

Decolonising mathematics: how and why it makes science better (and enables students to solve harder problems)

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Extended summary

Normal vs formal math

Today, professional mathematicians just assume that mathematics means formal mathematics. But there is an increasing dissatisfaction with this assumption, amidst a growing demand for decolonisation, most notably in South Africa (“Rhodes must fall”), India, and various Muslim countries.¹ A fundamental question is being raised: is formal mathematics the only way to do mathematics? Is it the wrong mathematics? Just a bad piece of colonial knowledge used by the coloniser to capture and rule the colonised mind?

Traditional **normal mathematics** accepted *both* empirical proofs and reasoning, just like science. It existed for thousands of years before colonialism. Colonial education, which came as 100% church education, just replaced it with Western **formal mathematics** (which prohibits the empirical, and is founded on a metaphysics of infinity allied to church dogmas of eternity²). Colonialism just claimed that Western (formal) mathematics is superior without the slightest critical examination, the way it indiscriminately declared everything Western as superior, and even globalised the demonstrably inferior Gregorian calendar³ by indoctrinating the colonised into it through colonial education.

Decolonisation of mathematics is NOT an indiscriminate rejection of everything Western, as it is often caricatured. Decolonisation means the practice and current teaching of mathematics urgently needs to be *critically re-examined*. This must be done *by the colonised*, because the coloniser never did it, and will try to stall any critical analysis even today. The colonised must critically compare normal and formal mathematics *independently of the consent or approval of the West* or of the “(formal) mathematics community” subordinate to it, which, even today, desperately defends its right to make uncritical claims of superiority by censoring and suppressing any attempt at a critical re-examination. Decolonisation of mathematics is an important act of resistance especially relevant to Palestine, a colonised state where even a discussion on decolonisation yet needs to be initiated.

- 1 For an example of the decolonisation debate in Malaysia, see “Decolonisation: Conversations in the Sun”, archived clips posted at <http://ckraju.net/blog/?p=61>.
- 2 CKR, “Eternity and Infinity: the Western misunderstanding of Indian mathematics and its consequences for science today.” *American Philosophical Association Newsletter on Asian and Asian American Philosophers and Philosophies* 14(2) (2015) pp. 27–33. <http://ckraju.net/papers/Eternity-and-infinity-Pages-from-APA.pdf>.
- 3 CKR, “A tale of two calendars”, in *Multicultural knowledge and the university*, ed. Claude Alvares, Multiversity, 2014, pp. 112-119. <https://www.youtube.com/watch?v=MvpuC7Dg4e0> (45K views).

The censored critique

I will explain in detail the point of view in my widely censored article^{4, 5} that to decolonise mathematics one must first stand up to its **false history and bad philosophy**, and why decolonisation of mathematics makes science better and also makes mathematics easier.

False history of Greek origins

First the false history. The myths of an early Greek origin of formal mathematics (Euclid, Pythagoras, etc.) are indefensible⁶ lies. Hence, in the last decade, no one has dared claim my Euclid challenge prize to provide the slightest serious (primary) evidence for “Euclid”. These myths serve a political purpose of hiding the church connection: for the undeniable fact is that the book *Elements* was used as a church text for centuries. Therefore, its “authoritative” understanding (and the related myths and superstitions) *had* to be theologically correct and fit the political agenda of the church and especially its post-Crusade theology of reason against Muslims (who have yet to understand the trick).

Bad Western philosophy

Formal mathematics really began at the turn of the 20th c., with the collapse of the centuries-old Western myth about deductive proofs in “Euclid’s” *Elements*. The truth was finally admitted that the book *Elements* does not have a valid deductive proof of even a single one of its propositions. Hilbert, Russell, Birkhoff etc. then undertook to repair the broken myth by rewriting the book *Elements* to force it to fit the false myth about it. In the process, they uncritically assumed the bad church philosophy that deductive proofs are infallible and superior to empirical/inductive proofs. It is this philosophy on which formal mathematics is founded today.

I will explain my arguments that actually normal mathematics is superior, because contrary to Western beliefs, deductive proofs are MORE fallible than empirical/inductive proofs. Thus, (1) an invalid deductive proof may be persistently mistaken for a valid proof (as happened for centuries with the proofs in “Euclid’s” *Elements*). The philosophical doubt about the validity of a purported deductive proof may persist, so the doubt can only be settled inductively. Invalid deductive proofs are more persistent than errors of observation because the mind is more easily deceived than the senses: given a complex task of deduction, such as a game of chess against a machine, *every* human

- 4 CKR, “To decolonise math stand up to its false history and bad philosophy.” The most accurate version of the censored article is on my blog at <http://ckraju.net/blog/?p=117>. It was reproduced by many others worldwide most of whom also later took it down, but survives in some other locations, where the editors took an independent stand, such as The Wire, <https://thewire.in/75896/to-decolonise-maths-stand-up-to-its-false-history/>. The strange editorial ground for censorship given by the *Conversation* South Africa editor was that the racist academic practices prevailing in South Africa disallowed a non-White to build up an original and large connected body of knowledge and therefore it was illegal for me to “site” any of my own published work. The editor stipulated that a non-White must necessarily only cite and parrot the work and opinions of the White/Western master as befits a slave. A similar colonial “rule” is also tacitly followed by Wikipedia, and at the core of colonial indoctrination through education, that non-Western or non-White sources are unreliable and must be distrusted, and only the West/Whites are “reliable”. Such “rules” and teaching effectively amount to a *carte blanche* for the West/Whites to not only tell the most absurd lies, as they have been doing for centuries, but to get away with them indefinitely, by banning any critique in the traditional manner of the church.
- 5 See, also, CKR, “Mathematics and Censorship”, on Kafila, <https://kafila.online/2017/06/25/mathematics-and-censorship-c-k-raju/> and CKR, “Black Thoughts Matter: Decolonized Math, Academic Censorship, and the “Pythagorean” Proposition”, *Journal of Black Studies*, 48(3) pp. 256-278. . The censored article is also due to appear in *Rhodes Must Fall in Oxford*.
- 6 CKR., *Euclid and Jesus: How and why the church changed mathematics and Christianity across two religious wars*, Multiversity, Penang, 2012.

almost *always* makes a mistake. Likewise, it is very hard for the human mind to verify the validity of a complex computer-generated mathematical proof, such as that of the four colour theorem. Even if someone claims to have validated it, why should others trust him as infallible?

(2) A formal mathematical theorem is NOT valid knowledge (and certainly not eternal truth, as in the Platonic superstition) for it may begin with unsound or factually false premises. Indeed, **any nonsense conclusion whatsoever can be deductively proved as a formal mathematical theorem** from appropriately chosen premises. Even if the premises were to be empirically verified, the theorem would be no more valid than the fallible empirical proofs used to verify the premises. Further, all current formal mathematics begins with an assumed metaphysics of infinity which it is impossible to check empirically, and which may and does lead to absurd conclusions (considered in more detail below), such as the Banach-Tarski paradox (a theorem of set theory but not valid knowledge in reality). In any case, why should anyone (except a colonised mind) superstitiously believe that the Western authorities who postulated that metaphysics of infinity are infallible?

(3) It is a fatal objection to formal mathematics that its proofs are all based on the naïve Western assumption that two-valued “Aristotelian” logic is universal. However, two-valued logic (or even truth-functional multi-valued logic) is neither culturally universal (e.g. Buddhist logic)⁷ nor empirically certain (e.g. quantum logic⁸) so the belief in the universality of two-valued logic is just another Western superstition which sits at the core of formal mathematics. [This superstition arose from the parochial church misunderstanding that logic binds God⁹ so that God cannot create an illogical world (though he can create the facts of his choice, but not the logic of his choice!).] There has been not the whisper of an answer to this fatal objection in the last two decades¹⁰ so it is time to assume the formal mathematics community cannot answer it, and move on.

“It works”, but what exactly works?

Unable to answer these solid objections, the typical lame-duck defence of formal mathematics is to say that “it works”. This fallaciously confounds normal and formal mathematics, by neglecting to identify what exactly “works”. In fact, what “works” for all practical applications to science (and worked for thousands of years) is traditional normal mathematics. No one ever showed that the metaphysics of formal mathematics, built around normal mathematics, adds anything to the practical value of normal mathematics. For example, in a grocer's shop, Bertrand's Russell's 378 page proof (or any other formal proof) of $1+1=2$ in integers adds nothing to the practical value of $1+1=2$ from traditional normal arithmetic, still taught in KG and primary school.¹¹

Again, the formal theory of the calculus, as currently taught, requires formal real numbers and limits, but all practical applications of calculus, such as sending a man to the moon, are achieved by numerical computations done on high-speed computers which use floating-point numbers and cannot represent even a single formal real number. Floats are countably finite (not “uncountably

7 CKR,, “Logic”, article for *Encyclopedia of Non-Western Science, Technology and Medicine*, Springer, 2008, 2014, 2016. pp. 2564–2570. <http://ckraju.net/papers/Nonwestern-logic.pdf>.

8 CKR, “Quantum mechanical time”, chp. 6b, in: *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994.

9 CKR, “The Religious Roots of Mathematics”, *Theory, Culture & Society* **23** Jan-March 2006, Special Issue ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillips, pp. 95–97.

10 CKR, “Mathematics and Culture”, in *History, Culture and Truth: Essays Presented to D. P. Chattopadhyaya*, ed. Daya Krishna and K. Satchidananda Murthy, Kalki Prakash, New Delhi, 1999, pp. 179–193. Also, CKR, “Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhāā”, *Philosophy East and West*, **51**:3 (2001) pp. 325–362.

11 See the cartoon on Russell in CKR, “Mathematics, decolonization and censorship”, Kafila, <https://kafila.online/2017/06/25/mathematics-and-censorship-c-k-raju/>

infinite”) and do not even constitute a field.¹² It is time to declare that it is the purported (metaphysical) exactitude of metaphysical “real” numbers which is truly erroneous.

Stephen Hawking and the gross misuse of the metaphysics of formal mathematics

Normal mathematics, unlike formal mathematics, adopts exactly the methods of science, so it is commonsense that “it works better” for science. However, to save the story of formal mathematics, the bogey is often raised that any change from formal to normal mathematics will somehow hurt science, or at least “advanced” applications to science.

In fact, the key “advantage” of otherwise inferior formal mathematics is solely a political one: because it prohibits the empirical, it forces reliance instead on an authoritatively postulated metaphysics of infinity. Since mathematical authority *de facto* lies in the West, this allows Western authority to be misused to slip in political church dogmas of eternity into science. A classic example of this is the way the authority of Stephen Hawking and his singularity theory has been persistently misused to justify the utterly bogus claim that “physics has proved the truth of Judeo-Christian theology”.¹³ Eliminating such widely propagandised and authoritative but superstitious and false claims clearly only improves science.

University calculus vs Schwartz theory: does either work for physics?

Thus, the conclusions of singularity theory depend tightly on a particular way of doing calculus. Physics is formulated using differential equations. On the university calculus a discontinuous function is declared not differentiable, so the differential equations of physics seemingly fail at a discontinuity such as a physical shock wave, whether in air or in spacetime. This “failure of the equations of physics” has no particularly catastrophic consequences with shocks in air or water but the corresponding “failure of the equations of general relativity” at a discontinuity has been interpreted as the beginning or end of the world, exactly in accord with Judeo-Christian theology! Indeed, this “failure of the equations of physics or of physics itself” is a purely metaphysical claim: the inability to differentiate a discontinuous function is *mere metaphysical preference, not a fact or eternal truth “up there”*. Using the authority of science to pass off this metaphysical preference as proof of creation in complete accord with Judeo-Christian theology is one of the biggest scientific scandals of recent times. Metaphysics, unlike science, is completely flexible: for example, the Schwartz theory of distributions (or Mikusinski’s theory etc.) allows one to differentiate a discontinuous function (or any Lebesgue integrable function) as many times as one wants.

Of course, the Schwartz theory cannot be directly used to study shocks and other discontinuities since it is linear, and the differential equations of physics are non-linear resulting in what is called the problem of products of distributions. The difficulties in defining the product, such as the failure of the “Leibniz rule” or the associative law, are the source of Riemann’s historic error with shocks: differential equations which are equivalent for smooth solutions are inequivalent for shocks. However, the actual problem of applying formal mathematics to physics is *not* that there is no definition of the product of distributions, but that there is a surfeit of metaphysics: there are some 50

12 CKR, “Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhāsā”, *Philosophy East and West*, 51:3 (2001) pp. 325–362.

13 The claim that “physics has established the truth of Judeo-Christian theology”, using Penrose-Hawking singularities, was made e.g. by F. J. Tipler (*Physics of Immortality*, Macmillan, preface) who boasts of several publications in *Nature*. For a full discussion, see CKR, *The Eleven Pictures of Time* (Sage, 2003) More recently in Cape Town, Hawking’s co-author G. F. R. Ellis fled from a public debate on this issue, and preferred instead to organize a racist lynch campaign of wild accusations against me using the racist press.

different definitions. The “choose what you like” approach of metaphysics results in bad physics, for science is required to be empirically refutable, and one cannot jump at will from one definition to another depending on subjective preference. For example, the Hahn-Banach product used in quantum renormalization theory¹⁴ fails for shock waves in fluids. The only definition of the product which does work in all cases, and serves also to handle spacetime singularities (even within formal mathematics),¹⁵ is mine obtained by using an extension of the Schwartz theory based on Non-Standard Analysis.¹⁶

Even after all this debauch of technicalities, one has to resort to the empirical, so a far simpler solution to this problem of formal university calculus is just to revert to calculus as normal mathematics, as proposed in decolonisation. A key aspect of calculus, the way it actually originated as normal mathematics in India, is non-Archimedean arithmetic (see below), and this is the only essential feature of non-standard analysis which is required for the above solution. That is, contrary to the bogey that any change in mathematics may damage science: switching back to normal mathematics demonstrably improves science, by using a version of the calculus which can handle discontinuities, without declaring them to be the end of the world (or its beginning).

But decolonisation does get rid of the dirty Western trick of smuggling superstitions into science by misusing mathematical authority. One dirty trick leads to another: recently in South Africa, I offered to debate the matter of the bad mathematics of singularity theory with Hawking’s co-author, Ellis. But formal mathematicians persistently refuse to settle the matter by open debate. Afraid that this scandalous misuse of mathematics/science would be exposed, Ellis desperately avoided the debate by resorting to the age-old trick of hurling all sorts of falsehoods through the racist press which did not permit a proper rejoinder. That the claim of “superior” formal mathematics *has* to be justified with such dirty tricks, shows it cannot be justified in an open academic debate.

Teaching normal math

As regards the millions of students who are lifelong victims of the teaching of formal mathematics, it has been demonstrated over the last decade that getting rid of the useless metaphysics of infinity in current university calculus, and teaching calculus without limits, the way it originated, as normal mathematics, makes calculus easy enough to teach in five days.¹⁷ Making calculus easier has the advantage that it enables students to solve harder practical problems not covered in usual university calculus courses. For example, the first serious science experiment with the simple pendulum, involves non-elementary elliptic integrals to reconcile theory with experiment.¹⁸ Such integrals are not covered in standard calculus courses. There is no such artificial restriction to elementary integrals in decolonised “calculus without limits” which treats them on par with elementary integrals. The course has been tried out in 10 universities, in 3 countries, and also makes calculus (hence probability and statistics) accessible to social scientists.

14 CKR, “On the Square of x^{-n} ” *J. Phys. A: Math. Gen.* **16** (1983) pp. 3739–53. Also, “Distributional Matter Tensors in Relativity.” In: *Proceedings of the Fifth Marcel Grossman Meeting on General Relativity*, Perth, 1988, ed. D. Blair and M. J. Buckingham, R. Ruffini (series ed.), World Scientific, Singapore, 1989, pp. 421–23. arxiv:0804:1998.

15 “Distributional Matter Tensors in Relativity.” cited above.

16 CKR, “Products and Compositions with the Dirac Delta Function.” *J. Phys. A: Math. Gen.* **15** (1982) pp. 381–96.

CKR, “Junction Conditions in General Relativity.” *J. Phys. A: Math. Gen.* **15** (1982) pp. 1785–97.

17 CKR, “Teaching Mathematics with a Different Philosophy. 1: Formal mathematics as biased metaphysics”. *Science and Culture* **77** (2011) pp. 275–80. <http://www.scienceandculture-isna.org/July-aug-2011/03%20C%20K%20Raju.pdf>. arxiv:1312.2099. “Teaching Mathematics with a Different Philosophy. 2: Calculus without limits”. *Science and Culture*, **77** (2011) pp. 281–86. . arxiv:1312.2100.

18 Suvrat Raju, “The pendulum project”, <http://ckraju.net/11picsoftime/pendulum.pdf>. Also, CKR, “Time: What is it That it can be Measured” *Science & Education*, **15**(6) (2006) pp. 537–551. http://ckraju.net/papers/ckr_pendu_2.pdf.

This calculus without limits course requires a preparatory school-geometry course, which discards both the foolish metaphysics of invisible points etc. and the inferior Western compass box (and various inferior axiomatic geometries such as Hilbert's synthetic geometry, and Birkhoff's metric geometry etc.), and reverts to the original teaching of empirical string geometry, to enhance practical value.

The Indian origin of calculus

Apart from shocks and singularities, to understand other concrete ways in which advanced physics itself is improved, by rejecting the metaphysics of formal mathematics, it is important to critically reconsider the calculus in more detail. Of course, first, it is necessary to dismantle the false Western myth that the calculus was invented by Newton and Leibniz. To the contrary, there is ample evidence that the calculus was invented a thousand years earlier in India to calculate precise (but not exact) trigonometric values by numerically solving differential equations. Over a thousand years, this technique developed into a method of using high order polynomial interpolations to calculate trigonometric values for sine, cosine, arctangent etc. precise to ten decimal places.¹⁹ This is now well known.

Non-Archimedean arithmetic and zeroism

What is little known is that the use of infinite series in the Indian calculus also involved a method of summing infinite series, as demonstrated in Nilakantha's first-ever formula²⁰ for the sum of *infinite* geometric series. (Finite geometric series were known for thousands of years, and are found in the eye of Horus and the Yajurveda 17.2.) This little-known Indian method of summing infinite series avoided limits by used non-Archimedean arithmetic. That arithmetic arose very naturally as part of the 7th c. Brahmagupta's algebra or *avyakt ganit*²¹ of polynomials and rational functions which naturally gives rise to non-Archimedean arithmetic. This is to be used *together* with a philosophy of discarding infinitesimals today called zeroism.²² This philosophy of zeroism is contrary to the Western superstition that mathematics, in so far as it relates to the real world of science, can ever be exact. The possibility of doing calculus with non-Archimedean arithmetic was completely unknown to arithmetically challenged Europeans, when calculus first reached Europe, and was not grasped by Newton etc. who struggled to find a valid way to sum infinite series. This possibility was vaguely understood by some mathematicians in the West only after the advent of non-standard analysis in the 1960's. Of course, unlike non-standard analysis, in which infinities and infinitesimals enters only at an intermediate stage, non-Archimedean arithmetic has permanent infinities and infinitesimals.

Transmission of calculus to Europe

Jesuits stole the Indian calculus, translated it in their Cochin college, and brought it to Europe, in the 16th c. They were motivated by the desperate European need for precise trigonometric values for the solution of the foremost scientific challenge then facing Europe: the European navigational problem. As several European governments themselves acknowledged, through huge prizes, the

19 CKR., *Cultural Foundations of Mathematics: the nature of mathematical proof and the transmission of calculus from India to Europe in the 16th c. CE*, Pearson Longman, 2007.

20 Nilakantha. *Aryabhatiyabhasya*, commentary on Ganita 22. See, *Cultural Foundations of Mathematics*, cited above.

21 CKR, "Calculus", in: *Encyclopedia of Non-Western Science, Technology and Medicine*, Springer, 2016. pp. 1010–1015. <http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-1-final.pdf>. "Calculus transmission", pp. 1016–1022. <http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-2-final.pdf>

22 CKR, "Zeroism", in: *Encyclopedia of Non-Western Science, Technology and Medicine*, Springer, 2016, pp. 4604–4610. <http://ckraju.net/papers/Springer/zeroism-springer-f.pdf>.

problem remained unsolved till 1762.²³ The need for better navigation also motivated the Gregorian reform of the primitive Julian calendar, also based on imported information.

However, on my epistemic test, a bad student copying from a good one, in an exam, does not understand what he copies. Likewise, leading European minds failed to understand the imported Indian calculus (or even the imported calendrical and astronomical knowledge). Thus, Tycho Brahe (Astronomer Royal to the Holy Roman Empire, and a natural recipient of Indian texts, after the Jesuit general Clavius) was content with adding his name to the Indian astronomical model of Nilakantha, while his assistant Kepler could only dabble unsuccessfully in infinite series. On the Western superstition since Plato that mathematics involves eternal truths,²⁴ and must hence be exact, Descartes declared it beyond the human mind to sum infinite series, and Galileo (who had access to Jesuit archives) concurred with him against his student Cavalieri. Descartes was alluding to Fermat (whose famous challenge problem to European mathematicians on “Pell’s equation” is an easy solved exercise in an Indian text from centuries earlier) and Pascal who accepted the imported Indian methods also in probability (where limits are still problematic).²⁵

Newton’s fluxions and his confusion about time

Newton too failed to understand the calculus or how to sum the infinite sine series (credit for which he claimed on the fanatical Doctrine of Christian Discovery²⁶), so his argument that Leibniz failed to understand the calculus equally applies to him. On the Thomist superstition that God wrote the eternal laws of nature in the exact language of mathematics, Newton tried to make the calculus “perfect” on his doctrine of fluxions. As the name “fluxion” shows, this involved the utterly confused belief that time itself “flows”.²⁷ Newton thought it enough to postulate that this “flow” of “absolute, true, and mathematical time” had an even tenor, “without regard to anything external”. That is, to make calculus “perfect”, Newton sought to make time metaphysical (against his mentor Barrow who rightly sought a physical definition of time).²⁸

The origin of special relativity

This wretched piece of obvious conceptual confusion (about both time and the calculus) was the real reason for the failure of Newtonian physics a century ago: Newtonian physics failed to provide a physical definition of time measurement.²⁹ This conceptual defect in Newtonian physics was corrected by Poincare who invented and developed the theory of special relativity, by providing a physical way of measuring time (which improved on Barrow), by **postulating the constancy of the speed of light**. (His deep analysis gets suppressed because of the glorification of Einstein who

23 More details in *Cultural Foundations of Mathematics*, cited above.

24 E.g. Plato, Meno. More details on all the above points in *Euclid and Jesus* and in *Cultural Foundations of Mathematics*, both cited above.

25 CKR, “Probability in Ancient India”, *Handbook of the Philosophy of Science*, vol 7, *Philosophy of Statistics*, ed. Prasanta S. Bandyopadhyay and Malcolm R. Forster. General Editors: Dov M. Gabbay, Paul Thagard and John Woods. Elsevier, 2011, pp. 1175–1196. <http://ckraju.net/papers/Probability-in-Ancient-India.pdf>.

26 CKR, “The meaning of Christian Discovery”, *Frontier Weekly* 47 (29) Jan 25 - 31, 2015. <http://www.frontierweekly.com/archive/vol-number/vol/vol-47-2014-15/47-29/47-29-The%20Meaning%20of%20Christian%20Discovery.html>. Also, video “The discovery of India”, <https://www.youtube.com/watch?v=EI4IPuA1bKw>

27 I. Newton, *The Mathematical Principles of Natural Philosophy*, A. Motte’s translation revised by Florian Cajori, Encyclopedia Britannica, Chicago, 1996, p. 8.

28 CKR, “Time: What is it That it can be Measured” *Science & Education*, 15(6) (2006) pp. 537–551. http://ckraju.net/papers/ckr_pendu_2.pdf.

29 CKR, *Time: Towards a Consistent Theory* (Kluwer Academic, Dordrecht, 1994), chp 3a “The Michelson-Morley experiment”, and chp. 3b “Einstein’s time”.

plagiarised the theory,³⁰ without properly understanding it.)

Correcting Newtonian gravitation

Einstein's error in understanding special relativity has the following effect on science. As is well known, Newton's "laws" of motion, and "law" of gravitation are not independently refutable: they come as a package deal.³¹ Therefore, when the "laws" of motion were corrected to make them compatible with special relativity, Newtonian gravitation too should have been similarly corrected (for, to measure acceleration, one must measure time). For some peculiar historical reasons this happened only recently in the Lorentz-covariant retarded gravitation theory (RGT) proposed by this author.³² Needless to say, Lorentz covariant RGT is quite different from the naive theories of retarded gravitation proposed a century ago to try to explain the perihelion advance of Mercury. Laplace's stability argument against those theories fails because it naively assumed ordinary differential equations, while RGT uses functional differential equations (FDEs)³³ as proper for a Lorentz covariant theory. (The use of FDEs is a theoretical requirement, not a hypothesis, and certainly not "Atiyah's hypothesis" as was made out in the usual Western credit grab on the centenary of Einstein's relativity paper.³⁴) The velocity-dependent component of the RGT force ensures stability.

Experimental predictions of RGT

Unlike Newtonian gravitation, a Lorentz covariant force cannot depend purely on position, but must also be velocity dependent. The velocity dependence is tiny, at the v/c level, but effects due to the rotation of the earth on nearby satellites and spacecraft are observable today. In particular, the so-called NASA flyby anomaly³⁵ for various spacecraft is an effect of earth's rotation at precisely the v/c level predicted by RGT, the only theory which also explains its other qualitative features. (It is not clear at this time whether the recently observed³⁶ deviation of 'Oumuamua from its expected trajectory could be explained as a similar effect of the sun's rotation on the interstellar asteroid.)

30 CKR, *The Eleven Pictures of Time*, Sage, 2003, chp. 5 "In Einstein's shadow".

31 CKR, *Time: Towards a Consistent Theory* (Kluwer Academic, Dordrecht, 1994), chp. 2 "Newton's time".

32 CKR, "Retarded gravitation theory" in: Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro (ed.), *Sixth International School on Field Theory and Gravitation*, American Institute of Physics, New York, 2012, pp. 260–276. http://ckraju.net/papers/retarded_gravitation_theory-rio.pdf.

33 CKR, "Functional Differential Equations. 4: Retarded gravitation", *Physics Education* (India) **31**(2) April-June, 2015, [http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-\(2\).pdf](http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-(2).pdf).

34 The claim advanced by Michael Atiyah in his lecture on the centenary of Einstein's special relativity paper was christened "Atiyah's hypothesis" in a prominent article written for the *Notices of the AMS* long AFTER Atiyah was informed of this author's book *Time: Towards a Consistent Theory*, cited above, about which he should anyway have known. This author's prior published work was acknowledged only later: *Notices of the American Mathematical Society* **54**(4) (2007) p. 472. http://ckraju.net/atiyah/Belated_mention_NoticesApril2007.pdf. The editor of the *Notices of AMS* fully approved the twisting of AMS published ethics that the works of non-Western authors need only be acknowledged by Westerners if and when their attempt to steal credit was contested. The AMS endorsed this tacit reinterpretation of its ethics for non-Westerners on grounds of editorial freedom.

35 John D. Anderson et al. Anomalous orbital energy changes observed during spacecraft flybys of earth. *Phys. Rev. Lett.*, 100:091102, 2008.

36 Marco Micheli et al. Non-gravitational acceleration in the trajectory of 1I/2017 U1 ('Oumuamua). *Nature* 27 June 2018. doi: 10.1038/s41586-018-0254-4.

RGT vs GRT

The tiny difference between RGT and Newtonian gravitation (at the level of the solar system) involves an important theoretical issue: that tiny difference is too large to be explained by general relativity theory (GRT), and especially the Kerr metric cannot account for an effect of this order³⁷ due to relativistic frame drag. Historically, Einstein rushed to claim credit for the equations of GRT from Hilbert,³⁸ in 5 days, without examining the possibility (indeed necessity) of first constructing a theory of gravitation consistent with *special* relativity. (I will not go into the subsequent historical scandal concerning Einstein and Hilbert.)

There is also the important issue of the failure of Newtonian gravitation for spiral galaxies.³⁹ Even granting the attempts to save the conceptually confused Newtonian theory by hypothesizing dark matter, and further hypothesizing its peculiar distribution, there is the question of accurate estimates of the amount of dark matter which must use the more accurate RGT theory.

The RGT theory can be easily empirically tested using a pair of counter-rotating artificial satellites.⁴⁰ These efforts are on.

37 Anderson et al. cited above.

38 Jagdish Mehra, *Einstein, Hilbert and the theory of gravitation*, D. Reidel, 1974.

39 FDEs-4, cited above.

40 . CKR, *Functional Differential Equations: a new mathematical paradigm in physics* (to appear). Appendix: The two-satellite experiment