Calculus: the real story

C. K. Raju

Centre for Studies in Civilisations
New Delhi
web:ckraju.net
Roman arithmetic

Q. Can you write 1788 in Roman numerals?
Q. Can you write 1788 in Roman numerals?
A. MDCCLXXXVIII.
Roman arithmetic

Q. Can you write 1788 in Roman numerals?

A. MDCCLXXXVIII.

Roman and early Greek way of representing numerals is inferior: requires 12 symbols instead of 4.
Roman arithmetic

- Q. Can you write 1788 in Roman numerals?
- A. MDCCLXXXVIII.
- Roman and early Greek way of representing numerals is inferior: requires 12 symbols instead of 4.
- Challenge: can you write $10^{53}$ in Roman numerals?
Indian place value system

Large numbers

- Indian arithmetic used superior decimal place value system.
Indian place value system
Large numbers

- Indian arithmetic used superior decimal place value system.
- Names of numbers up to $10^{12}$ found e.g. in Yajurveda 17.2. (Similar to names in current use.)
Indian place value system

Large numbers

- Indian arithmetic used superior decimal place value system.
- Names of numbers up to $10^{12}$ found e.g. in Yajurveda 17.2. (Similar to names in current use.)
- Names of 53 places ($10^{53} = tallakshana$) in Buddhist Lalitavistara sutta.
Indian arithmetic used superior decimal place value system.

Names of numbers up to $10^{12}$ found e.g. in Yajurveda 17.2. (Similar to names in current use.)

Names of 53 places ($10^{53} =$ tallakshana) in Buddhist Lalitavistara sutta.

Numbers such as $10^{29}$ in Jaina literature.
हुमा मैठ्छ्रमुशिःष्टाकाघेनावः सुन्त्रेकोऽछु दशं छु दशं छु शुरुषु
शुरुषु सूहस्वेतृङ्गुत्र्युत्र्युत्युत्युत्युत्कर्म नियुत्युत्कर्म नियुत्त्वकर्म प्रयुत्त्वकर्मार्नुर्दश्च
न्युर्दश्च समुद्रश्च मध्यश्चान्तेश्च परार्थाश्चता मैठ्छ्रमुशिःष्टाकाघेनावः
सत्यमुनुत्रामुशिःष्टालोके २
Shukla Yajurveda 17.2

एका च दश च दश च शतं च शतं च सहस्त्रं च सहस्त्रं च अन्युतं च अन्युतं च नियुतं च नियुतं च प्रयुतं च अन्तुदं च न्यंतुदं च समुदः च मध्यं च अन्त्यः च परार्थः च
Efficient arithmetic

- Place value system led to efficient arithmetic,
Efficient arithmetic

- Place value system led to efficient arithmetic,
- as in the methods of addition, subtraction, multiplication, and division which you learn in elementary school.
Efficient arithmetic

- Place value system led to efficient arithmetic,
- as in the methods of addition, subtraction, multiplication, and division which you learn in elementary school.
- These methods originated in India called *pātigaṇita* (slate arithmetic) by Brahmagupta (7th c.) (*Brāhmasphuṭasiddhānta*),
Efficient arithmetic

- Place value system led to efficient arithmetic,
- as in the methods of addition, subtraction, multiplication, and division which you learn in elementary school.
- These methods originated in India called pātigaṇita (slate arithmetic) by Brahmagupta (7th c.) (Brāhmasphuṭasiddhānta),
- followed by Śridhar (Pātigaṇita), Mahavira (Gaṇitasārasangraha), Bhaskara (Lilāvatī).
Transmission to Europe via Arabs

- Indian methods today called “algorithms”,

Calculus: the real story
C. K. Raju
Transmission to Europe
via Arabs

- Indian methods today called “algorithms”,
Transmission to Europe via Arabs

- Indian methods today called “algorithms”,
- His Latin name was Algorismus.
Transmission to Europe via Arabs

- Indian methods today called “algorithms”,
- His Latin name was Algorismus.
- Why did Arabs and Europeans import Indian arithmetic?
Roman arithmetic-2

- Can you multiply XXIX and LXXVIII?
Roman arithmetic-2

- Can you multiply XXIX and LXXVIII?
- (Note: Not allowed to convert to decimals perform the multiplication, and reconvert. Do it the way Romans did, for they did not know the decimal place value system.)
Can you multiply XXIX and LXXVIII?
(Note: Not allowed to convert to decimals perform the multiplication, and reconvert. Do it the way Romans did, for they did not know the decimal place value system.)
Romans did multiplication by repeated addition.
Roman arithmetic-2

- Can you **multiply** XXIX and LXXVIII?
- (Note: **Not allowed** to convert to decimals perform the multiplication, and reconvert. Do it the way Romans did, for they did not know the decimal place value system.)
- Romans did multiplication by repeated addition.
- Simpler question: How do you **add** XXIX and LXXVIII?
The Roman abacus

Addition

- Lacking place value, Roman arithmetic used an abacus.
Lacking place value, Roman arithmetic used an abacus.

Step 1: write out XXIX in its full form: XXVIII.
The Roman abacus

Addition

- Lacking place value, Roman arithmetic used an abacus.
- Step 1: write out XXIX in its full form: XXVIII.
- Step 2: Use counters (coins) for each of L, X, V, I.
The Roman abacus

Addition

- Lacking place value, Roman arithmetic used an abacus.
- Step 1: write out XXIX in its full form: XXVIII.
- Step 2: Use counters (coins) for each of L, X, V, I.
- Step 3. Pool together all counters used for XXIX and LXXVIII.
The Roman abacus

Addition

- Lacking place value, Roman arithmetic used an abacus.
- Step 1: write out XXIX in its full form: XXVIII.
- Step 2: Use counters (coins) for each of L, X, V, I.
- Step 3: Pool together all counters used for XXIX and LXXVIII.
- Step 4: Simplify 7 I’s = 1 V and 2 I’s. 3 V’s = 1 X and 1 V. 5 X’s = 1 L, 2 L’s = 1 C.
The Roman abacus
Addition

▶ Lacking place value, Roman arithmetic used an abacus.
▶ Step 1: write out XXIX in its full form: XXVIII.
▶ Step 2: Use counters (coins) for each of L, X, V, I.
▶ Step 3. Pool together all counters used for XXIX and LXXVIII.
▶ Step 4: Simplify 7 I’s = 1 V and 2 I’s. 3 V’s = 1 X and 1 V. 5 X’s = 1 L, 2 L’s = 1 C.
▶ Answer: So, sum is CVII.
The Roman abacus

Addition

- Lacking place value, Roman arithmetic used an abacus.
- Step 1: write out XXIX in its full form: XXVIII.
- Step 2: Use counters (coins) for each of L, X, V, I.
- Step 3. Pool together all counters used for XXIX and LXXVIII.
- Step 4: Simplify 7 I’s = 1 V and 2 I’s. 3 V’s = 1 X and 1 V. 5 X’s = 1 L, 2 L’s = 1 C.
- Answer: So, sum is CVII.
- Whew!
Roman arithmetic-3

- To multiply XXIX with LXXVIII you must add LXXVIII to itself 29 times by repeating the previous steps 29 times.
To multiply XXIX with LXXVIII you must add LXXVIII to itself **29** times by repeating the previous steps 29 times.

Hence, Greek/Roman arithmetic with abacus was excessively inefficient and inferior.
To multiply XXIX with LXXVIII you must add LXXVIII to itself 29 times by repeating the previous steps 29 times.

Hence, Greek/Roman arithmetic with abacus was excessively inefficient and inferior.

This inefficiency not a trivial matter.
To multiply XXIX with LXXVIII you must add LXXVIII to itself 29 times by repeating the previous steps 29 times.

Hence, Greek/Roman arithmetic with abacus was exceedingly inefficient and inferior.

This inefficiency not a trivial matter.

Without efficient techniques of calculation how could the Greeks do science?
Roman arithmetic-3

- To multiply XXIX with LXXVIII you must add LXXVIII to itself 29 times by repeating the previous steps 29 times.
- Hence, Greek/Roman arithmetic with abacus was excessively inefficient and inferior.
- This inefficiency not a trivial matter.
- Without efficient techniques of calculation how could the Greeks do science?
- Hence, inefficiency of Graeco-Roman arithmetic is compelling non-textual evidence against wild claims of Greek scientific achievement.
Roman arithmetic-3

- To multiply XXIX with LXXVIII you must add LXXVIII to itself 29 times by repeating the previous steps 29 times.
- Hence, Greek/Roman arithmetic with abacus was excessively inefficient and inferior.
- This inefficiency not a trivial matter.
- Without efficient techniques of calculation how could the Greeks do science?
- Hence, inefficiency of Graeco-Roman arithmetic is compelling non-textual evidence against wild claims of Greek scientific achievement
- by Crusading and racist historians based on very late texts (in another language from another place).
Ending false history

- Recall that false history was used to justify racist domination and exploitation (e.g. Kant)
Ending false history

- Recall that false history was used to justify racist domination and exploitation (e.g. Kant)
- and used to impose colonial domination and exploitation (Macaulay).
Ending false history

- Recall that false history was used to **justify racist domination and exploitation** (e.g. Kant)
- and used to impose **colonial domination and exploitation** (Macaulay).
- (Not true that history written by victors. This false history written by military losers fetched a huge empire.)
Ending false history

- Recall that false history was used to justify racist domination and exploitation (e.g. Kant)
- and used to impose colonial domination and exploitation (Macaulay).
- (Not true that history written by victors. This false history written by military losers fetched a huge empire.)
- Exposing falsehoods of Western history important to end racist and colonial inequity today.
Transmission of Arithmetic from India to Europe

- Because Graeco-Roman arithmetic was inferior
Transmission of Arithmetic
from India to Europe

- Because Graeco-Roman arithmetic was inferior
- therefore efficient Indian arithmetic was imported by West
Transmission of Arithmetic
from India to Europe

- Because Graeco-Roman arithmetic was inferior
- therefore efficient Indian arithmetic was imported by West
- first via Arabs in Cordoba (10th c.)
Transmission of Arithmetic
from India to Europe

- Because Graeco-Roman arithmetic was inferior
- therefore efficient Indian arithmetic was imported by West
- first via Arabs in Cordoba (10th c.)
- then via Florentine merchants (13th. c.)
But arithmetically challenged Westerners failed to understand the imported methods of arithmetic, for centuries.
Failure to understand imported knowledge

But arithmetically challenged Westerners failed to understand the imported methods of arithmetic, for centuries.

This story suppressed by Western historians,
Failure to understand imported knowledge

- But arithmetically challenged Westerners failed to understand the imported methods of arithmetic, for centuries.
- This story suppressed by Western historians,
- but the story emerges from the very terms “Arabic numerals”, and “zero”.

Calculus: the real story
C. K. Raju
The pope’s mistake

- First import of Indian arithmetic from Cordoba in 10th c. by Gerbert (later pope Sylvester II).
The pope’s mistake

- First import of Indian arithmetic from Cordoba in 10th c. by Gerbert (later pope Sylvester II).
- The pope was a learned man who wrote a book on the abacus.
The pope’s mistake

- First import of Indian arithmetic from Cordoba in 10th c. by Gerbert (later pope Sylvester II).
- The pope was a learned man who wrote a book on the abacus.
- He thought the only way to do arithmetic was by using an abacus.
The pope’s mistake

- First import of Indian arithmetic from Cordoba in 10th c. by Gerbert (later pope Sylvester II).
- The pope was a learned man who wrote a book on the abacus.
- He thought the only way to do arithmetic was by using an abacus.
- So, he thought the efficiency of Indian arithmetic was due to some magic in the shape of the numerals which he inscribed on the back of counters.
The pope’s mistake

- First import of Indian arithmetic from Cordoba in 10th c. by Gerbert (later pope Sylvester II).
- The pope was a learned man who wrote a book on the abacus.
- He thought the only way to do arithmetic was by using an abacus.
- So, he thought the efficiency of Indian arithmetic was due to some magic in the shape of the numerals which he inscribed on the back of counters.
- And continued to use an abacus: the infallible pope got a special abacus constructed for “Arabic numerals”!
The pope’s mistake

- First import of Indian arithmetic from Cordoba in 10th c. by Gerbert (later pope Sylvester II).
- The pope was a learned man who wrote a book on the abacus.
- He thought the only way to do arithmetic was by using an abacus.
- So, he thought the efficiency of Indian arithmetic was due to some magic in the shape of the numerals which he inscribed on the back of counters.
- And continued to use an abacus: the infallible pope got a special abacus constructed for “Arabic numerals”!
- (That mistake immortalised in the term “Arabic numerals”.)
European misunderstanding of Indian arithmetic
The mystery of zero

- The word “zero” comes from sifr or cypher (meaning mysterious code).
European misunderstanding of Indian arithmetic

The mystery of zero

- The word “zero” comes from sifr or cypher (meaning mysterious code).
- What is mysterious about zero?
European misunderstanding of Indian arithmetic

The mystery of zero

- The word “zero” comes from sifr or cypher (meaning mysterious code).
- What is mysterious about zero?
- Roman numerals are additive, like counters: XII means $X + I + I = 12$. 
European misunderstanding of Indian arithmetic

The mystery of zero

- The word “zero” comes from sifr or cypher (meaning mysterious code).
- What is mysterious about zero?
- Roman numerals are additive, like counters: XII means $X + I + I = 12$.
- Place value numerals are not additive: $120 \neq 1 + 2 + 0 = 3$. 
Europeans hence complained about this mysterious entity, 0, which has no value in itself, but adds any amount of value to the preceding number.
The mysterious zero contd

- Europeans hence complained about this mysterious entity, 0, which has no value in itself, but adds any amount of value to the preceding number.
- A contract for $120 could easily be changed to $1200.
The mysterious zero

contd

- Europeans hence complained about this mysterious entity, 0, which has no value in itself, but adds any amount of value to the preceding number.
- A contract for $120 could easily be changed to $1200.
- Hence, the Florentine law that contracts (cheques) must be written out in words.
Algorismus vs abacus

- Indian techniques of arithmetic introduced into the Jesuit syllabus ca. 1572.
Algorismus vs abacus

- Indian techniques of arithmetic introduced into the Jesuit syllabus ca. 1572.
- Thus confusion about Indian arithmetic persisted for six centuries.
Arithmetic: summary
Origin, transmission, misunderstanding

- India did not contribute only “zero”.
Arithmetic: summary
Origin, transmission, misunderstanding

- India did not contribute only “zero”.
- It contributed efficient arithmetic,
Arithmetic: summary
Origin, transmission, misunderstanding

- India did not contribute only “zero”.
- It contributed efficient arithmetic,
- which was hence transmitted to Europe, and replaced inefficient Graeco-Roman arithmetic.
Arithmetic: summary
Origin, transmission, misunderstanding

- India did not contribute only “zero”.
- It contributed efficient arithmetic,
- which was hence transmitted to Europe, and replaced inefficient Graeco-Roman arithmetic.
- Europeans misunderstood that arithmetic in various ways, and that confusion persisted for over six centuries.
Transmission via Toledo

- The story of trigonometry is similar.
Transmission via Toledo

- The story of trigonometry is similar.
- Trigonometry originated in India, travelled to Arabs.
Transmission via Toledo

- The story of trigonometry is similar.
- Trigonometry originated in India, travelled to Arabs.
- Was imported to West via Toledo translations of Arabic texts ca. 1125.
Transmission via Toledo

- The story of trigonometry is similar.
- Trigonometry originated in India, travelled to Arabs.
- Was imported to West via Toledo translations of Arabic texts ca. 1125.
- The word sine derives from the Arabic \textit{jaib} (as in जेब meaning pocket).
What has a pocket to do with sines?
A pocketful of sines

- What has a pocket to do with sines?
- The term *jaib* was a misreading of *jiba*, written as just the consonantal skeleton *jb* (without *nuktas*, which were not common then).
A pocketful of sines

- What has a pocket to do with sines?
- The term *jaib* was a misreading of *jiba*, written as just the consonantal skeleton *jb* (without *nuktas*, which were not common then).
- *jiba* was from the Sanskrit *jya* or *jiva*, meaning chord.
Conceptual misunderstanding

- Amartya Sen has started telling this story today, hiding the fact that
Conceptual misunderstanding

- Amartya Sen has started telling this story today, *hiding the fact* that
- as usual transmission of knowledge was accompanied by misunderstanding.
Conceptual misunderstanding

- Amartya Sen has started telling this story today, hiding the fact that
- as usual transmission of knowledge was accompanied by misunderstanding.
- (Raju’s epistemic test: those who copy usually make mistakes.)
Conceptual misunderstanding

- Amartya Sen has started telling this story today, hiding the fact that
- as usual transmission of knowledge was accompanied by misunderstanding.
- (Raju’s epistemic test: those who copy usually make mistakes.)
- The word “trigonometry” indicates a conceptual misunderstanding: the concepts relate to a circle (chord, jya) not a triangle.
Conceptual misunderstanding

- Amartya Sen has started telling this story today, hiding the fact that
- as usual transmission of knowledge was accompanied by misunderstanding.
- (Raju’s epistemic test: those who copy usually make mistakes.)
- The word “trigonometry” indicates a conceptual misunderstanding: the concepts relate to a circle (chord, jya) not a triangle.
- Correct term is circlometry. These concepts discussed in chapter on the circle in early Indian texts.
Measuring curved lines
Western confusion persists in school math

- Difference is non-trivial: even to do basic geometry we need to measure curved lines, not just straight lines.
Measuring curved lines

Western confusion persists in school math

- Difference is non-trivial: even to do basic geometry we need to measure curved lines, not just straight lines.
- even to define an angle of 1°.
Measuring curved lines
Western confusion persists in school math

- Difference is non-trivial: even to do basic geometry we need to measure curved lines, not just straight lines.
- Even to define an angle of 1°.
- Measuring curved lines impossible with the instruments in a school geometry-box today.
Measuring curved lines
Western confusion persists in school math

- Difference is non-trivial: even to do basic geometry we need to measure curved lines, not just straight lines.
- Even to define an angle of $1^\circ$.
- Measuring curved lines impossible with the instruments in a school geometry-box today.
- It is not explained how to construct a protractor: how do you divide the circumference of a semi-circle into 180 equal parts without a conceptual understanding of when those curved segments are equal?
Measuring curved lines

Western confusion persists in school math

- Difference is non-trivial: even to do basic geometry we need to measure curved lines, not just straight lines.
- Even to define an angle of $1^\circ$.
- Measuring curved lines impossible with the instruments in a school geometry-box today.
- It is not explained how to construct a protractor: how do you divide the circumference of a semi-circle into 180 equal parts without a conceptual understanding of when those curved segments are equal?
- Challenge: Try giving an axiomatic definition of an angle of $1^\circ$. 
Measuring curved lines
Western confusion persists in school math

- Difference is non-trivial: even to do basic geometry we need to measure curved lines, not just straight lines.
- Even to define an angle of 1°.
- Measuring curved lines impossible with the instruments in a school geometry-box today.
- It is not explained how to construct a protractor: how do you divide the circumference of a semi-circle into 180 equal parts without a conceptual understanding of when those curved segments are equal?
- Challenge: Try giving an axiomatic definition of an angle of 1°.
- Again, does the size of a protractor matter?
Measuring curved lines

Western confusion persists in school math

- Difference is non-trivial: even to do basic geometry we need to measure curved lines, not just straight lines.
- Even to define an angle of $1^\circ$.
- Measuring curved lines impossible with the instruments in a school geometry-box today.
- It is not explained how to construct a protractor: how do you divide the circumference of a semi-circle into 180 equal parts without a conceptual understanding of when those curved segments are equal?
- Challenge: Try giving an axiomatic definition of an angle of $1^\circ$.
- Again, does the size of a protractor matter?
- Why not? (Brings in essential properties of the circle.)
\textit{\textbf{\textit{\textit{\textit{\textbf{\textit{sulba sutra-s}}}}}}}

- **Superior** conceptual understanding available in Indian math since the \textit{sulba sutra-s}. 
śulba sūtra-s

- Superior conceptual understanding available in Indian math since the śulba sūtra-s.
- Use a flexible string (śulba)
Śulba sūtra-s

- Superior conceptual understanding available in Indian math since the śulba sūtra-s.
- Use a flexible string (śulba)
- to measure a curved arc empirically.
śulba sūtra-s

- Superior conceptual understanding available in Indian math since the śulba sūtra-s.
- Use a flexible string (śulba)
- to measure a curved arc empirically.
- Calculated value of $\pi$. This knowledge called non-eternal (सानित्य, Apastamba 3.12) and imperfect (सविशेषः, Katyayana, 2.12)
Superior conceptual understanding available in Indian math since the śulba sūtra-s. Use a flexible string (śulba) to measure a curved arc empirically. Calculated value of $\pi$. This knowledge called non-eternal (सानित्य, Apastamba 3.12) and imperfect (सविशेषः, Katyayana, 2.12) Just an absurd Western religious delusion that non-empirical (metaphysical) knowledge is somehow superior, perfect, eternal, and infallible.
Āryabhaṭa’s value of $\pi$

- Stated in Gaṇita 10
Āryabhaṭa’s value of π

- Stated in Gaṇita 10

- चतुराधिकं शतमष्टगुरं द्वाषष्टिस्तथा सहस्त्राशाम ।
  छसयुतद्वाय विशक्षभस्यासच्छो ब्रितपरिशाहः ॥ १०॥
āryabhaṭa’s value of $\pi$

- Stated in Gaṇita 10
- चतुराधिकं शतमष्टगुर्णं द्वाषष्टिस्तथा सहस्त्राशाम।
  अयुतद्वृय विश्कम्भस्यासन्नो ब्रित्तपरिशाह: ॥ १० ॥
- 100 plus 4 multiplied by 8, and added to 62,000: this is the near [asanna] measure of the circumference of a circle whose diameter is 20,000.
Āryabhaṭa’s value of $\pi$

- Stated in Gaṇīta 10

- चतुराधिकं शतमष्टगुरुं द्वाषष्टिस्तथा सहस्त्राशाम ।
  अयुतद्वय विशकम्भस्यासन्नो ब्रित्तपरिशाह ॥ १० ॥

- 100 plus 4 multiplied by 8, and added to 62,000: this is the near [asanna] measure of the circumference of a circle whose diameter is 20,000.

- Note: stating the length of a curved line (circumference).
Āryabhaṭa’s value of $\pi$

- Stated in Gaṇita 10
- चतुराधिकं शतमष्टगुरां द्वाषष्टिस्तथा सहस्त्राणां ।
  अश्रयुद्वय विश्कम्भस्यासन्नो ब्रित्तपरिशाहः ॥ १० ॥
- 100 plus 4 multiplied by 8, and added to 62,000: this is the near [asanna] measure of the circumference of a circle whose diameter is 20,000.
- Note: stating the length of a curved line (circumference).
- This value of $\pi$ repeated, a thousand years later, in 16th c. Europe by Simon Stevin.
Trigonometry again

- Sam Pitroda, Chairman of Knowledge commission told me, do trigonometry the MIT MOOC way.
Trigonometry again

- Sam Pitroda, Chairman of Knowledge commission told me, do trigonometry the MIT MOOC way.
- My position: something not knowledge unless one checks and cross-checks oneself.
Trigonometry again

► Sam Pitroda, Chairman of Knowledge commission told me, do trigonometry the MIT MOOC way.
► My position: something not knowledge unless one checks and cross-checks oneself.
► Using triangles (“geometry”) to do “trigonometry” very limited. (No way even to calculate $\pi$.)
Trigonometry again

- Sam Pitroda, Chairman of Knowledge commission told me, do trigonometry the MIT MOOC way.
- My position: something not knowledge unless one checks and cross-checks oneself.
- Using triangles (“geometry”) to do “trigonometry” very limited. (No way even to calculate $\pi$.)
- What is $\sin 1^\circ$? (My son’s question in 8th std.)
Trigonometry again

- Sam Pitroda, Chairman of Knowledge commission told me, do trigonometry the MIT MOOC way.
- My position: something not knowledge unless one checks and cross-checks oneself.
- Using triangles ("geometry") to do "trigonometry" very limited. (No way even to calculate $\pi$.)
- What is $\sin 1^\circ$? (My son’s question in 8th std.)
- Geometry useful only in cases of high symmetry (six values $15^\circ$, $30^\circ$, $45^\circ$, etc. taught in school).
Trigonometry again

- Sam Pitroda, Chairman of Knowledge commission told me, do trigonometry the MIT MOOC way.
- My position: something not knowledge unless one checks and cross-checks oneself.
- Using triangles (“geometry”) to do “trigonometry” very limited. (No way even to calculate $\pi$.)
- What is $\sin 1^\circ$? (My son’s question in 8th std.)
- Geometry useful only in cases of high symmetry (six values $15^\circ$, $30^\circ$, $45^\circ$, etc. taught in school).
- Numerical methods still needed to calculate $\sin 1^\circ$. 

Trigonometry again

- Sam Pitroda, Chairman of Knowledge commission told me, do trigonometry the MIT MOOC way.
- My position: something not knowledge unless one checks and cross-checks oneself.
- Using triangles ("geometry") to do "trigonometry" very limited. (No way even to calculate $\pi$.)
- What is $\sin 1^\circ$? (My son’s question in 8th std.)
- Geometry useful only in cases of high symmetry (six values $15^\circ$, $30^\circ$, $45^\circ$, etc. taught in school).
- Numerical methods still needed to calculate $\sin 1^\circ$.
- Hence, my software CALCODE.
Āryabhaṭa’s method
Difference/differential equations

- Āryabhaṭa’s striking shift to difference equations
Āryabhaṭa’s method
Difference/differential equations

- Āryabhaṭa’s striking shift to difference equations
- (only metaphysically different from differential equations).
Āryabhaṭa’s method

Difference/differential equations

- Āryabhaṭa’s striking shift to difference equations
- (only metaphysically different from differential equations).
- He solved them using only linear interpolation or the rule of 3.
Āryabhaṭa’s method
Difference/differential equations

- Āryabhaṭa’s striking shift to difference equations
- (only metaphysically different from differential equations).
- He solved them using only linear interpolation or the rule of 3.
- (Today wrongly called “Euler’s method” of solving ordinary differential equations, after Euler who, like other Westerners, never acknowledged his Indian (non-Christian) sources.)
Important thing is that Āryabhaṭa in Gaṇita 10 Āryabhaṭa stated sine differences, later called khanḍa-jyā
Sine differences, not sine values

- Important thing is that Āryabhaṭa in Gaṇita 10 stated sine differences, later called khanda-jyā.
- Differences 3.75° apart.
Sine differences, not sine values

- Important thing is that Āryabhaṭa in Gaṇīta 10 stated sine differences, later called khanḍa-jyā.
- Differences 3.75° apart.
- Differences can be directly used to interpolate.
Sine differences, not sine values

- Important thing is that Āryabhaṭa in Gaṇīta 10 Āryabhaṭa stated sine *differences*, later called *khanda-jyā*
- Differences 3.75° apart.
- Differences can be directly used to interpolate
- or calculate values at end points of intervals.
Āryabhaṭa’s table of sine differences

मक्खि भक्षि फखि धखि शखि अखि
डखि हस्स्व स्ककि किष्ण श्वकि कित्व
घ्वलकि किम्ब्र हक्क धकर किच
स्म श्रम ड्र कल प्त फ़ छ कलार्ध्या॥ १२॥
Āryabhaṭa’s numerical notation

- Numerical notation explained in Gītikā 2.
Āryabhaṭa’s numerical notation

- Numerical notation explained in Gītikā 2.
- *varga* (classified) letters in the *varga* (odd) places.
  (Thus, they have the values 1–25 in alphabetical order.)
Āryabhaṭa’s numerical notation

- Numerical notation explained in Gītikā 2.
- *varga* (classified) letters in the *varga* (odd) places. (Thus, they have the values 1–25 in alphabetical order.)
- *avarga* (unclassified) letters in the *avarga* (even) places. (They have values 30, 40, 50, 60, 70, 80, 90, 100, respectively.)
Āryabhaṭa’s numerical notation

- Numerical notation explained in Gītikā 2.
- *varga* (classified) letters in the *varga* (odd) places. (Thus, they have the values 1–25 in alphabetical order.)
- *avarga* (unclassified) letters in the *avarga* (even) places. (They have values 30, 40, 50, 60, 70, 80, 90, 100, respectively.)
- The nine vowels अ, इ, उ, ऊ, ल, ए, ओ, ए, औ denote the two nines of zeros (corresponding to the 18 places from $10^0$ to $10^{17}$): each vowel takes one *varga* and one *avarga* place.
Āryabhaṭa’s numerical notation

- Numerical notation explained in Gītikā 2.
- varga (classified) letters in the varga (odd) places. (Thus, they have the values 1–25 in alphabetical order.)
- avarga (unclassified) letters in the avarga (even) places. (They have values 30, 40, 50, 60, 70, 80, 90, 100, respectively.)
- The nine vowels अ, इ, उ, ऊ, ए, ओ, ए, ओ, ओ denote the two nines of zeros (corresponding to the 18 places from $10^0$ to $10^{17}$): each vowel takes one varga and one avarga place.
- Thus अ denotes the place of 1 as well as 10, इ denotes the place of 100 as well as 1000, etc.
Āryabhaṭa’s numerical notation (contd.)

- A consonant combined with a vowel denotes a number.
Āryabhaṭa’s numerical notation (contd.)

- A consonant combined with a vowel denotes a number.
- When the vowel is combined with an avara letter, it has a value 10 times what it has when combined with a varga letter.
Āryabhaṭa’s numerical notation (contd.)

- A consonant combined with a vowel denotes a number.
- When the vowel is combined with an *avarga* letter, it has a value 10 times what it has when combined with a *varga* letter.
- System is very compact and order-independent.
Āryabhaṭa’s numerical notation (contd.)

- A consonant combined with a vowel denotes a number.
- When the vowel is combined with an avara letter, it has a value 10 times what it has when combined with a vara letter.
- System is very compact and order-independent.
- E.g., ख्युघू = 4,320,000, since ख्र = 2, घ्र = 30, so that ख्यु = 320,000, while घ्र = 4, so that घू = 4,000,000.
Aryabhaṭa’s numerical notation (contd.)

- A consonant combined with a vowel denotes a number.
- When the vowel is combined with an *avarga* letter, it has a value 10 times what it has when combined with a *varga* letter.
- System is very compact and order-independent.
- E.g., ḵṛṣṇू = 4,320,000, since ḵṛ = 2, ṝṛ = 30, so that ḵṛṣṇ = 320,000, while ḷṛ = 4, so that ḷṛṣṇ = 4,000,000.
- It is also order-independent: could write above as ḷṛṛṣṇ.
Translation

- 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7—[these are the] Rsine [differences] [for the quadrant divided into as many equal parts, each part hence being 225′] [in] minutes.
Translation

- 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7—[these are the] Rsine [differences] [for the quadrant divided into as many equal parts, each part hence being 225′] [in] minutes.

- (Circumference of the circle in minutes is $360 \times 60 = 21,600$.)
Difference equation
not algebraic equation

▶ Āryabhaṭa’s method of calculating sine differences (Gaṇita 12)
Difference equation
not algebraic equation

- Āryabhaṭa’s method of calculating sine differences (Gaṇita 12)

Translation: (12) The Rsine of the first arc divided by itself and negated gives the second Rsine difference. That same first Rsine, when it divides successive Rsines gives the remaining [Rsine differences].

प्रथमाच्चापज्याठ्ठाक्रमे रसिन ख्रिखतं द्वितियार्थम्।
तत्प्रथमज्याठ्ठाशेषस्तैसर्नानि शेषार्थि॥ १२॥
Difference equation
not algebraic equation

- Āryabhaṭa’s method of calculating sine differences (Gaṇita 12)

Translation: (12) The Rsine of the first arc divided by itself and negated gives the second Rsine difference. That same first Rsine, when it divides successive Rsines gives the remaining [Rsine differences].
Mathematical translation

$R_i =$ sine values, $\delta_i = R_i - R_{i-1}$ sine differences. Then Āryabhaṭa’s rule consists of two parts

$$\delta_2 - \delta_1 = -\frac{R_1}{R_1}, \quad (1)$$

$$\delta_{n+1} - \delta_n = -\frac{R_n}{R_1}. \quad (2)$$
Mathematical translation

- \( R_i \) = sine values, \( \delta_i = R_i - R_{i-1} \) sine differences. Then Āryabhaṭa’s rule consists of two parts

\[
\delta_2 - \delta_1 = -\frac{R_1}{R_1},
\]

(1)

\[
\delta_{n+1} - \delta_n = -\frac{R_n}{R_1}.
\]

(2)

- Note 1: Second differences have been brought in.
Mathematical translation

- \( R_i = \) sine values, \( \delta_i = R_i - R_{i-1} \) sine differences.
  Then Āryabhaṭa’s rule consists of two parts

\[
\delta_2 - \delta_1 = -\frac{R_1}{R_1}, \quad (1)
\]

\[
\delta_{n+1} - \delta_n = -\frac{R_n}{R_1}. \quad (2)
\]

- Note 1: Second differences have been brought in.
- Note 2: Brahmagupta also uses 2nd differences for quadratic interpolation.
Āryabhaṭa’s method not an algebraic equation

- Gaṇita 12 cannot be used as an algebraic equation for the purpose of calculating sine differences.
Āryabhaṭa’s method not an algebraic equation

- Gaṇita 12 cannot be used as an algebraic equation for the purpose of calculating sine differences.
- $\delta_n - \delta_{n+1}$ can be calculated from $R_n$ using (2);
Āryabhaṭa’s method not an algebraic equation

- Gaṇita 12 cannot be used as an algebraic equation for the purpose of calculating sine differences.
- $\delta_n - \delta_{n+1}$ can be calculated from $R_n$ using (2);
- however, if we try to calculate $R_n$ by multiplying 2 by $R_1$ to obtain $R_1 \times (\delta_n - \delta_{n+1})$, that would result in incorrect values concerned.
Āryabhaṭa’s method not an algebraic equation

- Gaṇita 12 cannot be used as an algebraic equation for the purpose of calculating sine differences.
- \( \delta_n - \delta_{n+1} \) can be calculated from \( R_n \) using (2);
- however, if we try to calculate \( R_n \) by multiplying 2 by \( R_1 \) to obtain \( R_1 \times (\delta_n - \delta_{n+1}) \), that would result in incorrect values concerned.
- E.g. for \( n = 23 \), \( \delta_{23} = 22 \), \( \delta_{24} = 7 \), while \( R_1 = 225 \), so that we should have \( R_{23} = (\delta_{23} - \delta_{24}) \times R_1 = 15 \times 225 = 3735 \neq 3431 \) the 23rd sine value.
Āryabhaṭa’s method not an algebraic equation

- Gaṇita 12 cannot be used as an algebraic equation for the purpose of calculating sine differences.
- $\delta_n - \delta_{n+1}$ can be calculated from $R_n$ using (2);
- however, if we try to calculate $R_n$ by multiplying 2 by $R_1$ to obtain $R_1 \times (\delta_n - \delta_{n+1})$, that would result in incorrect values concerned.
- E.g. for $n = 23$, $\delta_{23} = 22$, $\delta_{24} = 7$, while $R_1 = 225$, so that we should have $R_{23} = (\delta_{23} - \delta_{24}) \times R_1 = 15 \times 225 = 3735 \neq 3431$ the 23rd sine value.
- Difference in each case, since no value is a multiple of 225.
Non-terminating processes

- Āryabhaṭa’s process is **non-terminating**
Non-terminating processes

- Āryabhaṭa’s process is non-terminating
- like his algorithm for extracting $\sqrt{2}$. 
Non-terminating processes

- Āryabhaṭa’s process is non-terminating
- like his algorithm for extracting $\sqrt{2}$.
- His sine values (precise to the minute) were made precise to the seconds (Vaṭeśvar, 10th c.)
Non-terminating processes

- Āryabhaṭa’s process is non-terminating like his algorithm for extracting $\sqrt{2}$.
- His sine values (precise to the minute) were made precise to the seconds (Vaṭeṣvar, 10th c.)
- then to the thirds (attempted, Govindasvamin, 9th c., achieved Madhava, 14th c.)
Non-terminating processes

- Āryabhaṭa’s process is non-terminating
- like his algorithm for extracting $\sqrt{2}$.
- His sine values (precise to the minute) were made precise to the seconds (Vaṭeṣvar, 10th c.)
- then to the thirds (attempted, Govindasvamin, 9th c., achieved Madhava, 14th c.)
- by extending Āryabhaṭa’s recursive process to infinite series.
Increasing precision

- Gradual progress clear from gradually increasing precision in the value of $\pi$. 
Increasing precision

- Gradual progress clear from gradually increasing precision in the value of \( \pi \).
- 7th c. Bhāskara I’s figure is 3438’,
Increasing precision

- Gradual progress clear from gradually increasing precision in the value of $\pi$.
- 7th c. Bhāskara I’s figure is 3438′,
- 9th c. Vaṭeśvara’s figure is 3437′ 44″.
Increasing precision

- Gradual progress clear from gradually increasing precision in the value of $\pi$.
- 7th c. Bhāskara I’s figure is 3438′.
- 9th c. Vaṭeśvara’s figure is 3437′ 44″.
- Later stated as Devo viśvasthalī bhṛguḥ, corresponding (in reverse order) to 34374448 or 3437′ 44″ 48‴.
Madhava’s value of $\pi$

- Value of $\pi$ also stated in older *bhūta saṃkhyā* system in Nīlakaṇṭha’s *ĀryabhaṭīyaBhaṣya*

Value of $\pi$ contd.

- Corresponds to value of $\pi = 3.141, 592, 653, 5922 \ldots$, accurate to 11 decimal places with the 12th and 13th places (92 respectively) differing slightly from their accurate value (89).
Value of $\pi$ contd.

- Corresponds to value of $\pi = 3.141, 592, 653, 5922 \ldots$,
- accurate to 11 decimal places
Value of $\pi$ contd.

- Corresponds to value of $\pi = 3.141, 592, 653, 5922 \ldots$,
- accurate to 11 decimal places
- with the 12th and 13th places (92 respectively) differing slightly from their accurate value (89).
Madhava’s sine table

श्रेष्ठं नाम वरिष्ठानां हिमाद्रिवेदः भावनः।
तपनो भानुसूक्तेऽ मध्यमं विद्धि दोहनम्॥
धिगाज्यो नाशनं कष्टं छन्नभोगाशयाम्बिका।
प्रिगाहारो नरेशोड्यं वीरो रशाजयोत्सुकः॥
...
छायालयो गजो नीलो निर्मलो नास्ति सत्कुले।
रात्रि दर्पशामभ्राकं नगस्तुल्लको बली॥
धीरो युवा कथालोलः पूज्यो नारीजनेभंगः।
कन्यागारे नागवल्ली देवो विश्वस्थली भृगुः॥
तत्परादिकलान्तास्तु महाज्या माधवोदिताः।
स्वस्वपुर्वेविशुद्धे तु शिष्टास्तत्स्थराडमौरिकः॥ २.९.४॥
# Madhava’s sine table

<table>
<thead>
<tr>
<th>No.</th>
<th>Kaṭapayāḍī</th>
<th>kalā (')</th>
<th>vikalā(’’')</th>
<th>tatparā(’’’')</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Śr̥ṣṭāḥ nāma vṛṣṭānāṃ</td>
<td>224</td>
<td>50</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>hīmaḍiṛiḍavēdabhāvanā</td>
<td>448</td>
<td>42</td>
<td>58</td>
</tr>
<tr>
<td>3</td>
<td>tāpno bānusūkṛkṣno</td>
<td>670</td>
<td>40</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>mādhymaṃ viddhi dohānam्</td>
<td>889</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>21</td>
<td>dhiīre yuva kathālōlaṃ</td>
<td>3371</td>
<td>41</td>
<td>29</td>
</tr>
<tr>
<td>22</td>
<td>pūjyaḥ naarijṇaṁbhāgaḥ</td>
<td>3408</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>23</td>
<td>kānḍyāgāre naṅgavallī</td>
<td>3430</td>
<td>23</td>
<td>11</td>
</tr>
<tr>
<td>24</td>
<td>devaḥ vīśvasthaliḥ bhūgaḥ</td>
<td>3437</td>
<td>44</td>
<td>48</td>
</tr>
</tbody>
</table>
### Accuracy of Madhava’s sine values

<table>
<thead>
<tr>
<th>No.</th>
<th>Mādhava’s sine value</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0654031452</td>
<td>0.0000000160</td>
</tr>
<tr>
<td>2</td>
<td>0.1305262297</td>
<td>0.0000000375</td>
</tr>
<tr>
<td>3</td>
<td>0.1950903240</td>
<td>0.0000000020</td>
</tr>
<tr>
<td>4</td>
<td>0.2588190035</td>
<td>-0.0000000416</td>
</tr>
<tr>
<td>21</td>
<td>0.9807852980</td>
<td>0.0000000176</td>
</tr>
<tr>
<td>22</td>
<td>0.9914448967</td>
<td>0.0000000353</td>
</tr>
<tr>
<td>23</td>
<td>0.9978589819</td>
<td>0.0000000587</td>
</tr>
<tr>
<td>24</td>
<td>1.0000000000</td>
<td>0.0000000000</td>
</tr>
</tbody>
</table>
Why so much precision?

- Precise sine values were needed for astronomical models.
Why so much precision?

- Precise sine values were needed for astronomical models
- Needed for the **two key means of wealth in India**

(Note: Gregorian calendar is a bad, unscientific, religious calendar which continues to ruin our economic interests to this day: see video and presentation: "A tale of two calendars").
Why so much precision?

- Precise sine values were needed for astronomical models
- needed for the **two key means of wealth in India**
- **overseas trade** (needs good navigation)
Why so much precision?

- Precise sine values were needed for astronomical models
- needed for the two key means of wealth in India
- overseas trade (needs good navigation)
- agriculture (needs a good calendar to tell the rainy season).
Why so much precision?

- Precise sine values were needed for astronomical models.
- Needed for the **two key means of wealth in India**.
- **Overseas trade** (needs good navigation).
- **Agriculture** (needs a good calendar to tell the rainy season).

(Note: Gregorian calendar is a bad, unscientific, religious calendar which continues to ruin our economic interests to this day: see video and presentation: “A tale of two calendars”.)
Indian methods of navigation

- Precise sine and arctangent values are needed
Indian methods of navigation

- Precise sine and arctangent values are needed
- to determine local latitude
Indian methods of navigation

- Precise sine and arctangent values are needed
- to determine local latitude
  - from the shadow of a gnomon (Laghu Bhaskariya III.2–3),
  - or the solar altitude at noon (Laghu Bhaskariya III.22-23) (also needs a good calendar).
  - Also needed to determine the size of the earth, and to solve longitude triangles from knowledge of latitude difference and departures (Maha Bhaskariya, II.3–4), (of Bhaskara I, 7th c.)
Indian methods of navigation

- Precise sine and arctangent values are needed
- to determine local latitude
  - from the shadow of a gnomon (Laghu Bhaskariya III.2–3),
  - or the solar altitude at noon (Laghu Bhaskariya III.22-23) (also needs a good calendar).
Indian methods of navigation

- Precise sine and arctangent values are needed
- to determine local latitude
  - from the shadow of a gnomon (Laghu Bhaskariya III.2–3),
  - or the solar altitude at noon (Laghu Bhaskariya III.22-23) (also needs a good calendar).
- Also needed to to determine the size of the earth, and
Indian methods of navigation

- Precise sine and arctangent values are needed
- to determine local latitude
  - from the shadow of a gnomon (Laghu Bhaskariya III.2–3),
  - or the solar altitude at noon (Laghu Bhaskariya III.22-23) (also needs a good calendar).
- Also needed to to determine the size of the earth, and
- use that to solve longitude triangles from knowledge of latitude difference and departures (Maha Bhaskariya, II.3–4), (of Bhaskara I, 7th c.)
Blunders of Columbus and Vasco

- Europeans then were poor and hoped to generate wealth by overseas trade.
Blunders of Columbus and Vasco

- Europeans then were poor and hoped to generate wealth by overseas trade.
- But **Europeans were navigationally challenged** then.
Blunders of Columbus and Vasco

- Europeans then were poor and hoped to generate wealth by overseas trade.
- But Europeans were navigationally challenged then.
- Columbus mistook Cuba for China (and then said his instrument was broken! It had only one moving part: a plumb line!)
Blunders of Columbus and Vasco

- Europeans then were poor and hoped to generate wealth by overseas trade.
- But Europeans were navigationally challenged then.
- Columbus mistook Cuba for China (and then said his instrument was broken! It had only one moving part: a plumb line!)
- Vasco da Gama hired an Indian navigator to bring him from Melinde in Africa to Calicut in India,
Blunders of Columbus and Vasco

- Europeans then were poor and hoped to generate wealth by overseas trade.
- But Europeans were navigationally challenged then.
- Columbus mistook Cuba for China (and then said his instrument was broken! It had only one moving part: a plumb line!)
- Vasco da Gama hired an Indian navigator to bring him from Melinde in Africa to Calicut in India,
- Recorded that the pilot was telling the distance by his teeth!
How calculus was transmitted

- Jesuits set up a college in Cochin to translate Indian texts and send them back to Europe
How calculus was transmitted

- Jesuits set up a college in Cochin to translate Indian texts and send them back to Europe
- on the Toledo model of mass translations using local Syrian Christians as intermediaries.
How calculus was transmitted

- Jesuits set up a college in Cochin to translate Indian texts and send them back to Europe
- on the Toledo model of mass translations using local Syrian Christians as intermediaries.
- This import solved the **latitude** and **loxodrome** problems which required precise sine values,
How calculus was transmitted

- Jesuits set up a college in Cochin to translate Indian texts and send them back to Europe
- on the Toledo model of mass translations using local Syrian Christians as intermediaries.
- This import solved the \textit{latitude} and \textit{loxodrome} problems which required precise sine values,
- also used Indian length of tropical year for Gregorian calendar reform of 1582 (not based on fresh observation, not immediately accepted by Protestants)
In 1581 Ricci was in Cochin and wrote that he was looking for “an honorable Moor or an intelligent Brahmin to tell him about Indian methods of timekeeping”. 
European blunders

Longitude problem

- However European navigational problem not fully solved for a peculiar reason.
European blunders

Longitude problem

- However European navigational problem not fully solved for a peculiar reason.
- Columbus underestimated the size of the earth by 40%.
European blunders

Longitude problem

- However European navigational problem not fully solved for a peculiar reason.
- Columbus underestimated the size of the earth by 40%.
- Clavius published Madhava’s sine table (to 10 decimal place precision) in 1608,
European blunders

Longitude problem

- However European navigational problem not fully solved for a peculiar reason.
- Columbus underestimated the size of the earth by 40%.
- Clavius published Madhava’s sine table (to 10 decimal place precision) in 1608,
- but did not know enough “trigonometry” to determine the size of the earth. (Any child can do it.)
European blunders

Longitude problem

- However European navigational problem not fully solved for a peculiar reason.
- Columbus underestimated the size of the earth by 40%.
- Clavius published Madhava’s sine table (to 10 decimal place precision) in 1608,
- but did not know enough “trigonometry” to determine the size of the earth. (Any child can do it.)
- Hence, Europe could not solve the longitude problem this way (used inaccurate “Dead reckoning”).
Calculus: the real story

C. K. Raju

Arithmetic
Trigonometry
Calculus
Development of calculus
Values of π
Calculus transmission
Infinite series
Contemporary issues
Pedagogy
Conclusions
European navigational problem

Due to this lack of understanding, European navigational problem persisted until late 18th c.
European navigational problem

- Due to this lack of understanding, European navigational problem persisted until late 18th c.
- with various European governments offering huge prizes for its solution
European navigational problem

- Due to this lack of understanding, European navigational problem persisted until late 18th c.
- with various European governments offering huge prizes for its solution
- the last being the British prize offered by an act in 1711.
European navigational problem

- Due to this lack of understanding, European navigational problem persisted until late 18th c.
- with various European governments offering huge prizes for its solution
- the last being the British prize offered by an act in 1711.
- Royal Society, French Royal Academy were set up for this purpose.
Precise sine values and values of $\pi$ were calculated using infinite series.
Infinite series

- Precise sine values and values of $\pi$ were calculated using infinite series.
- Europeans failed to understand Indian method of summing infinite series.
Infinite series

- Precise sine values and values of $\pi$ were calculated using infinite series.
- Europeans failed to understand Indian method of summing infinite series.
- Finite geometric series known in India since Yajurveda, as already shown.
Infinite series

- Precise sine values and values of $\pi$ were calculated using infinite series.
- Europeans failed to understand Indian method of summing infinite series.
- Finite geometric series known in India since Yajurveda, as already shown.
- Infinite geometric series had appeared in India by the 14th-15th c.
Infinite geometric series

- Sum of infinite (anantya) geometric series stated by Nīlakanṭha (in Āryabhaṭīyabhāṣya, Gaṇita 17).
Infinite geometric series

- Sum of infinite (anantya) geometric series stated by Nīlakanṭha (in Āryabhaṭīyabhāṣya, Gaṇīta 17).

- एवं यस्तुल्यच्छेदपरभागपरम्पराया अनन्ताया अपि संयोगः
  तस्यानन्तानाममपि कल्पमानस्य योगस्यादावयविनः
  परम्परांश्च्छेदादेकोनच्छेदांशास्मायं सर्वत्रापि समानमेव।

(Assuming \(d > 1\), so common ratio less than 1.)
Infinite geometric series

- Sum of infinite \((anantya)\) geometric series stated by Nīlakanṭha (in Āryabhaṭīyabhāṣya, Gaṇita 17).

- \textbf{The sum of an infinite [anantya] series, whose later terms (after the first) are got by dividing the preceding one by the same divisor everywhere, is equal to the first term \([a]\) multiplied by the common divisor \([d]\), and divided by one less than the common divisor.}

\[
\sum \frac{a}{d^n} = \frac{a}{d} \frac{1}{d-1} \quad (\text{Assuming } d > 1, \text{ so common ratio less than 1.})
\]
Infinite geometric series

▶ Sum of infinite (anantya) geometric series stated by Nīlakanṭha (in Āryabhaṭīyabhāṣya, Gaṇita 17).

▶ एवं यस्तुल्यच्छेदपरभागपरम्पराया अनन्ताया अर्थे संयोगः
तस्यानन्तानामपि कल्पनामान्य योगस्यावाक्यविनः
परम्परांशच्छेदादेकोनच्छेदांशास्मायं सर्वत्रापि समानमेव।

▶ The sum of an infinite [anantya] series, whose later terms (after the first) are got by dividing the preceding one by the same divisor everywhere, is equal to the first term [a] multiplied by the common divisor [d], and divided by one less than the common divisor.

▶ \[ a + \frac{a}{d} + \frac{a}{d^2} + \cdots = \frac{ad}{d-1}. \]
(Assuming \( d > 1 \), so common ratio less than 1.)
Infinite series for $\pi$

- The number today called $\pi$ (≡ ratio of circumference to diameter) requires an infinite series.
Infinite series for $\pi$

- The number today called $\pi$ (equal ratio of circumference to diameter) requires an infinite series.
- One such infinite series wrongly credited to Leibniz found in 16th c. *Yuktidīpikā* 2.271.

\[\text{व्यासे वारिधिनिहते रूपहते व्याससागराभिहते।} \]
\[\text{त्रिश्रादिविषमसंख्याभक्तमृगां स्वं प्रिथक् क्रमात् कुर्यात्॥ २.२७१॥} \]
Infinite series for $\pi$

- The number today called $\pi$ (ratio of circumference to diameter) requires an infinite series.
- One such infinite series wrongly credited to Leibniz found in 16th c. *Yuktidīpikā* 2.271.

यासे वारिधिनि हते रुपहते यासागराभिहते।
त्रिशरादिविषसंव्याभक्तमूर्गां स्वं प्रिथक्क्रमात् कुर्यात्॥ २.२७१॥
Infinite series for $\pi$

- The number today called $\pi$ (= ratio of circumference to diameter) requires an infinite series.
- One such infinite series wrongly credited to Leibniz found in 16th c. *Yuktidīpikā* 2.271.

> व्यासे वारिधिनिहते रुपहते व्याससागराभिहते।
> त्रिशरादिविषमसंव्याभक्तमृगां स्वं प्रिथक् क्रमात् कुर्यात्॥ २.२७१॥

- Translation: To the diameter multiplied by 4 alternately add and subtract in order the diameter multiplied by 4 and divided separately by the odd numbers 3, 5, etc.
“Leibniz” series contd.

- Mathematical translation: if $d$ is the diameter of the circle, then

$$\text{circumference} = 4d - \frac{4d}{3} + \frac{4d}{5} - \frac{4d}{7} + \cdots \ (3)$$
“Leibniz” series

contd.

- Mathematical translation: if $d$ is the diameter of the circle, then

  \[
  \text{circumference} = 4d - \frac{4d}{3} + \frac{4d}{5} - \frac{4d}{7} + \cdots. \quad (3)
  \]

- This corresponds to the value of π given by

  \[
  \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots. \quad (4)
  \]
“Leibniz” series contd.

- Mathematical translation: if \( d \) is the diameter of the circle, then

\[
\text{circumference} = 4d - \frac{4d}{3} + \frac{4d}{5} - \frac{4d}{7} + \cdots. \quad (3)
\]

- This corresponds to the value of \( \pi \) given by

\[
\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots. \quad (4)
\]

- What baffled all Western thinkers (Descartes, Galileo, Newton, Berkeley . . . ) was this: how to do this infinite sum “perfectly”.
Some more blunders

- E.g. Descartes (Geometry Book 2, p. 544): “ratios of curved and straight lines are beyond the human mind”.

Traditional Indian math syllabus, since Sulba Sutras, taught children to measure a curved line with a flexible string and straighten it to compare with a straight line. (Sulba Sutras also gives value of π.)

Western geometry box has no instrument with which to measure a curved line.

Post-colonial Indians blindly accepted the Western practice of working with straight lines as superior without a critical evaluation.

See my article: “Towards equity in math education. 2: The Indian rope trick” (google, CKR, rope trick)
Some more blunders

- E.g. Descartes (Geometry Book 2, p. 544): “ratios of curved and straight lines are beyond the human mind”.

- Traditional Indian math syllabus, since śulba sūtra, taught children to measure a curved line with a flexible string and straighten it to compare with a straight line. (śulba sūtra also gives value of π.)
Some more blunders

- E.g. Descartes (Geometry Book 2, p. 544): “ratios of curved and straight lines are beyond the human mind”.

- Traditional Indian math syllabus, since śulba sūtra, taught children to measure a curved line with a flexible string and straighten it to compare with a straight line. (śulba sūtra also gives value of π.)

- Western geometry box has no instrument with which to measure a curved line.
Some more blunders

- E.g. Descartes (Geometry Book 2, p. 544): “ratios of curved and straight lines are beyond the human mind”.
- Traditional Indian math syllabus, since śulba sūtra, taught children to measure a curved line with a flexible string and straighten it to compare with a straight line. (śulba sūtra also gives value of π.)
- Western geometry box has no instrument with which to measure a curved line.
- Post-colonial Indians blindly accepted the Western practice of working with straight lines as superior without a critical evaluation.
Some more blunders

- E.g. Descartes (Geometry Book 2, p. 544): “ratios of curved and straight lines are beyond the human mind”.

- Traditional Indian math syllabus, since śulba sūtra, taught children to measure a curved line with a flexible string and straighten it to compare with a straight line. (śulba sūtra also gives value of π.)

- Western geometry box has no instrument with which to measure a curved line.

- Post-colonial Indians blindly accepted the Western practice of working with straight lines as superior without a critical evaluation.

- See my article: “Towards equity in math education. 2: The Indian rope trick” (google, CKR, rope trick)
Charitable interpretation of Descartes’ difficulty
The perils of perfection

- For most practical purposes, we can still use Āryabhaṭa’s value $\pi = 3.14159$. 
Charitable interpretation of Descartes’ difficulty
The perils of perfection

- For most practical purposes, we can still use Āryabhaṭa’s value $\pi = 3.14159$.
- Or take accuracy to 100 or 1000 or billion decimal places.
Charitable interpretation of Descartes’ difficulty
The perils of perfection

- For most practical purposes, we can still use Āryabhaṭa’s value $\pi = 3.14159$.
- Or take accuracy to 100 or 1000 or billion decimal places.
- What Descartes’ wanted was the perfect sum of the infinite series, which leaves nothing out.
Charitable interpretation of Descartes’ difficulty
The perils of perfection

- For most practical purposes, we can still use Āryabhaṭa’s value $\pi = 3.14159$.
- Or take accuracy to 100 or 1000 or billion decimal places.
- What Descartes’ wanted was the perfect sum of the infinite series, which leaves nothing out.
- He thought for this we must physically sum the series term by term, which would take an infinite amount of time.
Eternal truth and perfection in math
A mere religious belief

- But why is math perfect and eternal knowledge?
Eternal truth and perfection in math
A mere religious belief

- But why is math perfect and eternal knowledge?
- why not practically useful knowledge which is imperfect and non-eternal as śulba sūtra-s state?
Eternal truth and perfection in math
A mere religious belief

- But why is math perfect and eternal knowledge?
- why not practically useful knowledge which is imperfect and non-eternal as śulba sūtra-s state?
- Plato said in Meno that mathematics is part of the soul’s innate knowledge, and that proves the existence of the soul.
Eternal truth and perfection in math
A mere religious belief

► But why is math perfect and eternal knowledge?
► why not practically useful knowledge which is imperfect and non-eternal as śulba sūtra-s state?
► Plato said in Meno that mathematics is part of the soul’s innate knowledge, and that proves the existence of the soul.
► Proclus commented that mathematics derives from mathesis (meaning the soul recollecting its knowledge of past lives) and leads to the blessed life.
Eternal truth and perfection in math

A mere religious belief

- But why is math perfect and eternal knowledge?
- why not practically useful knowledge which is imperfect and non-eternal as śulba sūtra-s state?
- Plato said in *Meno* that mathematics is part of the soul’s innate knowledge, and that proves the existence of the soul.
- Proclus commented that mathematics derives from mathesis (meaning the soul recollecting its knowledge of past lives) and leads to the blessed life.
- He argued that math arouses the soul by sympathetic magic just because math has eternal truths.
Eternal truth and perfection in math
A mere religious belief

- But why is math perfect and eternal knowledge?
- why not practically useful knowledge which is imperfect and non-eternal as śulba sūtra-s state?
- Plato said in *Meno* that mathematics is part of the soul’s innate knowledge, and that proves the existence of the soul.
- Proclus commented that mathematics derives from mathesis (meaning the soul recollecting its knowledge of past lives) and leads to the blessed life.
- He argued that math arouses the soul by sympathetic magic just because math has eternal truths.
- These are religious beliefs which are not universal or compelling.
More religious beliefs
or add on superstitions

- Aquinas said *(Summa Theologica, First part of the second part, 91,1)* that God rules the world with eternal laws
More religious beliefs
or add on superstitions

- Aquinas said (*Summa Theologica*, First part of the second part, 91,1) that God rules the world with eternal laws
- (hence the common belief that those laws are written in the language of mathematics which contains eternal truths).

▶ This belief in eternal laws is just a Western superstition contrary to our repeated mundane experience that we create a bit of the future cosmos at each instant.

▶ (See, e.g., KL-paper on “Islam and science”, google).
More religious beliefs
or add on superstitions

- Aquinas said (\textit{Summa Theologica}, First part of the second part, 91,1) that God rules the world with eternal laws
- (hence the common belief that those laws are written in the language of mathematics which contains eternal truths).
- This belief in eternal laws is just a Western superstition contrary to our repeated mundane experience that we create a bit of the future cosmos at each instant.
More religious beliefs
or add on superstitions

- Aquinas said (*Summa Theologica*, First part of the second part, 91,1) that God rules the world with eternal laws
- (hence the common belief that those laws are written in the language of mathematics which contains eternal truths).
- This belief in eternal laws is just a Western superstition contrary to our repeated mundane experience that we create a bit of the future cosmos at each instant.
- (See, e.g., KL-paper on “Islam and science”, google).
Infallibility of deduction
Just another religious belief

- Like the West believed the pope to be infallible,
Infallibility of deduction

Just another religious belief

- Like the West believed the pope to be infallible,
- it also believes deductive proof to be infallible.
Infallibility of deduction

Just another religious belief

- Like the West believed the pope to be infallible,
- it also believes deductive proof to be infallible.
- Aquinas bowdlerized al Ghazali, saying that logic bound God.
Infallibility of deduction
Just another religious belief

- Like the West believed the pope to be infallible,
- it also believes deductive proof to be infallible.
- Aquinas bowdlerized al Ghazali, saying that logic bound God.
- Just another religious belief because Buddhists, for example, don’t accept 2-valued logic.
Infallibility of deduction
Just another religious belief

- Like the West believed the pope to be infallible,
- it also believes deductive proof to be infallible.
- Aquinas bowdlerized al Ghazali, saying that logic bound God.
- Just another religious belief because Buddhists, for example, don’t accept 2-valued logic.
- Deciding logic culturally means truths of math are mere cultural truths.
Infallibility of deduction
Just another religious belief

- Like the West believed the pope to be infallible,
- it also believes deductive proof to be infallible.
- Aquinas bowdlerized al Ghazali, saying that logic bound God.
- Just another religious belief because Buddhists, for example, don’t accept 2-valued logic.
- Deciding logic culturally means truths of math are mere cultural truths.
- And if nature of logic itself is decided by experience then deductive proofs are more fallible than inductive proofs.
Newton

- However, Newton accepted all those Western religious beliefs about math.
Newton

- However, Newton accepted all those Western religious beliefs about math.
- He was right that Leibniz was a confused “second inventor” of calculus, but so was he.
Newton

- However, Newton accepted all those Western religious beliefs about math.
- He was right that Leibniz was a confused “second inventor” of calculus, but so was he.
- Newton thought \( \frac{d}{dt} \) could be “perfectly” defined as “fluxion”
Newton

- However, Newton accepted all those Western religious beliefs about math.
- He was right that Leibniz was a confused “second inventor” of calculus, but so was he.
- Newton thought $\frac{d}{dt}$ could be “perfectly” defined as “fluxion”
- if time (“absolute”, “true”, and “mathematical”, or the time known to his God) flowed on!
Newton

- However, Newton accepted all those Western religious beliefs about math.
- He was right that Leibniz was a confused “second inventor” of calculus, but so was he.
- Newton thought $\frac{d}{dt}$ could be “perfectly” defined as “fluxion”
- if time (“absolute”, “true”, and “mathematical”, or the time known to his God) flowed on!
- Newtonian physics failed because he made the notion of time metaphysical.
Time: Towards a Consistent Theory

by

C. K. Raju

Kluwer Academic Publishers
Does calculus require metaphysics?

- Newton’s fluxions abandoned today as totally confused.
Does calculus require metaphysics?

- Newton’s fluxions abandoned today as totally confused.
- (How can time itself flow? e.g. Śriharṣa and his Western copycat (McTaggart’s paradox)).
Does calculus require metaphysics?

- Newton’s fluxions abandoned today as totally confused.
- (How can time itself flow? e.g. Śriharṣa and his Western copycat (McTaggart’s paradox)).
- However, belief persists that calculus requires metaphysical “real” numbers \( \mathbb{R} \) for existence of limits.
Does calculus require metaphysics?

- Newton’s fluxions abandoned today as totally confused.
- (How can time itself flow? e.g. Śriharṣa and his Western copycat (McTaggart’s paradox)).
- However, belief persists that calculus requires metaphysical “real” numbers $\mathbb{R}$ for existence of limits.
- That belief is taught in school and undergraduate calculus courses today (without actually teaching about $\mathbb{R}$ except to math majors!).
Most practical applications of the calculus, such as sending a rocket to Mars
Most practical applications of the calculus, such as sending a rocket to Mars

still done the Āryabhaṭa way by numerically solving differential equations
Most practical applications of the calculus, such as sending a rocket to Mars
still done the Āryabhaṭa way by numerically solving differential equations
on computers which use floating point numbers (computers cannot use $\mathbb{R}$)
Most practical applications of the calculus, such as sending a rocket to Mars
still done the Āryabhaṭa way by numerically solving differential equations
on computers which use floating point numbers (computers cannot use $\mathbb{R}$)
which have a different arithmetic (no associative “law”, etc.)
Calculus: the real story

C. K. Raju

Arithmetic

Trigonometry

Calculus

Development of calculus

Values of $\pi$

Calculus transmission

Infinite series

Contemporary issues

Pedagogy

Conclusions

R not adequate

- Calculus with limits is limited.
Calculus with limits is limited.
Discontinuous functions cannot be differentiated.
Not adequate

- Calculus with limits is limited.
- Discontinuous functions cannot be differentiated.
- But need to do so arises in physics and engineering (since Heaviside, or before the formalization of $\mathbb{R}$).
\textbf{R} not adequate

- Calculus with limits is limited.
- Discontinuous functions cannot be differentiated.
- But need to do so arises in physics and engineering (since Heaviside, or before the formalization of \( \mathbb{R} \)).
- This is typically done today using Schwartz distributions.
Calculus with limits is limited.

Discontinuous functions cannot be differentiated.

But need to do so arises in physics and engineering (since Heaviside, or before the formalization of \( \mathbb{R} \)).

This is typically done today using Schwartz distributions.

Not good enough for physics since products of distributions arise because differential equations of physics are non-linear.
R not adequate

- Calculus with limits is limited.
- Discontinuous functions cannot be differentiated.
- But need to do so arises in physics and engineering (since Heaviside, or before the formalization of \( \mathbb{R} \)).
- This is typically done today using Schwartz distributions.
- Not good enough for physics since products of distributions arise because differential equations of physics are non-linear.
- Similarly issues in renormalization problem of quantum field theory.
Divergent series

- In simplified form, problems correspond to the need to sum divergent series such as

\[
\sum_{n=1}^{\infty} (-1)^{n+1} = \frac{1}{2}
\]

(Raju sum).

- (Corresponds to \( \theta \cdot \delta = \frac{1}{2} \delta \), where \( \theta \) is the Heaviside function, and \( \delta \) is the Dirac delta.)

- \[1 + 2 + 3 + 4 + \cdots?\]
Divergent series

- In simplified form, problems correspond to the need to sum divergent series such as
- \(1 - 1 + 1 - 1 + \cdots = \frac{1}{2}\) (Raju sum).
Divergent series

- In simplified form, problems correspond to the need to sum divergent series such as
- \[ 1 - 1 + 1 - 1 + \cdots = \frac{1}{2} \text{ (Raju sum)}. \]
- (Corresponds to \( \theta \cdot \delta = \frac{1}{2} \delta \), where \( \theta \) is the Heaviside function, and \( \delta \) is the Dirac delta.)
Divergent series

- In simplified form, problems correspond to the need to sum divergent series such as
- \(1 - 1 + 1 - 1 + \cdots = \frac{1}{2}\) (Raju sum).
- (Corresponds to \(\theta \cdot \delta = \frac{1}{2}\delta\), where \(\theta\) is the Heaviside function, and \(\delta\) is the Dirac delta.)
- \(1 + 2 + 3 + 4 + \cdots = \)
Divergent series

- In simplified form, problems correspond to the need to sum divergent series such as
- \( 1 - 1 + 1 - 1 + \cdots = \frac{1}{2} \) (Raju sum).
- (Corresponds to \( \theta \cdot \delta = \frac{1}{2} \delta \), where \( \theta \) is the Heaviside function, and \( \delta \) is the Dirac delta.)
- \( 1 + 2 + 3 + 4 + \cdots = -\frac{1}{12} \) (Ramanujam sum).
Non-”Archimedean” arithmetic

These problems may be partly handled using Non-Standard Analysis and empirical inputs.
Non-”Archimedean” arithmetic

- These problems may be partly handled using Non-Standard Analysis and empirical inputs.
- However, the key feature needed is only non-Archimedean arithmetic (nothing to do with Archimedes)
Non-”Archimedean” arithmetic

- These problems may be partly handled using Non-Standard Analysis and empirical inputs.
- However, the key feature needed is only non-Archimedean arithmetic (nothing to do with Archimedes)
- involving infinities and infinitesimals.
Non-”Archimedean” arithmetic

- These problems may be partly handled using Non-Standard Analysis and empirical inputs.
- However, the key feature needed is only non-Archimedean arithmetic (nothing to do with Archimedes)
- involving infinities and infinitesimals.
- Infinitesimals must be discarded
Non-”Archimedean” arithmetic

- These problems may be partly handled using Non-Standard Analysis and empirical inputs.
- However, the key feature needed is only non-Archimedean arithmetic (nothing to do with Archimedes)
- involving infinities and infinitesimals.
- Infinitesimals must be discarded
- corresponds to limits by order counting (when formal limits exist).
Non-”Archimedean” arithmetic

- These problems may be partly handled using Non-Standard Analysis and empirical inputs.
- However, the key feature needed is only non-Archimedean arithmetic (nothing to do with Archimedes)
- involving infinities and infinitesimals.
- Infinitesimals must be discarded
- corresponds to limits by order counting (when formal limits exist).
- Needs zeroism not formalism.
Indian non-Archimedean arithmetic

- Brahmagupta used the term *avyakta* (unexpressed number) for polynomials
Indian non-Archimedean arithmetic

- Brahmagupta used the term *avyakta* (unexpressed number) for polynomials
- such as $3x + 2$ which acquire a value only when $x$ is specified.
Indian non-Archimedean arithmetic

- Brahmagupta used the term *avyakta* (unexpressed number) for polynomials
- such as $3x + 2$ which acquire a value only when $x$ is specified.
- Ratios of such numbers are *avyakta* fractions ($\frac{3x+2}{4x+3}$)
Indian non-Archimedean arithmetic

- Brahmagupta used the term *avyakta* (unexpressed number) for polynomials such as \(3x + 2\) which acquire a value only when \(x\) is specified.
- Ratios of such numbers are *avyakta* fractions \(\frac{3x+2}{4x+3}\).
- These are today called rational functions, and constitute a non-Archimedean ordered field.
Brahmagupta used the term *avyakta* (unexpressed number) for polynomials such as $3x + 2$ which acquire a value only when $x$ is specified. Ratios of such numbers are *avyakta* fractions ($\frac{3x+2}{4x+3}$). These are today called rational functions, and constitute a non-Archimedean ordered field. Discarding of infinitesimals equivalent to limits by order counting used by Nīlakanṭha to sum geometric series or...
Indian non-Archimedean arithmetic

- Brahmagupta used the term *avyakta* (unexpressed number) for polynomials
  - such as $3x + 2$ which acquire a value only when $x$ is specified.
- Ratios of such numbers are *avyakta* fractions ($\frac{3x+2}{4x+3}$)
- These are today called rational functions, and constitute a non-Archimedean ordered field.
- Discarding of infinitesimals equivalent to limits by order counting used by Nīlakanṭha to sum geometric series or in *Yuktidīipikā* to accelerate convergence of slowly convergent series, like “Leibniz” series.
Contemporary changes

- Fully correcting Newton’s error leads to (a) a paradigm shift in physics (functional differential equations)
Contemporary changes

- Fully correcting Newton’s error leads to (a) a paradigm shift in physics (functional differential equations)
- and (b) a new theory of gravitation called “Retarded gravitation theory”
Contemporary changes

- Fully correcting Newton’s error leads to (a) a paradigm shift in physics (functional differential equations)
- and (b) a new theory of gravitation called “Retarded gravitation theory”
- and (c) a modified electrodynamics.
Contemporary changes

- Fully correcting Newton’s error leads to (a) a paradigm shift in physics (functional differential equations)
- and (b) a new theory of gravitation called “Retarded gravitation theory”
- and (c) a modified electrodynamics.
- (See the series of expository articles in *Physics Education*, India)
Math pedagogy

- Many people find math difficult today.
Math pedagogy

- Many people find math difficult today.
- On the principle that phylogeny is ontogeny
Math pedagogy

- Many people find math difficult today.
- On the principle that phylogeny is ontogeny
- these difficulties in the classroom are a replay of European difficulties with imported Indian math.
Math pedagogy

- Many people find math difficult today.
- On the principle that phylogeny is ontogeny
- these difficulties in the classroom are a replay of European difficulties with imported Indian math.
- Solution is to go back to the way that math developed in India, and reject the way it was misunderstood in Europe.
5-day course on calculus

- Teaching calculus the way it developed in India makes it very easy
5-day course on calculus

- Teaching calculus the way it developed in India makes it very easy
- Substance of fat book on calculus (and more)
5-day course on calculus

- Teaching calculus the way it developed in India makes it very easy
- Substance of fat book on calculus (and more)
- can be taught in a mere five days as I have demonstrated with 8 groups in 5 universities in 3 countries.
Calculus: the real story
C. K. Raju

Arithmetic
Trigonometry
Calculus
Development of calculus
Values of π
Calculus transmission
Infinite series
Contemporary issues
Pedagogy
Conclusions
Calculus: the real story

C. K. Raju

Arithmetic
Trigonometry
Calculus

Development of calculus
Values of π

Calculus transmission
Infinite series
Contemporary issues
Pedagogy

Conclusions
The big picture

1. Much of basic school math developed in India and was transmitted to Europe:
1. Much of basic school math developed in India and was transmitted to Europe:
   - that includes arithmetic,
1. Much of basic school math developed in India and was transmitted to Europe:
   ▶ that includes arithmetic, trigonometry,
1. Much of basic school math developed in India and was transmitted to Europe:
   - that includes arithmetic, trigonometry, calculus,
The big picture

1. Much of basic school math developed in India and was transmitted to Europe:
   - that includes arithmetic, trigonometry, calculus, and probability.
The big picture

1. Much of basic school math developed in India and was transmitted to Europe:
   - that includes arithmetic, trigonometry, calculus, and probability.

2. This math was misunderstood in Europe.
The big picture

- 1. Much of basic school math developed in India and was transmitted to Europe:
  - that includes arithmetic, trigonometry, calculus, and probability.

- 2. This math was misunderstood in Europe.

- 3. That inferior understanding was given back as “superior” and globalised during colonialism,
The big picture

1. Much of basic school math developed in India and was transmitted to Europe:
   - that includes arithmetic, trigonometry, calculus, and probability.

2. This math was misunderstood in Europe.

3. That inferior understanding was given back as “superior” and globalised during colonialism,
   - and is still taught today.
Notorious claims of “superiority”

- The West is notorious for its claims of superiority:
Notorious claims of “superiority”

- The West is notorious for its claims of superiority:
- claim of colonial “superiority” was preceded by
Notorious claims of “superiority”

- The West is notorious for its claims of superiority:
  - claim of colonial “superiority” was preceded by
  - claims of racist “superiority”,

Notorious claims of “superiority”
Notorious claims of “superiority”

- The West is notorious for its claims of superiority:
- claim of colonial “superiority” was preceded by
- claims of racist “superiority”,
- and claims of Christian “superiority” (as in US Supreme Court judgment: that native Indians in America lost their right to land after being discovered by Christians)
Is formalism rigorous?

or just metaphysical

- Claim that Western formalism is superior
Is formalism rigorous?

or just metaphysical

- Claim that Western formalism is superior
- is similarly supercilious and religious.
Is formalism rigorous?

or just metaphysical

- Claim that Western formalism is superior
- is similarly supercilious and religious.
- The time has come to be critical and to discard it,
Is formalism rigorous?
or just metaphysical

- Claim that Western formalism is superior
- is similarly supercilious and religious.
- The time has come to be critical and to discard it,
- to focus on the practical value of math,
Is formalism rigorous?

Claim that Western formalism is superior
is similarly supercilious and religious.
The time has come to be critical and to discard it,
to focus on the practical value of math,
and go back to our roots.
References

References

2. A longer reading list and list of videos is posted at http://ckraju.net/papers/Reading-list-Bengaluru.html