculations (required for commerce, navigation etc.), and unable also to accept them because they did not fit in the metaphysical frame of what European then regarded as valid.

Studying the historical dynamics of this cultural interaction is of contemporary interest[7] since it affords insight also into the current mathematical predicament where computers have again enhanced the ability to calculate in a way that is of practical value, but stretches far beyond what is regarded as epistemologically secure. (E.g. the solutions of stochastic differential equations driven by Lévy motion can be readily calculated on a computer, but the solutions cannot

[3] C. K. Raju, "Mathematics and Culture", in *History, Culture and Truth: Essays presented to D. P. Chattopadhyaya*, ed. Daya Krishna and Satchidananda Murthy, New Delhi, 1999. Reproduced in *Philosophy of Mathematics Education, **11***: `www.exe.ac.uk/~PErnest/pome11/art18.htm`

[4] C. K. Raju, "Why deduction is MORE fallible than induction: Ending the tyranny of Western metaphysics in mathematics and science", Invited talk at the International Conference on Methodology and Science, Vishwabharati, Santiniketan, Dec 2004. Abstract at `www.IndianCalculus.info/Santiniketan.txt`

[5] C. K. Raju, "How should 'Euclidean' geometry be taught?", in Nagarjuna G., ed. *History and Philosophy of Science: Implications for Science Education*, Homi Bhabha Centre, Bombay , 2001, 241–260.

[6] ". . . Alternative epistemology. . . ", paper cited in note 1 above.

[7] C. K. Raju, "Math Wars and the Epistemic Divide in Mathematics", Paper presented at Episteme 1, Goa, Dec 2004. `www.hbcse.tifr.res.in/episteme/themes/ckraju_finalpaper`

be formally proved to exist, although the calculated solution can nevertheless be of practical value for understanding financial markets.) Because of the same old problem with non-representability, made manifest by computers, floating point numbers on a computer do not obey the most elementary "laws", such as the associative law, that numbers in formal number systems like the real number system are required to obey. However, instead of modifying the philosophy of mathematics, or even the formal mathematics of numbers, all results of computations are deemed to be "erroneous"!

Europeans eventually managed to assimilate the imported mathematics—in the case of the calculus this required the semi-formalisation of real numbers by Dedekind, using set theory, which itself was formalised only in the 1930's. A similar shift in the notion of number from concrete to abstract was required earlier by the shift from abacus to algorismus.

On the principle that phylogeny is ontogeny, these historical difficulties faced in Europe are today reflected, in fast forward mode, in the learning of mathematics in the K-12 classroom: especially arithmetic, algebra, trigonometry, and calculus—all of which first came into Europe with a foreign epistemology. Thus, elementary mathematics (e.g. numbers) can no longer be taught in a "mathematically correct" or clear way at an elementary level (since, e.g., the formal set theory required for this cannot be taught at an elementary level), and the process of learning mathematics mimics the complex historical process of the assimilation of this mathematics in Europe. This constitutes a substantial regress from the situation prevailing some 1200 years ago, when the same elementary mathematics could be easily taught, as in the Indian "slate-arithmetic" (*Pāṭī Gaṇita*) texts.

Any stable way out of these present-day difficulties must address the central thread common to the thousand-year old European difficulties with arithmetic, calculus, and now computers—the problem of non-representability. A simple way out is to revert to the original philosophy of non-representables, as implicitly used in the *śulba sūtra*, as inherent in Indian arithmetic, trigonometry, and calculus, and as explicitly articulated in Nagarjuna's *śūnyavāda*) ("zero-ism", Zen),[8] This would also help to resolve the problem with representability made manifest by computers. I argue that this way out is to be preferred to repeatedly forcing mathematics to fit into Platonic idealism, and related religious beliefs, inherently contrary to mathematics-as-calculation, merely on the basis of a largely mythical and racist view of history which locates the origins of mathematics in "Greece". Such a change in the philosophy of mathematics would also be most appropriate to the changes in the understanding of mathematics and number that may be expected to accompany the more recent enhancement in the capability to calculate, using computers.

[8] *Mūlamādhyamakakārikā*, Sanskrit text and Eng. trans. in David J. Kalupahana, *The Philosophy of the Middle Way*, SUNY, New York, 1986. C. K. Raju, "The Mathematical Epistemology of Śūnya", interventions during the Seminar on Concept of Śūnya, Delhi, 1997, in: *The Concept of Śūnya* ed. A. K. Bag and S. R. Sarma, IGNCA, INSA, Delhi, 2002, 168–81.