# Calculus without Limits: the Theory

## A Critique of the History of Mathematics
## Part 1: Euclid and all that

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# Outline

# Introduction

- We saw that teaching calculus with limits involves practical difficulties.

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- It offers no particular practical advantage.
- And is maintained through an incorrect claim (“rigor”)
- Which involves culturally specific beliefs.
- This intrusion of culture into a secular science is maintained by an appeal to history.

# Euclid

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- That other cultures which lacked this notion did not really do mathematics,
- and the knowledge they had was inferior.

# The stock claim

contd.



- This is stated quite explicitly, for example, by the historian Rouse Ball

  *The history of mathematics cannot with certainty be traced back to any school or period before that of the. . . Greeks. . . . Though all early races. . . knew something of numeration yet the rules. . . were neither deduced from nor did they form part of any science.*[1]

[1] W. W. Rouse Ball, *A Short Account of the History of Mathematics*, Dover, New York, 1960, pp. 1–2, emphasis mine.

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- Does anyone know what that proof was which was supposedly found by Pythagoras?
- Was it a “deductive” proof or did it involve the empirical?

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- Another example is Elements 1.4 (Side-angle-side) theorem.
- Which proved the equality of two triangle by "applying" one triangle to another.
- Empirical proofs of the Pythagorean theorem are very easy, and were known to other cultures, such as Egyptian and India.

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- Does anyone know the evidence that “Euclid” was the author of the *Elements*?

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- They all claim to be based on the lectures of Theon.
- Euclid's name is also not mentioned in any commentaries on the Elements,
- They all speak anonymously of the "author of the Elements".

- This is confirmed by Sir Thomas Heath, a leading authority on “Euclid”. *Elements*.[2]

> *All our Greek texts of the Elements up to a century ago. . . purport in their titles to be either ‘from the edition of Theon’. . . or ‘from the lectures of Theon’.*

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  > *All our Greek texts of the Elements up to a century ago. . . purport in their titles to be either ‘from the edition of Theon’. . . or ‘from the lectures of Theon’.*

- Euclid’s name does not appear even in Greek commentaries on the *Elements* because the Greek commentaries “commonly speak of the writer of the *Elements* instead of using his name.”

[2] Sir Thomas Heath, *A History of Greek Mathematics*, Dover, New York, 1981, p. 360.

# The evidence for Euclid

## A single remark by Proclus

*Not much younger than these [pupils of Plato] is Euclid, who put together the Elements,. . . bringing to irrefutable demonstration the things which had been only loosely proved by his predecessors. This man [must have] lived in the time of the first Ptolemy; for Archimedes, who followed closely the first [Ptolemy? book?] makes mention of Euclid, and further they say that Ptolemy once asked him if there were a shorter way to study geometry. . . to which he replied that there was no royal road to geometry. He is therefore younger than Plato's circle, but older than Eratosthenes and Archimedes; for these were contemporaries, as Eratosthenes somewhere says."*

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- The vague remark is attributed to Proclus
- who came 800 years after “Euclid’s” supposed date.
- and says that no one else mentioned “Euclid” earlier.
- And even that remark itself comes from a manuscript which comes from another 800 years after Proclus.

# “Proclus’ remark”

## contd.

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- attributed to Archimedes but coming from 1800 years after him.
- The citation is regarded as spurious since it is isolated
- and it was not the custom in Archimedes’ time to make such citations.

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contd.

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- Since the author of the “Proclus’ remark” knew of the spurious Arhicmedes citation
- which comes from a thousand years after Proclus
- the “Proclus’ remark” is itself spurious.
- So, there is no evidence that “Euclid” even existed.

# Does it matter?

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- This was a time when a religious war was going on.
- How does a religious war concern mathematics?

# Mathematics and religion

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- Mathematics derives from “mathesis” meaning learning.
- Plato (Meno) explained that “all learning is recollection of eternal ideas in the soul”.
- Plato (Republic) prescribed the teaching of mathematics to make people virtuous, since it improves their soul.

# Proclus on the *Elements*



- This is repeated by Proclus in his Commentary on the *Elements* from a thousand years later.

> *learning ($\mu\alpha'\theta\eta\sigma\iota\zeta$ [mathesiz]) is recollection of the eternal ideas in the soul; and this is why the study that especially brings us the recollection of these ideas is called the science concerned with learning ($\mu\alpha'\theta\eta\mu\alpha\theta\iota\kappa\eta'$ [mathematike])*[3]

[3] Proclus, *Commentary*, p. 38.

# Proclus on the *Elements*

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- Proclus' Neoplatonist beliefs were under attack by the Christian church.
- And he defended them using mathematics.
- But present-day historians maintain that he did not know what he was writing about!

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- These texts came from the mass translation of an Arabic library captured during the Crusades.
- At time the Christian church was fighting a religious war (Crusades) against the Muslim Arabs?
- It had a 800 year old tradition of burning books.
- How was it possible to learn from the books of the enemy?

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- The text was also reinterpreted in line with Christian theology.

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- Wy not make mathematics easy by eliminating the theology
- and focussing on its practical applications?

# Further reading



- *Is Science Western in Origin?* (Multiversity, Penang, 2009)
- "Towards Equity in Math Education 1. Good-Bye Euclid!", *Bhartiya Samajik Chintan* **7** (4) (New Series) (2009), pp. 255–264.

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- "Teaching racist history", *Indian Journal of Secularism* **11**(4) (2008) pp. 25-28.