Pre-colonial appropriations of Indian ganita: epistemic lessons

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Abstract

Nearly 30 years ago, a group of influential intellectuals in India decided: let us tell our own stories. The West has been telling our stories for us for far too long. I participated in the initial discussions, and in the resulting Project of History of Indian Science, Philosophy and Culture, and eventually authored one of its over 100 volumes.[1]

My PHISPC volume documented the origin of the calculus in India, in the 5th c., with Aryabhata, in connection with the two key sources of Indian wealth: agriculture (which is monsoon-driven, hence requires a good calendar), and overseas trade (which required celestial navigation), both of which require astronomy and precise "trigonometric" values. Ganita texts related to the Indian calculus were translated by Jesuits in their Cochin college, in the 16th c., and taken to Europe. Astronomy, and the precise trigonometric values derived using calculus, were important for the European navigational problem, the foremost scientific challenge in Europe until the end of the 18th c. European governments offered large prizes for its solution from the 16th to the 18th c. (British longitude prize, 1712) since European dreams of wealth all rested overseas.

While examining the evidence for calculus transmission, such as opportunity, motivation, circumstantial and documentary evidence,[2] my PHISPC volume emphasized a novel technique, the **epistemic test**: like students who cheat in an exam, those who copy often fail to understand what they copy; therefore in a situation where transmission is possible, failure to understand is proof of copying.

Most present-day school mathematics, arithmetic,[3] algebra,[4] trigonometry and calculus,[5] probability and statistics,[6] originated in India in pre-colonial times, and was transmitted to Europe. Three different models were used to appropriate it; all deny credit to Indians. In each case, the epistemic test exposes the theft. The need to correct the faulty Western understanding (which persists e.g. in calculus, and probability) results in the contemporary value of this revised history.

1. Arithmetic first travelled from India to Baghdad where, in the 9th c., al Khwarizmi ("Algorithmus") wrote Hisab al Hind. When Gerbert (Pope Sylvester II) imported it from Cordoba, in the 10th c., Europeans were accustomed to the primitive Roman abacus, so Gerbert foolishly got an abacus (apices) constructed for what he called "Arabic" numerals. This was not only historically wrong, but exhibited lack of understanding that the efficiency of Indian arithmetic was from "algorithms", and that this efficiency was destroyed by using the abacus: Europeans thought it was only about a change in the shape of the numerals! Next, Florentine merchants (Fibonacci, 12th c., *Liber abaci*) again imported Indian arithmetic via Africa, for efficient arithmetic offered a comparative advantage for commerce. But Florentines accustomed to Roman arithmetic failed to understand the Indian place value system, and passed a law against zero (which term derives from sifr or cypher meaning mysterious code). In the third round, Clavius received ganita texts directly from India, and introduced fractions in the Jesuit syllabus, ca. 1575, as practical mathematics.[7] He also received inputs from his student Matteo Ricci, who searched for Indian calendrical methods in Cochin.[8] Clavius authored the Gregorian reform of the primitive Julian calendar, since this reform was needed not only for the date of Easter, but to determine latitude at sea during daytime. But fractions were so little known to Europeans then, that the Gregorian reform stuck to the old primitive system of leap years, so even now it gets the (tropical) year approximately right not from year to year, but only on a thousand year average![9]

2. As regards algebra, Western epistemic errors are clear from the very word "surd" from Latin surdus meaning "deaf". Actually, Indians extracted square roots, using the diagonal or karna, as in the sulba sutra.[10] But the term "bad karna" for $\sqrt{2}$ was mistranslated as bad ear, hence the absurd "surd". What the West never understood was that Brahmagupta's avyakt ganit or polynomial arithmetic involving avyakt fractions resulted in non-Archimedean arithmetic (which was used instead of "real" numbers in India to first sum infinite series such as the infinite geometric series).[11] In non-Archimedean arithmetic there are infinities and infinitesimals (but no limits).[12] This has great contemporary value for the teaching of calculus today.[13] Nor did Europeans understand the related idea of zeroism or sunyavada. Hence, also Bhaskara's "division by zero" was not an error, any more than division by an infinitesimal.

3. As regards "trigonometry", the very term and the way it is wrongly taught in school as relating to a triangle, shows up the epistemic error. In Indian ganita texts, such as the युक्तिभाषा, it was related to the circle, or circle-metry, hence jya (meaning chord) or jiva (meaning half-chord). This went into Arabic as jiba, since there is no v sound in Arabic. It was written without nukta-s as just the consonantal skeleton, "jb". The unofficial (Mozharab) translators of Arabic texts at Toledo misread it as jaib meaning pocket, or fold from whence the Latin sinus or the present-day sine according to OED. When Jesuits stole ganita texts from India, for the related precise trigonometric values, precise to the 9th decimal place,[14] since precise values were required to determine latitude and longitude, these were published by the Jesuit general Clavius in his name.[15] Laughably, Clavius did not know enough trigonometry to correctly determine the radius of the earth, as found in so many Indian ganita texts.[16] As the 7th c. Brahmagupta[17] succinctly put it, "भूव्यासस्य अज्ञानाद् व्यर्थं देशान्तरं" ("ignorance of the radius of the earth makes longitude [calculations] futile"). Hence the European longitude problem persisted for another couple of centuries. (Granting that, in retrospect, Picard determined the radius correctly, by the end of the 17th c., it was not credible/known to the contemporary European navigator, as clear from the British longitude prize of 1713, set up by a parliamentary act.)

4. In the case of calculus, Europeans did not understand how to sum the infinite series of the imported Indian calculus, such as the infinite ("Taylor") series for sine, or the infinite "Leibniz" series for π found in the युक्तिदीपिका 2.271, करणपद्धति VI, I etc. Alluding to it, Descartes[18] declared the ratios of curved and straight lines to be beyond the human mind. As Newton correctly pointed out, Leibniz did not understand how to sum the series named after him, though Newton's fluxions were a laugh well meriting Berkeley's polemic that they were "ghosts of departed quantities".[19] The Jesuits also passed on the Indian astronomical model of Nilakantha to Tycho Brahe then the Astronomer Royal to the Holy Roman Empire. It is likely that Kepler, used Indian observations, since Tycho's masonry instruments were far too inaccurate, and he kept those "observations" secret, until his death, even from his assistant, the nearly blind Kepler.[20] Parts of Indian ganita texts were later passed on to Galileo (Cavalieri), Fermat (e.g. challenge problem), Pascal, etc. and eventually reached Protestant Europe where credit for the calculus was falsely given to Newton and Leibniz.

5. In the case of probability, formulae for the binomial expansion are found in the earliest texts,[21] while "Pascal's" triangle (मेरु प्रस्तार), which first appeared in Europe about a century before Pascal (on the title page of the Arithmetic of Apianus and in China in the 14th c.[22]) is found Halayuddha's commentary. The related theory of permutations and combinations is found in numerous texts, from even before Pingala (e.g. in the Jain Bhagvati sutra), in the सुश्रुत संहिता, Varahamihira, Sridhara's पाटीगणित (72), Mahavira's गणित सार संग्रह (vi.218), and most flamboyantly in the लीलावती[23] of Bhaskar II who asks his daughter to challenge the "evil lads of astronomers" to tell how many 5 digit numbers are there whose digits sum to 13. The use of probabilities to analyse games of chance is found in the Mahabharata, e.g., in the story of Nala and Damayanti (or the famous द्युत क्रीडा). The former also has an example of sampling used to count the number of fruits in a tree[24] (वन पर्व, 72). The Western epistemic problem with probabilities is clear from the fact that the method of limits used in calculus fails for probabilities, since, on the "law" of large numbers, relative frequency converges to probability only in a probabilistic sense, so that probability cannot be defined as the limit of relative frequency without begging the question. This has resulted in much present-day confusion about the meaning of probability since the frequentist interpretation cannot be replaced by the subjectivist in the case of quantum mechanics and Popper's propensities make no sense to most physicists. Another problem is that the logic (rather than boolean algebra or σ algebra), on which (Kolmogorov) probabilities are-defined today, need not be two-valued or even truth functional, resulting in quantum probabilities where joint distributions need not exist.

In the case of arithmetic Europeans merely credited Arabs ("Arabic numerals"), ignorantly but non maliciously, since this transmission took place before the Crusades. In the case of calculus, Europeans applied the post-Crusade "Doctrine of Christian discovery". On this obnoxious doctrine, the first Christian to spot a piece of land becomes its "discoverer", hence owner, as in the absurd claim that Vasco da Gama "discovered" India, discounting the millions of people already living here, for thousands of years. Contrary to what some apologists might say, Vasco da Gama did not even discover the sea route for he hired an Indian navigator who knew the 5000-year old sea-route between Africa and India. Vasco, and subsequent Indologists did not even fully *understand* the instrument the navigator used, and Vasco took back with him, until I explained its sophisticated construction, using the two-scale principle, but applied to two *harmonic* scales.[25] The genocidal[26] doctrine of Christian discovery was applied to grab lands in Americas and Australia but could not be applied in India. For 250 years after Vasco, in India, the to-be coloniser was militarily too weak to grab any significant land, but they could easily grab an undervalued possession: knowledge. On this doctrine of Christian discovery, which is accepted law in the US, applied to the case of a "possession" such as knowledge, credit hence ownership for a discovery or invention must be handed over to Christians.

As distinct from these pre-Crusade and post-Crusade models of appropriation, there is a little-known third model of appropriation. During the Crusades the church suddenly switched from its medieval policy of burning heretical books to a new policy of learning from books. Accordingly, Arabic books captured at Toledo were mass translated into Latin starting 1125 CE (which led to the howlers about "sine" and "trigonometry"). To justify this change of policy as theologically correct, the origin of most knowledge in these Arabic books was attributed to the early Greeks (whom Eusebius called the only "friends of Christians"). On the strength of this wild fantasy, the knowledge in Arabic books

was declared a Christian inheritance. The philosophers in Islam continued the Egyptian mystery tradition, which they incorrectly called the "theology of Aristotle" and which is today called Neoplatonism in the West. Hence they attributed logic to Aristotle though there is no evidence for the use of the Aristotelian syllogism in Greece, and it is very similar to the Nyaya syllogism which could very well have been transmitted from India to Arabs in the 9th c. and was wrongly attributed to Aristotle.[27] .

The church used the Bible to advocate slavery as a moral duty for centuries, quite openly until the mid nineteenth century. The resulting institutionalised racism survived emancipation (obviously) and even the end of apartheid. Though Indians have debated William Jones' Aryan thesis only in the context of Indology, several thinkers including George James, Cheikh Anta Diop, Martin Bernal,[28] and Afrocentrists have pointed out that the racist "Aryan model" was systematically used after 1785 to appropriate the achievements of ancient Egypt to Greeks. While the church traditionally (from the time of Orosius) systematically used false history as a weapon to claim religious superiority, and racists extended that same false history (with some modifications) to claim superiority of Whites, colonial historians pitched in to claim the civilisational superiority of the West.[29]

That is, Western historians have collectively produced highly chauvinistic and utterly fraudulent accounts of scientific achievements due to early Greeks. These claims, such as the claim about "Euclid", rely on wild conjectures based on isolated and suspect passages in extremely late texts.[30] E.g. the isolated passage used to date "Euclid" (and infer his intentions) is contrary to the entire text of Proclus in which it is found. As just one more example, the 12th c. Arabic text *Almagest* is attributed to an early Greek, "Claudius" Ptolemy (but not to a Greek translation of an Egyptian astronomy text at the time of Ptolemy II). This ignores that early Greeks were such a superstitious lot, according to Plato's *Apology*, that Socrates was given the death penalty precisely on the charge of heresy for doing astronomy.[31] Western historians failed to ask the basic question: how did the text survive? Why was the text of the *Almagest* copied out for over a thousand years? If it was done for the utility of astronomy then the text would have been repeatedly updated: like all scientific texts it would be accretive. If so, a 12th c. text reflects 12th c. knowledge: this is anyway evident from the fact that the star list in the *Almagest* is headed by the current pole star, which was nowhere near the pole in the 2nd c. Western apologists falsely claim that these are problems common to all history. However, in the Indian case later commentaries reproduce the original text in full, therefore one can see the later day changes, as in the progression in the precision of trigonometric values in India from sexagesimal minutes in the time of Aryabhata to seconds to thirds by the time of Madhava. There is no such continuous chain of texts in the Greek case.

Because these late texts come to us from the hands of priests skilled in forgery and textual manipulation, we cannot take isolated passages in them seriously, and must check any claims against non-textual evidence. We have seen that Greeks and Romans had abysmally bad calendars just because their astronomy was hopelessly bad and because they could not manage fractions. This was a persistent inferiority: Europeans had difficulty with fractions until the 16th c. The *Almagest* text too talks of the difficulty of multiplication, exactly like 9th c. Arabic zijes, and states the duration of the tropical year in a clumsy way, which is still incorrect. But even this wrong value was never used in the Hilarius calendar reforms of the fifth century, after the date of Easter had manifestly slipped. Therefore, the non-textual evidence shows that knowledge of astronomy was missing among the early Greeks and Romans.

Without knowledge of fractions how exactly did Greeks calculate trigonometric values? Faith-based Western historians, have no difficulty in imagining that like fairy godmother (sexagesimal) fractions suddenly appeared and suddenly disappeared in Western tradition. Further, dishonest historians like David Pingree and van der Waerden even assert that Indian trigonometric values derived from the Greeks by anachronistically attributing all knowledge in a 12th c. text to a mythical "Claudius" in the 2nd c. Granting some similarity between the texts, given that the source (actual) Greek text are from so late, it is far more probable that the Byzantine Greek text accreted knowledge of Indian astronomy via Arabic texts. For example, the Arabic *Almagest*, begins with what appears to be a paraphrase of a long drawn debate in Indian astronomy (starting from Varahamihira's 6th c. objection in his पंचसिद्धान्तिका, against Aryabhata) about the movement of the earth. The Greek bishop Severus Sebokht (8th c.) acknowledged the superiority of Indian astronomy, because Indians had superior arithmetic. More arguments can be given. But, of course, faith-based (or dishonest) historians can overcome mountains of facts, by piling on the hypotheses. There are of course examples of indubitably Indian texts, such as the पंचतंत्र, which were translated from Sanskrit to Persian (7th c.) to Arabic (8th c.) to Byzantine Greek (11th c.) to Latin (12th c.), and absorbed into Aesop's fables etc.[32]

Therefore, it is time to stand on its head a whole lot of earlier Indology, and to recognize that the contribution of Indian mathematics was not merely zero. It is time also to challenge the Western authority, on which fantasies about Greek achievements are based, to provide primary facts, from contemporaneous texts, and to also engage with the non-textual evidence.

1 C. K. Raju, *Cultural Foundations of Mathematics: the nature of mathematical proof and the transmission of calculus from India to Europe in the 16th c. CE*, Pearson Longman, 2007.
2 C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhāsā", *Philosophy East and West*, **51**(3) (2001) pp. 325–362. http://ckraju.net/papers/Hawaii.pdf
3 For a quick account, see my talk at MIT. Video: https://youtu.be/IaodCGDjqzs, abstract: http://ckraju.net/papers/Calculus-story-abstract.html, presentation: http://ckraju.net/papers/presentations/MIT.pdf.
4 E.g. H. T. Colebrooke, *The Algebra of Brahmegupta and Bhascara*, John Murray, London, 1817. The word algebra derives from the al jabr wa'al muqabala of al Khwarizmi. which translated the work of Brahmagupta.
5 *Cultural Foundations of Mathematics*, cited above.
6 C. K. Raju, "Probability in Ancient India", *Handbook of the Philosophy of Science*, vol 7, *Philosophy of Statistics*, ed. Prasanta S. Bandyopadhyay and Malcolm R. Forster. General Editors: Dov M. Gabbay, Paul Thagard and John Woods. Elsevier, 2011, pp. 1175–1196. http://ckraju.net/papers/Probability-in-Ancient-India.pdf.
7 Christoph Clavius, *Arithmeticae practicae* [Practical arithmetic], Dominici Basae. Rome, 1583. Also, "A method of promoting mathematical studies in the schools of the Society", unpublished , 1575?. E. C. Phillips, "The proposal of Father Christopher Clavius, SJ, for improving the teaching of mathematics" *Bulletin of the American Association of Jesuit Scientists* **XVIII**(4) (1941) pp. 203-206.
8 Letter by Matteo Ricci to Petri Maffei on 1 Dec 1581. *Goa* **38** I, ff 129r–30v, corrected and reproduced in *Documenta Indica*, **XII**, 472-477 (p. 474).
9 C. K. Raju, "A tale of two calendars", in *Multicultural knowledge and the university*, ed. Claude Alvares, Multiversity, 2014, pp. 112-119. Also, video: https://www.youtube.com/watch?v=MvpuC7Dg4e0.
10 मानव शुल्ब सूत्र 10.10.
11 नीलकंठ, आर्यभटीयभाष्य, commentary on गणित 17.
12 For an elementary construction of this non-Archimedean field, see E. A. Moise, *Elementary Geometry from an Advanced Standpoint*, Addison Wesley, Reading, Mass., 1968, appendix.
13 E.g., C. K. Raju, "Teaching Mathematics with a Different Philosophy. 1: Formal mathematics as biased metaphysics". *Science and Culture* **77** (2011) pp. 275–80. arxiv:1312.2099. 2: Calculus without limits". *Science and Culture*, **77** (2011) pp. 281–86. arxiv:1312.2100. This reports on a pedagogical experiment in Universiti Sains Malaysia with 4 groups of students, pure math, applied math, post-graduate math, and non-math.
14 युक्तिदीपिका 2.9.5. For a translation and explanation see *Cultural Foundations of Mathematics*, chp. 3.
15 Christophori Clavii Bambergensis, *Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000...*, Ioannis Albini, 1607.
16 Such as महाभास्करीय II.3-4
17 ब्राह्मस्फुटसिद्धांत, chapter 11, तन्त्रपरीक्षाध्याय, verses 15-16
18 René Descartes, *The Geometry*, trans. David Eugene and Marcia L. Latham, Chicago, Encyclopaedia Britannica, 1990, Book 2, p. 544.
19 For a detailed discussion of fluxions, see *Cultural Foundations of Mathematics*, cited above.
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21 Pingala, *Chandahsutra* viii.28. ed. Sri Sitanath, Calcutta, 1840. For a translation and worked example, see B. B. Dutta and A. N. Singh, *History of Hindu Mathematics*, Asia Publishing House, 1962, vol. 1, pp. 75--76.
22 Joseph Needham, *The Shorter Science and Civilisation in China*, vol. 2 (abridgement by C. A. Ronan). Cambridge University Press, 1981, p. 55.
23 The verse is numbered differently in different editions. In K. V. Sarma's critical edition of the क्रियाक्रमकरी, a commentary on the text, this is at 269. This was the probable source text for Pascal.
24 For a full discussion see "Probability in ancient India", cited above.
25 See *Cultural Foundations of Mathematics*, cited above, chp. 5 on "Kamal or rapalagai"
26 E.g. C. K. Raju, "Meaning of Christian discovery", *Frontier Weekly*, **47** (29) 2015, Jan 25-31, http://www.frontierweekly.com/archive/vol-number/vol/vol-47-2014-15/47-29/47-29-The%20Meaning%20of%20Christian%20Discovery.html%20Christian%20Discovery.html. Also video: "Discovery of India", https://www.youtube.com/watch?v=EI4lPuAIbKw.
27 C. K. Raju, "Logic", article in *Encyclopedia of Non-Western Science, Technology and Medicine* (ed. Helaine Selin), Springer, 2008, 2014, 2016, pp. 2564–2570. http://ckraju.net/papers/Nonwestern-logic.pdf.
28 Martin Bernal, *Black Athena: The Afroasiatic roots of classical civilization. Vol. 1 The fabrication of ancient Greece 1785-1985*. Free association books, London, 1987. Bernal (personal communication, 9 Jan 2010) asked me to take up the issue of math and science.
29 *Is Science Western in Origin?* Cited above.
30 See, e.g. my 5 lectures at the University of South Africa on "Not out of Greece", posted online at http://ckraju.net/unisa/.
31 http://ckraju.net/hps-aiu/extract-from-Plato-Apology.txt.
32 For full details, see C. K. Raju, *Euclid and Jesus: How and why the church changed mathematics and Christianity across two religious wars*, Multiversity, Penang, 2012.