SGT University: calculus without limits

Pre-test

Answer all questions.
Blank answer fetches 0. Wrong answer gets negative marks.
Classes refer to NCERT texts. You may have learnt from a different text in school.

1. **Points.** You were taught about points in class VI.
   (a) Define a point.
   (b) What is the size of a point?
   (c) Can something with no size be seen? If something is invisible, how do you know where it is?
   (d) What is the difference between a fraction and a rational number?

2. **Numbers.** You were taught "real" numbers in class IX and class X.
   (a) Define a real number.
   (b) Write down the EXACT value of $\sqrt{2}$. If x is that exact value, show by direct calculation that $x^2=2$. (Note: this should not be 1.9999999999, but exactly 2.)
   (c) Can a complex number be written as the ratio of two integers? If i is the complex number such that $i^2=-1$ then is i irrational? Is it rational?
   (d) Are there numbers which are neither rational nor irrational? If your answer is yes, go back and re-check your definition at (a). If your answer is no, explain how -1 can have a real square root.

3. **Sets.** You were taught about sets in class X.
   (a) Define a set.
   (b) If you defined a set as a "collection of objects", define "collection" and define "object". Is "object" the same as in object-oriented programming? If not, what is the difference?
   (c) Let $R = \{ x \mid x \notin x \}$. Is it true that $R \in R$? Is it true that $R \notin R$?
   (d) Can a set have an infinite number of elements? How can you be sure?

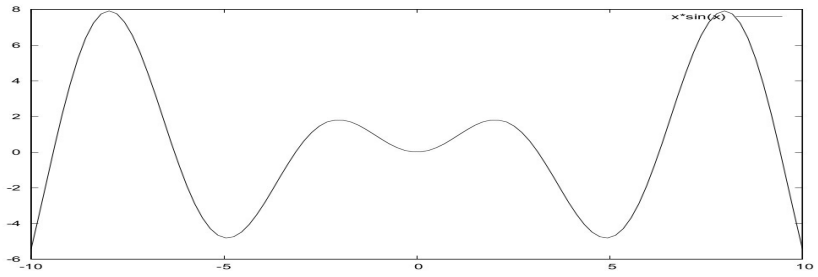
4. **Trigonometry.** You were taught about trigonometric functions in class IX.
   (a) Define sin(x).
   (b) Use that definition to calculate sin (0.3°).
   (c) Is sin (92°) defined? According to my calculator, sin (92°) = 0.9993. Is this right? Explain.
   (d) Define a radian. Exactly how many degrees is 1 radian?

5. **Limits.** You were taught about limits in class XI and XII.
   (a) Define limit.
   (b) According to my calculator $\sqrt{2}$ = 1.414213562373095048801688724209 7. Does the sequence 1, 1.4, 1.41, 1.414... have a limit?
   (c) What is the infinite sum of all natural numbers, 1+2+3+4+... ? Can it be a negative number?
   (d) What is the infinite sum 1-1+1-1+1-1+...?

6. **Derivative.** You were taught about derivatives in class XI and XII.
   (a) Define the derivative of a function.
   (b) Let N denote the set of natural numbers, and let f : N → N be given by $f(x)=x^2$.

What is the derivative of f ?

(c) Define the tangent to a curve at a point. Consider the function x sin(x) whose graph is



displayed. Write the equation of the tangent to the curve at x=0. At how many points does this line intersect the curve? Can you list these points?

(d) What is the derivative of atanh(x) (hyperbolic arc tangent) with respect to x?

7. **Integral.** You were taught about integrals in class XII.
(a) Define the integral of a function.
(b) Shown below is a piece of land with irregular boundaries. How will you calculate its area?

(c) Calculate $\int_{-1}^{-2} \frac{dx}{x}$.

(d) Calculate $\int \frac{1}{\sqrt{((1-x^2)(1-4x^2))}} dx$.

8. **Applications.** You learnt about Newton's laws of motion and the simple pendulum from class VIII to class XI.
(a) At approximately what angle should you throw a cricket ball so that it travels the furthest distance?
(b) Will the answer change if you use a tennis ball instead of a cricket ball?
(c) The formula for the time period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Therefore, the time period of a simple pendulum is independent of amplitude. Is this true or false?
(d) Did you ever experimentally verify any of your answers above?