Two plus two is not always four: why zero so confused Europeans

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Contrary to the belief that mathematics is universal, Indian arithmetic had a distinctive philosophy which is best understood in a historical perspective.

From the time of the Yajurveda, the Indian system of numeration utilised the place value-system, the names of places being roughly those still in current use. As opposed to this, the Greek and Roman system of numeration were adapted to the abacus, a very inefficient way of doing arithmetic (and a clear indication of the falsity of tall claims of Greek or Hellenic scientific achievements). In the 9th c. CE, Indian arithmetic texts of Brahmagupta, Mahāvīra etc. were imported and "translated" in the Baghdad House of Wisdom, notably by al Khwārizmī. This knowledge was later imported into Europe, starting from the 11th c. The Florentine merchants valued the efficiency of the Indian arithmetic techniques for commerce, and these techniques came to be known in Europe as the "algorismus" or "algorithmus" after al Khwārizmī's Latinized name. However, the algorismus took some 5 centuries to become widespread, and nearly 8 centuries to become fully accepted in Europe. (The exchequer, as the name indicates, *was* an abacus, and did not fully switch to algorismus techniques until about the 19th c.) The broad outlines of this story are well known.

The question here is why did it take so many centuries for Europe to understand and accept the Indian techniques of arithmetic? One way to understand this is that the Greek/Roman system of numeration was additive: XXIII meant X + X + I + I + I, and from this perspective, 20 was interpreted as 2 + 0 and felt to be the same as 2 or 2000. Hence, from *sifr* (= cypher), zephyr, to zero, as connoting something mysterious and incomprehensible. Hence, also, 13th c. Florence first passed the law, distrusting zero, that in a financial instrument (such as a cheque) numbers *must* be written also in words (to avoid putting any number of zeros at the end). While this difficulty is comprehensible, it is hard to understand why it took so many centuries to resolve. Zero (or the place value system) may have been confusing for Europeans, but why was it *so* confusing? This suggests that there are deeper issues here.

To understand these issues, I first go back to the representation of non-ratio numbers like $\sqrt{2}$ and π in Indian texts. From the *sulba sūtra*-s, through Āryabhaṭa and down to Nīlakanṭha, the use of words like सविशेष (*saviśeṣa*), सानित्य (*sānitya*), or आसन्न (*āsanna*) indicates a philosophy of representing numbers that is contrary to the Pythagorean, Platonic or Neoplatonic belief in the perfection of mathematics. This difficulty is reflected also in the Pythagorean analogy between mathematics and music. Indian music and European music have handled the problem of the Pythagorean comma in fundamentally different ways.

Most importantly, this difficulty is related to the *Śūnyavāda* philosophy which accepts non-representability as a fundamental limitation, deriving from the nature of time. In this context, *śūnya* means not zero, but anything non-representable, which is discarded in the course of a representation or calculation. The believed perfection of mathematics in Europe did not however allow the smallest quantity to be set aside in a calculation. The problem involved should not be confounded with rounding: these difficulties appear in full-blown form over the problem of representing infinitesimals, with the transmission of the calculus from India to Europe, and the reactions of European mathematicians such as Descartes, Galileo, Leibniz, Newton and Berkeley. At a slightly higher level of technicality, in formal mathematics, these difficulties persist to this day in the renormalization problem of quantum field theory, for example, or the related problem of defining the square of the Dirac δ in the Schwartz theory of distributions (or its extension using nonstandard analysis).

Based on this understanding of the clash between the Indian and Western philosophies of arithmetic, I examine some of the difficulties of calculation in current computer arithmetic. It is well known that neither integers nor rational nor real numbers can be correctly represented on a computer: it is elementary that a C program can produce $20,000 + 20,000 = -25536$ or $2 + 2 = 4.00000000000000001$, for example. In fact, a computer perforce does integer or floating point arithmetic according to fundamentally different algebraic "laws": the associative "law", for instance, fails for floating point arithmetic. I also take up the curious case of zero in the Java language, where present-day confusion about zero is demonstrated in the way zero as an int behaves differently from zero as a float! *Śūnyavāda* provides a novel way out.