Calculus without Limits: the Theory

A Critique of the History of Mathematics
Part 2: The Real History of the Calculus

C. K. Raju

Inmantec, Ghaziabad
and
Centre for Studies in Civilizations, New Delhi

Recap

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- (same as problem of defining product of distributions)

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- Notion of deductive proof is not a universal notion: empirical proofs were invariably used in math prior to the 20th c.
- No universal notion of logic: mathematical proof offers no certainty.
- Brings in a theological component into math which is unnecessary.
- Supported by distorted history developed during the Crusades: Pythagoras, Euclid etc. Big stories, nil evidence.

The argument

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- These trigonometric values were badly required for European navigation.
- Hence Indian calculus texts were translated and sent to Europe by Jesuits based in Cochin in the 16th c.
- However, during the Inquisition, its non-Christian origin was not acknowledged, and it was attributed to Newton and Leibniz.

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- This led to the theory of limits which is a more difficult way to do calculus.
- The Indian way of doing calculus better suited to computer technology, and should be used to teach calculus in place of limits.
- Will present a brief account. Full account in my book.


PEARSON LONGMAN
History of Science, Philosophy and Culture in Indian Civilization
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Volume X Part 4
Cultural Foundations of Mathematics
The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. CE
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- in widely distributed palm-leaf manuscripts from Kerala etc.
- (first reported in a British journal by Whish in 1835).
- Some of the texts are *Yuktibhāṣā*, *Tantrasangraha*, *Karaṇapaddhati*, or Nīlakanṭha's *Bhaṣya* on *Āryabhaṭīya*

The sine series

- The “Taylor” series expansion for the sine stated in two verses, as follows (numbers refer to published *Yuktidīpikā*, critical ed. K. V. Sarma).

निहत्य चापवर्गेण चापं तत्तत्फलानि च ।
हरेत् समूल्युग्वर्गैस्त्रिज्यावर्गहतैः क्रमात् ॥ ४४० ॥
चापं फलानि चाधोऽधो न्यस्योपर्युपरित्यजेत् ।
जीवाप्त्यै, संग्रहोऽस्यैव विद्वान् इत्यदिना क्रितः ॥ ४४१ ॥

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- Place the arc and the successive results so obtained one below the other, and subtract each from the one above.
- These together give the *jīvā*, as collected together in the verse beginning with *"vidvān"* etc.

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- Let r = radius of the circle, s = arc and let t_n = the nth expression obtained by applying the above rule.

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- Numerator: multiply the arc s by its square s^2, this multiplication being repeated n times to obtain $s \cdot \prod_1^n s^2$.
- Denominator: multiply the square of the radius, r^2, by $[(2\,k)^2 + 2\,k]$ ("the squares of successive even numbers increased by that number") for successive values of k, repeating this product n times to obtain $\prod_{k=1}^{n} r^2\,[(2k)^2 + 2k]$.

Math translation: contd.

- Thus, the nth iterate is obtained by

$$t_n = \frac{s^{2n} \cdot s}{(2^2 + 2) \cdot (4^2 + 4) \cdot \cdots \cdot [(2n)^2 + 2n] \cdot r^{2n}}. \tag{1}$$

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- The rule further says:

$$\text{jīvā} = s - t_1 + t_2 - t_3 + t_4 - t_5 + \ldots \qquad (2)$$

$$= s - \frac{s^3}{r^2 \cdot (2^2+2)} + \frac{s^5}{r^4 (2^2+2)(4^2+4)} - \cdot \qquad (3)$$

Math Translation contd.

- Substituting
  (1) jīvā $\equiv r \sin\theta$,
  (2) $s = r\,\theta$, so that $s^{2n+1} / r^{2n} = r\,\theta^{2n+1}$, and noticing that
  (3) $[(2k)^2 + 2k] = 2k \cdot (2k+1)$, so that
  (4) $(2^2+2)(4^2+4)\cdots[(2n)^2 + 2n] = (2n+1)!$,

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  (4) $(2^2+2)(4^2+4)\cdots[(2n)^2 + 2n] = (2n+1)!$,
- and cancelling r from both sides, we see that this is entirely equivalent to the well-known expression

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots . \tag{4}$$

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- where $c = 5400'$ was a quarter of the circumference of the standard circle (with circumference $21,600'$).

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- where $c = 5400'$ was a quarter of the circumference of the standard circle (with circumference $21,600'$).
- That is $c = \frac{\pi}{2} r$ for this circle.

Series for numerical calculation

- The coefficients of this series were calculated and stored in the verse beginning *vidvān* etc.

विद्वांस् तुन्नबलः कवीशनिचयः सर्वार्थशीलस्थिरो
निर्विद्धाङ्गनरेन्द्ररुङ् निगदितेष्वेषु क्रमात् पञ्चसु ।
आधस्त्याद् गुणितादभीष्टधनुषः कृत्या विहृत्यान्तिम-
स्याप्तं शोध्यमुपर्युपर्यथ घनेनैवं धनुष्यन्ततः ॥ ४३७ ॥

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- Here *vidvān* etc. are reverse sexagesimal *kaṭapayādi* expressions for five numbers

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- and subtract from the next number. [Continue this process with the result so obtained and the next number.]
- Multiply [the final result] by the cube of the arc divided by quarter of the circumference, and subtract from the arc.

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- Again multiply the result by $(s/c)^2$ and subtract from the next number.
- Multiply the final result by $(s/c)^3$ and subtract from the arc s, to obtain the *jyā*. Thus, the result my be expressed by the formula (same as (**??**))

$$r \sin \theta = s - \left(\frac{s}{c}\right)^3 \left[a_3 - \left(\frac{s}{c}\right)^2 \left[a_5 - \left(\frac{s}{c}\right)^2 \left[a_7 - \left(\frac{s}{c}\right)^2 \left[a_9 - \left(\frac{s}{c}\right)^2 a_{11}\right]\right]\right]\right]. \quad (6)$$

Kaṭapayādi system

- Each consonant corresponds to a number according to Sanskrit alphabet. (For conjoint consonants only the last matters.)

1	क	ट	प	य
2	ख	ठ	फ	र
3	ग	ड	ब	ल
4	घ	ढ	भ	व
5	ङ	ण	म	श
6	च	त		ष
7	छ	थ		स
8	ज	द		ह
9	झ	ध		ळ
0	ञ	न		

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- (The even values of k are used for the cosine series.)
- In sexagesimal notation, a number is interpreted using first (*kalā*), second (*vikalā*), and third (*tatparā*) sexagesimal minutes. Thus,

- *vidvān* $=$ *vi dvā n* $= 4\ 4\ 0 = 0''44'''$ (for *dvā* use the conjoint consonant rule),

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- *tunnabala* = *tu nna ba la* = 6 0 3 3 = $33''06'''$, *kavīśanicaya* = *ka vī śa ni ca ya* = 1 4 5 0 6 1 = $16'05''41'''$, *sarvārthaśilasthira* = *sa rva rth śī la sthi ra* = 7 4 7 5 3 7 2 = $273'57''47'''$, *nirviddhaṅganarendraru* = *ni rvi ddha ṅga na re ndra ru* = 0 4 9 3 0 2 2 2 = $2220'39''40'''$.

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- More accurate than 7th c. Bhāskara I's figure of $3438'$,
- or 9th c. Vaṭeśvara's figure of $3437' \, 44''$.)

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  सङ्गमग्रामजो माधवः पुनरत्यासन्नां परिधिसंख्यामुक्तवान् –
  विबुधनेत्रगजाहिहुताशनत्रिगुणवेदभवारणबाहवः ।
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- Mādhava of Saṅgamagrāma spoke the approximate [*āsanna*] number of the circumference of a circle: *vibudha* [33] *netra* [2] *gaja* [8] *ahi* [8] *hutāśana* [3] *tri* [3] *guṇa* [3] *veda* [4] *bhavāraṇa* [27] *bāhavaḥ* [28], i.e., [2,827,433,388,233] is the measure of a circle of diameter *nava* [9] *nikharva* [100,000,000,000].

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- Corresponds to value of $\pi = 3.141, 592, 653, 5922 \ldots,$

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Value of π contd.

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- with the 12th and 13th places (92 respectively) differing slightly from their accurate value (89).

Madhava's sine table

श्रेष्ठं नाम वरिष्ठानां हिमाद्रिर्वेदभावनः।
तपनो भानुसूक्तज्ञो मध्यमं विद्धि दोहनम्॥
धिगाज्यो नाशनं कष्टं छन्नभोगाशयाम्बिका।
म्रिगाहारो नरेशोऽयं वीरो रणाजयोत्सुकः॥
...
छायालयो गजो नीलो निर्मलो नास्ति सत्कुले।
रात्रौ दर्पणमभ्राङ्गं नागस्तुङ्गनखो बली॥
धीरो युवा कथालोलः पूज्यो नारीजनैर्भगः।
कन्यागारे नागवल्ली देवो विश्वस्थली भृगुः॥
तत्परादिकलान्तास्तु महाज्या माधवोदिताः।
स्वस्वपूर्वविशुद्धे तु शिष्टास्तत्खण्डमौर्विकाः॥ २.९.५॥

Madhava's sine table

Table: Mādhava's sine values

No.	Kaṭapayādi	kalā (′)	vikalā(″)	tatparā(‴)
1	श्रेष्ठं नाम वरिष्ठानां	224	50	22
2	हिमाद्रिर्वेदभावनः	448	42	58
3	तपनो भानुसूक्तज्ञो	670	40	16
4	मध्यमं विद्धि दोहनम्	889	45	15
...	...	...	...	...
21	धीरो युवा कथालोलः	3371	41	29
22	पूज्यो नारीजनैर्भगः	3408	20	11
23	कन्यागारे नागवल्ली	3430	23	11
24	देवो विश्वस्थली भृगुः	3437	44	48

Accuracy of Madhava's sine values

Table: Accuracy of Mādhava's sine table.

No.	Mādhava's sine value	Difference
1	0.0654031452	0.0000000160
2	0.1305262297	0.0000000375
3	0.1950903240	0.0000000020
4	0.2588190035	−0.0000000416
...	...	...
...	...	...
21	0.9807852980	0.0000000176
22	0.9914448967	0.0000000353
23	0.9978589819	0.0000000587
24	1.0000000000	0.0000000000

Āryabhaṭa's sine table

- मखि भखि फखि धखि णखि ञखि
  ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।
  घ्लकि किग्र हक्य धकि किच
  स्ग श्झ ङ्व क्ल प्त फ छ कलार्धज्या ॥१२॥

Āryabhaṭa's numerical notation

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- *avarga* (unclassified) letters in the *avarga* (even) places. (They have values 30, 40, 50, 60, 70, 80, 90, 100, respectively.)

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- The nine vowels अ, इ, उ, ऋ, ऌ, ए, ओ, ऐ, औ denote the two nines of zeros (corresponding to the 18 places from 10^0 to 10^{17}): each vowel takes one *varga* and one *avarga* place.

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- Thus अ denotes the place of 1 as well as 10, इ denotes the place of 100 as well as 1000, etc.

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Translation

- 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7—[these are the] Rsine [differences] [for the quadrant divided into as many equal parts, each part hence being 225′] [in] minutes.

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- (Circumference of the circle in minutes is $360 \times 60 = 21,600$.)

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- (Brahmagupta tried to go back to older 6 sine values 15° apart using quadratic interpolation, with second differences.)

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- Translation: (12) The Rsine of the first arc divided by itself and negated gives the second Rsine difference. That same first Rsine, when it divides successive Rsines gives the remaining [Rsine differences].

Mathematical translation

- R_i = sine values, $\delta_i = R_i - R_{i-1}$ sine differences. Then Āryabhaṭa's rule consists of two parts

$$\delta_2 - \delta_1 = -\frac{R_1}{R_1}, \qquad (8)$$

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- while Āryabhaṭa is precise to the minute (so that $\delta_1 - \delta_2 = 225 - 224 = 1$).

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- E.g. for $n = 23$, $\delta_{23} = 22$, $\delta_{24} = 7$, while $R_1 = 225$, so that we should have $R_{23} = (\delta_{23} - \delta_{24}) \times R_1 = 15 \times 225 = 3735 \neq 3431$ the 23rd sine value in the *Sūrya Siddhānta* (or *Āryabhaṭīya*).

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- Difference in each case, since no value is a multiple of 225.

Āryabhaṭa and Euler

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- E.g. Bhaskara II used infinitesimal techniques to calculate (a) the volume of a sphere, and (b) for instantaneous velocity of a planet.

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- Further details in my book.

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- No limits: but order counting (which is adequate).
- **Principle:** In comparison with a constant (*rūpa*), for large n, we may neglect any quantity which is $O\left(\frac{1}{n}\right)$.

Why trigonometric values were needed

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- which required accurate planetary models.
- Note: Greek and Roman calendars inaccurate and slipshod. (Inaccurate length of the tropical year until 1582; months of varying length, unrelated with the motion of the moon).

The "delayed" monsoon of 2004

- Dire anticipation of drought

THE TIMES OF INDIA

New Delhi, Friday, July 30, 2004 www.timesofindia.com Late City 18 pages Invitation Price Rs. 1.50

International All set to break into movies, Naomi is taking acting classes Page 10

India Court order on Bhandarkar's bail plea today Page 8

Sport Harbhajan plots Jayasuriya's downfall for final Page 17

Drought grips half the country

276 of 524 Met Districts Get Deficient Or Scanty Rainfall

By Samiran Chakrawertti

New Delhi: Weather watching just got grimmer. The shadow of drought has spread over half the country. In Mausam Bhavan language, 276 out of 524 districts — or 54% of all meteorological districts — have had deficient or scanty rainfall for the period from June 1 to July 21, while 18% had excess rainfall. Neither is good for crops. Only 28% of the country has had normal rain.

Experts say that if things stay this way, the country could face a repeat of the drought situation of 2002, when 267 districts were affected by meteorological drought. A host of crops are wilting — among them paddy, pulses, groundnut, soya, sugarcane, cotton and coarse cereals.

The states that are worst hit are Haryana, Punjab, Rajasthan, Madhya Pradesh, Gujarat and Maharashtra. Things are pretty alarming in these states: as many as 150 districts out of a total of 181 have had deficient or scanty rainfall. The driest among these states are Rajasthan and Gujarat.

In Rajasthan, all 32 districts have had scanty rain, while in Gujarat it's 24 out of 25.

Rainfall is classified as excess (+20%), normal (+19 to -19%), deficient (-20 to -59%) or scanty (below 60%), depending on the variation in rainfall from the seasonal average.

Normal rain may not do: P8

WITHER REPORT

54% of the country's districts recorded below-normal rainfall till July 21

Worst-Hit States	No. Of Districts Deficient	Scanty
MP	24	10
Rajasthan	21	11
Gujarat	21	3
Maharashtra	21	0
Uttar Pradesh	19	9
Kerala	11	0
Andhra	11	0
Tamil Nadu	10	1
Haryana	8	8
Punjab	6	6

Graphic: Arpit

Plan ahead, say experts

New Delhi: Faced with erratic monsoon this year, agricultural experts have recommended creation of a buffer stock of seed, feed and fodder reserves. They have also urged the government to decide whether the crop pattern needs to be changed for the rabi season. P8

India Inc has no place for sonset clause

BOYS' CLUB: DAUGHTERS YET TO GET DUE

Surrender by

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- It was in time according to the traditional calendar.
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- Wind regime depends upon sidereal motion and lunar position.

2009

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2009

- Same phenomenon repeated in 2005 and in 2006.
- And again in 2009.

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M F HUSAIN GETS NOSTALGIC ABOUT AUG 15, 1947; RULES OUT RETURN TO INDIA 13

SAIN AT W QUA

Govt scrambles to save kharif, prays for rabi

Deficient Rain In 31 Out Of 36 Met Zones This Month

Rajeev Deshpande & Nitin Sethi | TNN

New Delhi: An unexpectedly dry first fortnight of August has starkly set out the big task before a drought-battered government — to salvage whatever remains of the kharif crop. At the same time, it's praying for rain even now so that moisture in the soil would help successful sowing of the rabi (winter) crop.

As things stood on Friday, the country has had deficient rainfall in all parts, barring Orissa, Karnataka and southern Gujarat (see map). The latest farm and rain statistics in the week August 6-12 show as many as 31 out of 36 Met sub-divisions were in either "deficient" (8 sub-divisions) or "scanty" (23) rainfall category.

Cabinet secretary K M Chandrashekhar, whose office is overseeing drought mitigation, told TOI, "The effort is to try and save the crop in the field at all costs with additional power and diesel subsidy while tackling urban problems like drinking water. The government is working on a war footing."

An emergency subsidy of Rs 1,000 crore is being given for diesel for farm pumpsets. Additional power will be

- Monsoon worsens in August, raising doubts about IMD's recently revised forecast
- In week gone by, rain deficit is 85% in east Rajasthan, 81% in AP's Rayalseema, 88% in Marathwada and 86% in west UP
- Except for Saurashtra and south Karnataka, all other Met sub-divisions got below normal rain in Aug 6-12. As many as 177 districts declared drought-hit
- Paddy sowing down by 25%, worst in UP (-28%) and Bihar (-14%). Total foodgrain sowing down by 11%, coarse cereals by 14%. Sugarcane sowing near-normal, but hit by poor rain in UP, Maharashtra

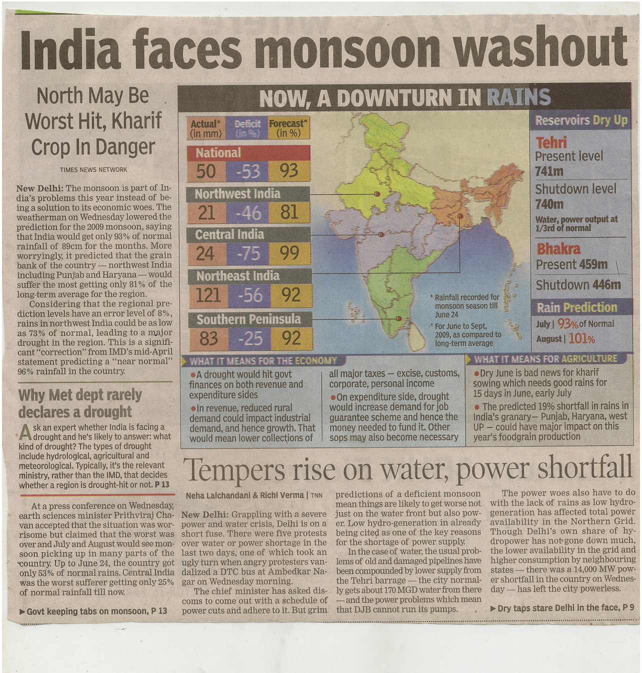
Wages of DROUGHT

Rainfall from June 1 to Aug 12 (% deviation from long term average)

-28% -51% -35% -42% -66% -68% -31% -85% -53% -40% -33 -37% -39% -49% -26% -31% -32% +46% -28% +9% -34% -47% -18% -59% -22% -46% 51% +9% -23%

OPERATION SALVAGE

- Govt provides **Rs 1,000cr to subsidize diesel** for pumpsets. Plans extra power for rain-deficit states and hundreds of new tubewells
- **Food stocks to be released in market.** Sugar, pulses to be imported. Move to



India faces monsoon washout

North May Be Worst Hit, Kharif Crop In Danger

TIMES NEWS NETWORK

New Delhi: The monsoon is part of India's problems this year instead of being a solution to its economic woes. The weatherman on Wednesday lowered the prediction for the 2009 monsoon, saying that India would get only 93% of normal rainfall of 89cm for the months. More worryingly, it predicted that the grain bank of the country — northwest India including Punjab and Haryana — would suffer the most getting only 81% of the long-term average for the region.

Considering that the regional prediction levels have an error level of 8%, rains in northwest India could be as low as 73% of normal, leading to a major drought in the region. This is a significant "correction" from IMD's mid-April statement predicting a "near normal" 96% rainfall in the country.

At a press conference on Wednesday, earth sciences minister Prithviraj Chavan accepted that the situation was worrisome but claimed that the worst was over and July and August would see monsoon picking up in many parts of the country. Up to June 24, the country got only 53% of normal rains. Central India was the worst sufferer getting only 25% of normal rainfall till now.

▶ Govt keeping tabs on monsoon, P 13

Why Met dept rarely declares a drought

Ask an expert whether India is facing a drought and he's likely to answer: what kind of drought? The types of drought include hydrological, agricultural and meteorological. Typically, it's the relevant ministry, rather than the IMD, that decides whether a region is drought-hit or not. P 13

NOW, A DOWNTURN IN RAINS

	Actual* (in mm)	Deficit (in %)	Forecast^ (in %)
National	50	-53	93
Northwest India	21	-46	81
Central India	24	-75	99
Northeast India	121	-56	92
Southern Peninsula	83	-25	92

* Rainfall recorded for monsoon season till June 24

^ For June to Sept, 2009, as compared to long-term average

Reservoirs Dry Up

Tehri
Present level 741m
Shutdown level 740m
Water, power output at 1/3rd of normal

Bhakra
Present 459m
Shutdown 446m

Rain Prediction
July | 93% of Normal
August | 101%

WHAT IT MEANS FOR THE ECONOMY

- A drought would hit govt finances on both revenue and expenditure sides
- In revenue, reduced rural demand could impact industrial demand, and hence growth. That would mean lower collections of all major taxes — excise, customs, corporate, personal income
- On expenditure side, drought would increase demand for job guarantee scheme and hence the money needed to fund it. Other sops may also become necessary

WHAT IT MEANS FOR AGRICULTURE

- Dry June is bad news for kharif sowing which needs good rains for 15 days in June, early July
- The predicted 19% shortfall in rains in India's granary— Punjab, Haryana, west UP — could have major impact on this year's foodgrain production

Tempers rise on water, power shortfall

Neha Lalchandani & Richi Verma | TNN

New Delhi: Grappling with a severe power and water crisis, Delhi is on a short fuse. There were five protests over water or power shortage in the last two days, one of which took an ugly turn when angry protesters vandalized a DTC bus at Ambedkar Nagar on Wednesday morning.

The chief minister has asked discoms to come out with a schedule of power cuts and adhere to it. But grim predictions of a deficient monsoon mean things are likely to get worse not just on the water front but also power. Low hydro-generation in already being cited as one of the key reasons for the shortage of power supply.

In the case of water, the usual problems of old and damaged pipelines have been compounded by lower supply from the Tehri barrage — the city normally gets about 170 MGD water from there — and the power problems which mean that DJB cannot run its pumps.

The power woes also have to do with the lack of rains as low hydrogeneration has affected total power availability in the Northern Grid. Though Delhi's own share of hydropower has not gone down much, the lower availability in the grid and higher consumption by neighbouring states — there was a 14,000 MW power shortfall in the country on Wednesday — has left the city powerless.

▶ Dry taps stare Delhi in the face, P 9

Drought almost certain now, admits IMD

GoM To Ready Contingency Plan

Prasad Kulkarni | TNN

Pune/New Delhi: There is every possibility of 2009 being a "drought year" with weather officials saying that only rainfall 30% in excess of normal for the remainder of the monsoon from mid-August to September — a near impossibility — can now stave off the spectre of drought.

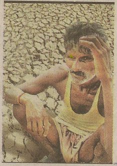
PM Manmohan Singh on Thursday set up a group of ministers to chalk out a contingency plan to deal with the situation while a committee of secretaries may be asked to look into import of sugar. Agriculture minister Sharad Pawar said the government would decide next week on whether to release food stocks into the open market to stabilise prices.

With the cumulative shortfall in rainfall touching 29%, the monsoon has defied predictions that it could revive to the extent that it significantly lowers the deficit in regions like north-west, central and north-east India.

A senior IMD official at Pune said the country was more than likely to end up with a drought unless the remaining 45 days of monsoon saw much more than average rainfall — something that would be contrary to the Met's own predictions.

Explaining the scenario, IMD's deputy director general for weather forecasting A B Mazumdar said, "The rainfall should be at least 30% in excess of normal rains for the remaining 45 days of the monsoon season."

The average rainfall of the June-to-September monsoon season in the country is 890mm and normal rain for the mid-August to end-of-September period is 324mm.



Running Dry

- Monsoon cannot revive enough to lower the deficit in large parts of India
- Only 30% above average rain in remaining 45 days of season can prevent drought
- Drought year is declared when 20% to 40% of the country's total area is rainfall deficient and June-Sept shortfall is 10% or more

'N India's groundwater vanishing'

Mumbai: Using Nasa satellite data, scientists have found that groundwater levels in northern India have been declining by as much as 33cm (one foot) per year over the past decade.

Attributing the loss almost entirely to human activity, Nasa's Jet Propulsion Laboratory said more than 108 cubic km of groundwater disappeared from aquifers in Haryana, Punjab, Rajasthan and Delhi between 2002 and 2008 — far more water than

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ANIL AMBANI WRITES ON THE IMPORTANCE OF TRUST IN SPEAKING TREE 16

BROAD'S FIVE-WICKET HAUL PUTS ENGLAND IN STRONG POSITION 23

As heavens open up, it's hell on city roads

TIMES NEWS NETWORK

New Delhi: For the second time this season, the heavens opened up over the capital and brought on hell for commuters and residents within a space of a few hours on Friday evening. As it turned dark and rain came down in sheets accompanied by strong winds, thrilled Delhiites soon witnessed the horror of familiar traffic chaos across the city. Several areas were completely flooded in the 74mm rain and

STALLED BY STORM
PAGES 3 & 4

many people spent hours on the road, stuck in traffic jams.

The IGI Airport proved to be the biggest embarrassment as part of the roof of the new departure terminal got blown off and a large part of the terminal got waterlogged. Many flights were delayed and diverted as visibility fell to between 100 and 300m. Several passengers also missed even delayed flights because they were stuck on the road.

The city lost several trees (NDMC said 100 trees were fully or partially damaged, while MCD said 14 trees were uprooted in its area) which blocked roads, aggravating

Yogesh Kumar

NOWHERE TO GO: A driver takes a break during a massive jam

RAIN & PAIN

What Came Down
- 74mm of rain in an afternoon storm with windspeeds up to 91 kmph (Palam)
- Temperature by 16°C (from 36°C to 20°C)
- Power demand by 1,500MW (from 4,000MW to 2,500MW)

What Went Up
- Stress levels and chaos. Just one car helpline got 600 breakdown calls by 9pm

5 Slowest Trips*
- Gurgaon-SP Marg. Time: **5 hrs**
- New Friends-South Ex: **3 hrs**
- IP Extn-India Gate. **2.15hrs**
- Delhi Gate-Mandi House. **2 hrs**
- IGI-Domestic Terminal. **1.30 hrs**

*Based on commuter feedback

Airport Down Under
- Part of new 1D terminal roof blown away. Operations stalled

Forecast
- Isolated showers over northwest India in next 48 hrs

Yet again, underpasses and flyovers were a sore point as acute waterlogging in and around them led to traffic diversions. Traffic lights also

roads and overflowing sewers and drains also saw water entering many houses.

The maximum temperature early Friday afternoon —

two hours because of the rain. Met officials said a cyclonic circulation over northwest India led to the thundersquall.

short stories

DELHI FINALLY RAIN-SURPLUS

8 mm

Delhi's rain surplus till Friday evening

NEW DELHI: The Capital's rain shortage is over. The late spurt in monsoon activity had left the city with an 8mm rain surplus by Friday evening. Against 578.1mm rain, which is considered 'normal' for Delhi by September 11, the city had recorded 586mm rain by Friday evening. The city received 9.9 mm rain on Friday. The Met office has predicted the same amount of rain on Saturday. Thanks to the wet weather, the temperature, too, has been hovering between 24 and 20 degrees Celsius. HTC

Roof collapse kills mother, child

GHAZIABAD: Two persons, including a three-year-old child, were killed and four other members of the family were critically injured on Friday when the ceiling of their house collapsed due to heavy rains. The victims were residents of Indirapuri Colony in the Loni area of Ghaziabad. The incident happened around 4 am. Police said that following incessant rains, the roof of house number B-5 came down, leaving the family members buried under the debris. Rajni Garg (35) and her

It's raining trouble n

TOO MUCH OF A GOOD THING Unabated rainfall is breaking up roads, causing traffic jams and b

204.83 m Yamuna's danger mark in Delhi | 203 m Water level recorded on Friday | 207.49 m Record water level on June 6, 1978, when even Model Town was flooded

SONU MEHTA/HT PHOTO

The Yamuna is expected to breach its danger mark by Saturday afternoon.

Yamuna's bank rupture looks imminent

HT Correspondent

Flood Control department to be on its guard.

The moment we

cusecs of water (12 million litres per second — that's equal to

Yamuna's danger level was decided by the British in the

Dengu malari to take

BREEDING T multiply in wet

Jaya Shroff Bhalla
jaya.shroff@hindustantimes.com

NEW DELHI: The belated onset rains in the Capital is now s nalling a delayed outbreak dreaded mosquito-borne d eases — dengue and malari

"We have already receiv eight cases and are expecti the figures to rise. Delay in m soons only delays water-bo diseases, but this certainly d not mean that we should expect cases this year," s Dr NK Yadav, chief medical cer, Municipal Corporatio Delhi (MCD).

"We have intensified che ing. Soon after rains, ther bound to be increase in bre ing and hence a greater cha of water-borne infections."

Delhi doctors are a wor lot as queues of patients viral fever outside the cli and out-patient departm in hospitals are getting lo

"As soon as the rains s cases of mosquito-borne eases will start coming Sporadic rains provide a fect breeding ground for the dengue-causing *a aegypti* and malaria-cau

Celestial navigation

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- Europeans had no reliable ephemeris calculations then, and (c) could not be used, since Columbus had the wrong size of the globe,
- hence Europeans lacked a good estimate of the size of the globe until Picard in 1671.

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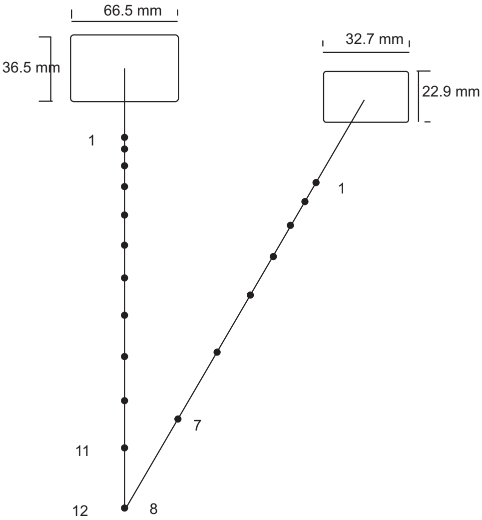
- Hence led to the longitude problem of European navigation.


66.5 mm
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11
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The *kamāl* or *rāpalagai*

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- Instrument has accuracy of 11 miles and a range of 670 miles.

Illustration of epistemological difficulties

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- The instrument was copied in Europe, in crude ways like the cross-staff, showing that it was never earlier understood (even by historians of science like Prinsep).

The European navigational problem

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- Various governments encouraged solution of this problem from 1530 to 1760.

Loxodromes and tables of secants

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- Mercator's sources of accurate trigonometric values unknown; he was arrested by the Inquisition, and had enough motive to hide his sources.

The Gregorian calendar reform of 1582

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- This required calendar reform of 1582 (practical compulsion to correct the date of Easter).

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- This involved "heaving the log", and was very inaccurate.
- By the time the size of the globe was accurately determined, globes had been banned aboard ships for nearly 2 centuries by Portugal. Hence this was not used.
- Eventually led to the development of the chronometer.

Roman Catholic church in India in the 16th c.

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- Set up school for local Syrian Christians based on "Prester John" strategy for winning the Crusades.
- Taken over by Jesuits around 1550 and converted into a college.
- Church in India replicated its earlier perceived winning strategies:
  - (a) destroyed all Hindu temples in Goa by 1543 (as in Alexandria, 4th c.),
  - (b) imposed Inquisition,
  - (c) tried to convert the Moghul king Akbar ca. 1580 a la Constantine
  - (d) burnt all local Bibles (older Aramaic bibles) at'Synod of Diamper' 1599 and...

Cochin and Toledo

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- His student Matteo Ricci was further trained in navigation in Coimbra before being despatched to India.
- He visited Cochin and wrote back that he was looking for an "honest Moor or an intelligent Brahmin" to explain Indian calendrical texts to him.

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- As compared to balance of probabilities (appropriate to history) I proposed a standard of evidence used in criminal law for proof beyond doubt.
- Involved consideration of (a) opportunity, (b) motivation, (c) circumstantial, (d) documentary, and (e) epistemic evidence.

Circumstantial trail

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- Kepler's fantastic accuracy of orbit of Mars explained away as a miracle. (He was nearly blind, and could not have made the observations. He decamped with Tycho's secret papers after Tycho's death.) Kepler's explanation not credible (he was a professional astrologer). Parameswar had made observations over 50 years to attain this accuracy.

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- Newton feels he has found a rigorous resolution through fluxions.

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- Introducing the study of navigation in schools in 1938.
- Induced islanders to become dependent on a British sextant which was less accurate than their indigenous instrument, and could not be made locally. Nobody bothered to compare the two.

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- and made hilarious blunders (he made an abacus with Arabic numerals).

Algorismus

- In the 10th c. when Arabic numerals went to Europe
- Gerbert of Aurillac (later Pope Sylvester) did not understand their function.
- and made hilarious blunders (he made an abacus with Arabic numerals).
- Europeans remained poor at arithmetic until 16th c.

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- But that is another story.

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- But a similar problem arose in pure mathematics.
- Descartes declared in his *Geometry* that measuring the ratios of curved and straight lines was beyond the capacity of the human mind.
- Indian mathematics accepted measurement of curved lines since *śulba sūtra*-s.

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- No agreement on metaphysics in Indian tradition—not even logic. (Buddhist and Jain logics are not 2-valued nor truth-functional.)
- Thus, present-day mathematical proofs are worthless from a Buddhist viewpoint.

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- When transmitted to Europe, this was not properly understood by Europeans dur to a variety of cultural factors
- The European difficulties with the calculus were globalised during colonialism.
- And are maintained today through uncritical imitation.