

Beyond zero: India's contribution to math and science

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Not zero

- ▶ Was India's contribution to math **zero**!?

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- ▶ This led to **large numbers** up to 10^{12} found in Yajurveda 17.2.

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- ▶ Was India's contribution to math **zero**!?
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- ▶ India contributed **efficient arithmetic**
- ▶ based on place value system.
- ▶ This led to **large numbers** up to 10^{12} found in Yajurveda 17.2.
- ▶ Names of 53 places ($10^{53} = tallakṣaṇa$) in Buddhist *Lalitavistara sutta*, chp. 12.

Efficient arithmetic

Shukla Yajurveda 17.2

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एका च दश च दश च शतं च शतं च सहस्रं च
सहस्रं च अयुतं च अयुतं च नियुतं च नियुतं च
प्रयुतं च अर्बुदं च न्यर्बुदं च समुद्रः च मध्यं च
अन्त्यः च परार्धः च

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- ▶ Large numbers needed for precise fractions for $\sqrt{2}$, π etc.

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- ▶ Large numbers needed for precise fractions for $\sqrt{2}$, π etc.
- ▶ (-8th c.) *śulba sūtra*
$$\sqrt{2} = 1.41421[56] = 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34}$$

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$$\sqrt{2} = 1.41421[56] = 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34}$$
- ▶ (5th c.) *Āryabhaṭīya* $\pi = 3.1416 = \frac{62832}{20000}$

Precise fractions and π

- ▶ (13th c.) Madhava (from Nīlakanṭha's *Āryabhaṭīyabhāṣya*)

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सङ्गमग्रामजो माधवः पुनरत्यासन्नां परिधिसंख्यामुक्तवान् -
विबुधनेत्रगजाहिहुताशनत्रिगुणावेदभवारणबाहवः ।
नवनिखर्वमिते वृतिविस्तरे परिधिमानमिदं जगदुर्बुधाः ॥

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- ▶ Corresponds to

$$\pi = 3.1415926535922 \dots = \frac{2,827,433,388,233}{9 \times 10^9}, \text{ (accurate to 11 places)}$$

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- ▶ (15th c.) *Karaṇapaddhati* (VI, 7) (reverse sexagesimal *kaṭapayādi* system) circumference =

$$\text{चरडांशुचन्द्राधमकुंभिपाल} = 31,415,926,536$$

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- ▶ i.e., $\pi = 3.1415926536$

Contrast: Inefficient Greek and Roman arithmetic

- ▶ In contrast, in Roman numerals, largest number is myriad = 10000.

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Contrast: Inefficient Greek and Roman arithmetic

- ▶ In contrast, in Roman numerals, largest number is myriad = 10000.
- ▶ No systematic notation for fractions

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- ▶ In contrast, in Roman numerals, largest number is myriad = 10000.
- ▶ No systematic notation for fractions
- ▶ Greeks and Romans could not say the precise length of the (tropical) year.

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- ▶ In contrast, in Roman numerals, largest number is myriad = 10000.
- ▶ **No systematic notation for fractions**
- ▶ Greeks and Romans could not **say** the precise length of the (tropical) year.
- ▶ Hence, Julian calendar had badly defective length of the (tropical) year at $365\frac{1}{4}$.

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- ▶ **No systematic notation for fractions**
- ▶ Greeks and Romans could not **say** the precise length of the (tropical) year.
- ▶ Hence, Julian calendar had badly defective length of the (tropical) year at $365\frac{1}{4}$.
- ▶ (Julian calendar reformed the earlier defective Roman calendar, and the Romans laughed at the Greek calends.)

Transmission of Indian Arithmetic

- ▶ Hence, Europeans themselves abandoned their native Roman system.

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- ▶ Hence, Europeans themselves abandoned their native Roman system.
- ▶ In 10th c. Christian Europe imported Indian arithmetic from Muslim Europe (Cordoba).

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- ▶ In the first attempt **Europeans failed to understand Indian arithmetic.**
- ▶ (Understood zero as blank space, but continued using inefficient abacus.)

८	५	१	८	५	१	
				१	५	13
				४	१	87
		५		१	१	4 019
५			५	१		400 520
			५	५	१	539
१				५	५	100 065

Transmission of arithmetic-2

Law against zero

- ▶ Indian arithmetic again imported by Florence via Africa in 12th c. (Fibonacci)

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- ▶ Florentine merchants understood comparative advantage of efficient arithmetic for commerce,

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- ▶ Indian arithmetic again imported by Florence via Africa in 12th c. (Fibonacci)
- ▶ Florentine merchants understood comparative advantage of efficient arithmetic for commerce,
- ▶ but were puzzled by zero,
- ▶ as suggested by its very name from *sifr* = *zephyr* = *cypher* = mysterious code.

Law against zero

contd.

- ▶ Hence, Florence passed a law against zero in 13th c.

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- ▶ Hence, Florence passed a law against zero in 13th c.
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- ▶ Roman numerals are additive XII = 10 + 1 + 1

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- ▶ Hence, Florence passed a law against zero in 13th c.
- ▶ Why?
- ▶ Roman numerals are additive XII = 10 + 1 + 1
- ▶ Indian numerals use place value: 130 \neq 1 + 3 + 0.

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- ▶ Florentines complained: zero has no value but adds any amount of value to the preceding number.

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- ▶ Indian numerals use place value: 130 \neq 1 + 3 + 0.
- ▶ Contracts were altered by adding 0's at the end.
- ▶ Florentines complained: zero has no value but adds any amount of value to the preceding number.
- ▶ Florentine law said numbers in financial contracts must also be written in words. We still follow it.

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Moral of the story

- ▶ (Raju's epistemic test) imported or copied knowledge is hard to understand.

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- ▶ Called *avyakt gaṇit* by Brahmagupta (Brāhma-sphuṭa-siddhānta, chp. 18)

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- ▶ (forcible solution by equating two quantities, one known, one unknown)
- ▶ Solves quadratic equations.

Probability and statistics

Probability and statistics

- ▶ Ṛgveda 10.2.34.1 is the *akṣa sūkta* or extraordinary “hymn on dice”.

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- ▶ Story of Nala and Damayanti in Mahabharata, relates probability (science of dice, *akṣa vidyā*) to sampling theory:

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- ▶ Ṛtuparṇa teaches Nala how to count the number of fruits in a tree.

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- ▶ Ṛtupaṇṇa teaches Nala how to count the number of fruits in a tree.
- ▶ No time for details. See my article on “Probability in Ancient India” in Elsevier *Handbook of Philosophy of Statistics* (or google for online version).

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- ▶ Ṛgveda 10.2.34.1 is the *akṣa sūkta* or extraordinary “hymn on dice”.
- ▶ Story of Nala and Damayanti in Mahabharata, relates probability (science of dice, *akṣa vidyā*) to sampling theory:
- ▶ Ṛtuparṇa teaches Nala how to count the number of fruits in a tree.
- ▶ No time for details. See my article on “Probability in Ancient India” in Elsevier *Handbook of Philosophy of Statistics* (or google for online version).
- ▶ Or article on “Probability” in Springer *Encyclopedia of Non-Western Science, Technology and Medicine*.

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- ▶ Binomial expansion in Piṅgala's *Chandahsūtra* 8.28.

Binomial expansion

- ▶ Binomial expansion in Piṅgala's *Chandaḥsūtra* 8.28.
- ▶ Used in Vedic theory of metre.

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- ▶ Binomial expansion in Piṅgala's *Chandaḥsūtra* 8.28.
- ▶ Used in Vedic theory of metre.
- ▶ *Meru prastāra* ("Pascal's triangle") in Halauyudha's 10th c. commentary.

Permutations and combinations

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- ▶ Theory of permutations and combinations

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- ▶ Theory of permutations and combinations
- ▶ in numerous texts (e.g. Suśruta, Varāhamihira, Śridhar, *Pāṭīganita* (72), Mahāvīra, *Gaṇita Sāra Sangraha* etc.

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- ▶ Used also in theory of Indian music.

Trigonometry and calculus

Trigonometry and calculus

- ▶ Both developed in India.

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- ▶ Word “sine” from Latin “sinus” is from fold, or “jaib” a misreading of “jiba” from Indian “jiva” or “jya”.

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- ▶ **Involves a conceptual error.**

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- ▶ Word “sine” from Latin “sinus” is from fold, or “jaib” a misreading of “jiba” from Indian “jiva” or “jya”.
- ▶ **Involves a conceptual error.**
- ▶ As the term “jya” suggests, sine relates to **the circle** (as in Indian texts): word **“trigonometry” is a conceptual misnomer.**

Āryabhaṭa's table of sine differences

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मखि भखि फखि धखि राखि जखि
डखि हस्मि स्ककि किष्ठा श्चकि किध्व ।
घलकि किग्र हक्य धकि किच
सा श्म ड्व क्ल प्त फ छ कलार्धज्या ॥ १२ ॥

- ▶ 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7—[these are the] Rsine [differences] [for the quadrant divided into as many equal parts, each part hence being 225'] [in] minutes.

- ▶ 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7—[these are the] Rsine [differences] [for the quadrant divided into as many equal parts, each part hence being 225'] [in] minutes.
- ▶ (Circumference of the circle in minutes is $360 \times 60 = 21,600$.)

Increasing precision

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Increasing precision

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- ▶ by the Āryabhaṭa school in Kerala to
- ▶ Madhava's sine table
- ▶ which used an infinite series for precise sine values.
- ▶ (Indians knew how to sum infinite series. Nīlakanṭha states the sum of **infinite geometric series**.)

The infinite series for sine

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- The “Taylor” series expansion for the sine stated in two verses, as follows.

निहत्य चापवर्गेण चापं तत्तत्फलानि च ।

हरेत् समूल्युग्वर्गेस्त्रिज्यावर्गहतैः क्रमात् ॥ ४४० ॥

चापं फलानि चाधोऽधो न्यस्योपर्युपरित्यजेत् ।

जीवाप्त्यै, संग्रहोऽस्यैव विद्वान् इत्यदिना क्रितः ॥ ४४१ ॥

Translation

- ▶ Multiply the arc by the square of the arc, and take the result of repeating that [any number of times].

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Translation

- ▶ Multiply the arc by the square of the arc, and take the result of repeating that [any number of times].
- ▶ Divide [each of the above numerators] by the squares of successive even numbers increased by that number [literally, the root] and multiplied by the square of the radius.

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- ▶ Divide [each of the above numerators] by the squares of successive even numbers increased by that number [literally, the root] and multiplied by the square of the radius.
- ▶ Place the arc and the successive results so obtained one below the other, and subtract each from the one above.

- ▶ Multiply the arc by the square of the arc, and take the result of repeating that [any number of times].
- ▶ Divide [each of the above numerators] by the squares of successive even numbers increased by that number [literally, the root] and multiplied by the square of the radius.
- ▶ Place the arc and the successive results so obtained one below the other, and subtract each from the one above.
- ▶ These together give the $jīvā$, as collected together in the verse beginning with “*vidvān*” etc.

Mathematical Translation

- ▶ Let r = radius of the circle, s = arc and let t_n = the n th expression obtained by applying the above rule.

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Mathematical Translation

- ▶ Let $r =$ radius of the circle, $s =$ arc and let $t_n =$ the n th expression obtained by applying the above rule.
- ▶ Numerator: multiply the arc s by its square s^2 , this multiplication being repeated n times to obtain

$$s \cdot \prod_1^n s^2.$$

Mathematical Translation

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- ▶ Let $r =$ radius of the circle, $s =$ arc and let $t_n =$ the n th expression obtained by applying the above rule.
- ▶ Numerator: multiply the arc s by its square s^2 , this multiplication being repeated n times to obtain

$$s \cdot \prod_1^n s^2.$$

- ▶ Denominator: multiply the square of the radius, r^2 , by $[(2k)^2 + 2k]$ (“the squares of successive even numbers increased by that number”) for successive values of k , repeating this product n times to obtain

$$\prod_{k=1}^n r^2 [(2k)^2 + 2k].$$

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Conclusions

Math translation: contd.

- ▶ Thus, the n th iterate is obtained by

$$t_n = \frac{s^{2n} \cdot s}{(2^2 + 2) \cdot (4^2 + 4) \cdot \dots \cdot [(2n)^2 + 2n] \cdot r^{2n}}. \quad (1)$$

Math translation: contd.

- ▶ Thus, the n th iterate is obtained by

$$t_n = \frac{s^{2n} \cdot s}{(2^2 + 2) \cdot (4^2 + 4) \cdot \dots \cdot [(2n)^2 + 2n] \cdot r^{2n}} \quad (1)$$

- ▶ The rule further says:

$$\begin{aligned} \bar{j}iv\bar{a} &= s - t_1 + t_2 - t_3 + t_4 - t_5 + \dots \quad (2) \\ &= s - \frac{s^3}{r^2 \cdot (2^2 + 2)} + \frac{s^5}{r^4(2^2 + 2)(4^2 + 4)} - \dots \quad (3) \end{aligned}$$

<+>Substituting (1) $j\bar{iv}\bar{a} \equiv r \sin \theta$, (2) $s = r \theta$, so that $s^{2n+1} / r^{2n} = r \theta^{2n+1}$, and noticing that (3) $[(2k)^2 + 2k] = 2k \cdot (2k + 1)$, so that (4) $(2^2 + 2)(4^2 + 4) \cdots [(2n)^2 + 2n] = (2n + 1)!$, and cancelling r from both sides, we see that this is equivalent to the well-known expression

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots \quad (4)$$

Details in my book *Cultural Foundations of Mathematics*, Pearson Longman, 2007.

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श्रेष्ठं नाम वरिष्ठानां हिमाद्रिवेदभावनः ।
तपनो भानुसूक्तज्ञो मध्यमं विद्धि दोहनम् ॥
धिगाज्यो नाशनं कष्टं छन्नभोगाशयाम्बिका ।
म्रिगाहारो नरेशोऽयं वीरो रणजयोत्सुकः ॥

...

छायालयो गजो नीलो निर्मलो नास्ति सत्कुले ।
रात्रौ दर्पणमभ्राङ्गं नागस्तुङ्गनखो बली ॥
धीरो युवा कथालोलः पूज्यो नारीजनैर्भगः ।
कन्यागारे नागवल्ली देवो विश्वस्थली भृगुः ॥
तत्परादिकलान्तास्तु महाज्या माधवोदिताः ।
स्वस्वपूर्वविशुद्धे तु शिष्टास्तत्खण्डमौर्विकाः ॥ २.९.५ ॥

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Table: Mādhava's sine values

No.	Kaṭapayādi	kalā (')	vikalā('')	tatparā('''')
1	श्रेष्ठं नाम वरिष्ठानां	224	50	22
2	हिमाद्रिर्वेदभावनः	448	42	58
3	तपनो भानुसूक्तज्ञो	670	40	16
4	मध्यमं विद्धि दोहनम्	889	45	15
...
21	धीरो युवा कथालोलः	3371	41	29
22	पूज्यो नारीजनैर्भगः	3408	20	11
23	कन्यागारे नागवल्ली	3430	23	11
24	देवो विश्वस्थली भृगुः	3437	44	48

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Accuracy of Madhava's sine values

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Table: Accuracy of Mādhava's sine table.

No.	Mādhava's sine value	Difference
1	0.0654031452	0.0000000160
2	0.1305262297	0.0000000375
3	0.1950903240	0.0000000020
4	0.2588190035	-0.0000000416
...
...
21	0.9807852980	0.0000000176
22	0.9914448967	0.0000000353
23	0.9978589819	0.0000000587
24	1.0000000000	0.0000000000

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- ▶ Why was so much accuracy needed?

Why calculus developed in India

- ▶ Why was so much accuracy needed?
- ▶ Because two key sources of wealth in India were

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- ▶ Why was so much accuracy needed?
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 1. **agriculture** and

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- ▶ Why was so much accuracy needed?
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 1. **agriculture** and
 2. **overseas trade**.

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- ▶ Because two key sources of wealth in India were
 1. agriculture and
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- ▶ That required an accurate calendar and accurate techniques of navigation,

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- ▶ Why was so much accuracy needed?
- ▶ Because two key sources of wealth in India were
 1. agriculture and
 2. overseas trade.
- ▶ That required an accurate calendar and accurate techniques of navigation,
- ▶ which required the science of astronomy.

How and why Europeans stole the calculus

- ▶ Because navigation then was THE major scientific problem of Europe

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How and why Europeans stole the calculus

- ▶ Because navigation then was THE major scientific problem of Europe
- ▶ Precise sine values were needed to determine **latitude, longitude, and loxodromes**.

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How and why Europeans stole the calculus

- ▶ Because navigation then was THE major scientific problem of Europe
- ▶ Precise sine values were needed to determine **latitude, longitude, and loxodromes**.
- ▶ Cochin-based Jesuits stole the calculus (and some calendrical knowledge).

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- ▶ Europeans understood the practical use of precise sine values

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- ▶ but failed to understand how Indians summed infinite series.

How and why Europeans stole the calculus

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- ▶ Cochin-based Jesuits stole the calculus (and some calendrical knowledge).
- ▶ Europeans understood the practical use of precise sine values
- ▶ but failed to understand how Indians summed infinite series.
- ▶ Unlike Arabs, Europeans then **did not acknowledge their non-Christian sources** (because of the Inquisition etc.).

Matteo Ricci's letter of 1581

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- ▶ In 1581 a student of Clavius (author of Gregorian reform, 1582), Matteo Ricci, was in Cochin and wrote that he was looking for “an honorable Moor or an intelligent Brahmin to tell him about Indian methods of timekeeping”.

... duas ou tres leguas e depois não tem mais nome, o mesmo se de ser em ba-
... e em Malaca e tem uns deques de los quais a seus nomes me con-
...; Gra não tem nome de agua doce mas q' e de agua sal-
gada e se chama de Gra e tambem se mette m^a g^a terra de m^a
... os nomes dos Reis são tao desacomunados q' não destes q' nunca agora sei
algua mais q' do Mogor q' se chama Hechabar, não outros os sabem, e tudo
que me parece q' será impossível saberse mas se de ser for uos d'algum nome
concedo ao bramae a intelligencia e seiba as cronicas dos tempos dos
quais eu puzere saber tudo
... e folgaria
tambem os outros q' a lerão, e vada q' eu não possa dar bom curso das cousas

Science

Astronomy

- ▶ Calendar and (celestial) navigation require astronomy: a key early Indian science.

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- ▶ Calendar and (celestial) navigation require astronomy: a key early Indian science.
- ▶ No time to go into it. Only two observations.

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- ▶ Calendar and (celestial) navigation require astronomy: a key early Indian science.
- ▶ No time to go into it. Only two observations.
 1. Indian astronomy NOT borrowed from Greeks (who were arithmetically challenged and could not do astronomy).

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- ▶ Calendar and (celestial) navigation require astronomy: a key early Indian science.
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 1. Indian astronomy NOT borrowed from Greeks (who were arithmetically challenged and could not do astronomy).
 2. Astronomy, NOT astrology. *Vedāṅga Jyotiṣa* from –1500 CE does not contain a single sentence on astrology. (My challenge since 2001.)

- ▶ Calendar and (celestial) navigation require astronomy: a key early Indian science.
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 1. Indian astronomy NOT borrowed from Greeks (who were arithmetically challenged and could not do astronomy).
 2. Astronomy, NOT astrology. *Vedānga Jyotiṣa* from –1500 CE does not contain a single sentence on astrology. (My challenge since 2001.)
 - ▶ Neither does ANY Indian astronomy text, across 3000 years, down to the 16th c. (including Varahamihira's *Pañcasiddhāntikā*).

Contrast: Kepler

- ▶ In contrast, Kepler earned a livelihood from astrology.

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Contrast: Kepler

- ▶ In contrast, Kepler earned a livelihood from astrology.
- ▶ Wrote that God created astrology as the natural profession for astronomers!

Indians against superstition-1

Eclipses NOT due to demons

- ▶ Lalla, Vateshvara etc. speak out against superstition, E.g. Lalla *Śiṣyadhīvr̥dhida*, chp. 20

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- ▶ “If an artful demon is the cause of an eclipse. . .

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- ▶ “If an artful demon is the cause of an eclipse. . .
 - ▶ then how is that that an eclipse can be determined by calculation?
 - ▶ Why does it always occur only on *poornima* or *amavasya*?”

Indians against superstition-2

Spherical Earth

- ▶ “If the earth is flat,

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Indians against superstition-2

Spherical Earth

- ▶ “If the earth is flat,
- ▶ why can't tall trees, alas, be seen at great distance?”

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Indians against superstition-3

Earth not supported

- ▶ “If the earth is supported by a serpent (śeṣa nāga), tortoise etc.

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Indians against superstition-3

Earth not supported

- ▶ “If the earth is supported by a serpent (śeṣa nāga), tortoise etc.
 - ▶ by whom are *they* supported in space?

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Indians against superstition-3

Earth not supported

- ▶ “If the earth is supported by a serpent (śeṣa nāga), tortoise etc.
 - ▶ by whom are *they* supported in space?
 - ▶ If they can stand unsupported why not the earth?”

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Contrast

- ▶ Contemporary Bible (5th c. Vulgate) says earth is flat. (E.g. Daniel:4-10-11:)

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Contrast

- ▶ Contemporary Bible (5th c. Vulgate) says earth is flat. (E.g. Daniel:4-10-11:)
- ▶ “behold a tree. . . and the height thereof was great. . . . and the sight thereof to the end of all the earth.

Contrast

- ▶ Contemporary Bible (5th c. Vulgate) says earth is flat. (E.g. Daniel:4-10-11:)
- ▶ “behold a tree. . . and the height thereof was great. . . . and the sight thereof to the end of all the earth.
- ▶ On Greek superstition, Atlas held up the earth. **No one contested it in all Greek literature.**

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Origin of experimental method

Payāsi's experiments

- ▶ The first record of the experimental method is found in India 2500 years ago.

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Origin of experimental method

Payāsi's experiments

- ▶ The first record of the experimental method is found in India 2500 years ago.
- ▶ In the *Dīgha Nikāya*.

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Conclusions

Origin of experimental method

Payāsi's experiments

- ▶ The first record of the experimental method is found in India 2500 years ago.
- ▶ In the *Dīgha Nikāya*.
- ▶ Payāsi performed some **30 experiments** with condemned felons to test the existence of the soul and life after death.

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- ▶ (weight of the soul?) “weigh the man, strangle him with a string and weigh him again”

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- ▶ (can you see the soul?) “put him in a pot, cover it with moist paste, and heat the pot till he is dead. Then make a small hole to see if his soul comes out”

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- ▶ and so on.

- ▶ (weight of the soul?) “weigh the man, strangle him with a string and weigh him again”
- ▶ (can you see the soul?) “put him in a pot, cover it with moist paste, and heat the pot till he is dead. Then make a small hole to see if his soul comes out”
- ▶ and so on.
- ▶ This was 2000 years before Francis Bacon.

Cultural pride?

Matter of pride?

- ▶ So, isn't all this a matter of great pride?

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Matter of pride?

- ▶ So, isn't all this a matter of great pride?
- ▶ **No**. It is a matter of great **shame!**

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Matter of pride?

- ▶ So, isn't all this a matter of great pride?
- ▶ **No**. It is a matter of great **shame!**
- ▶ Shame that we forget about calculus in India for 500 years,

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Matter of pride?

- ▶ So, isn't all this a matter of great pride?
- ▶ **No**. It is a matter of great **shame!**
- ▶ Shame that we forget about calculus in India for 500 years,
- ▶ until my 1998-2007 project. (Part of it done here in NMML.)

Theft of calculus story

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- ▶ Those who forget history repeat past mistakes.

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- ▶ Those who forget history repeat past mistakes.
- ▶ Calculus was stolen in the 16th c.,

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- ▶ Those who forget history repeat past mistakes.
- ▶ Calculus was stolen in the 16th c.,
- ▶ But the story that “calculus was stolen” was **itself stolen** in the 21st c.!

Theft of calculus story

- ▶ Those who forget history repeat past mistakes.
- ▶ Calculus was stolen in the 16th c.,
- ▶ But the story that “calculus was stolen” was **itself stolen** in the 21st c.!
- ▶ **Repeatedly**, in 2000, then 2004, then 2007 (as soon as my book on calculus theft was published).

Prof Raju's charge of plagiarism found correct

UK varsity warns lecturer

Shawn G. Robinson Alvest
Bhopal, November 7

THE SCIENTIFIC community in the United Kingdom (UK) has been left in a state of alarm after the charges of plagiarism levelled against a faculty member of the prestigious University of Exeter by the renowned City-based mathematician Professor C.K. Raju were found to be correct.

The university has issued a warning to the defunct faculty member Dennis Almeida who allegedly plagiarised the path-breaking works of Prof Raju in the field of mathematics and published them in his name.

A mathematician and physicist, Dr C.K. Raju is well known for his research in which he has claimed that European mathematicians did not invent calculus, which was perhaps transferred from India to the West. He was staggered to find his unpublished work plagiarised by the UK lecturers every this year.

Soon after the University of Exeter, who is Head of Centre for Computer Science with the Mahatma Chattrvedi National University of Mathematics, said the university's the



Prof C.K. Raju

vice DG Shreed Chandra Mohan took up the issue with the Vice-Chancellor of the University of Exeter in March this year.

Recently the VC of the University of Exeter Professor Vivek-Singh, wrote a letter to India that a plagiarism was taking place after the complaint of Dr Raju and after the enquiry was completed, a written warning has been issued to Almeida that he must adhere to moral-ethically accepted norms of authorship-acknowledgement in publications and any failure to follow would be referred to a tribunal constituted under the university's statutes, which could lead to his dismissal from the university.

Continued on page 4

UK varsity warns lecturer

Continued from page 1

Dr Raju had charged Almeida, Lecturer in Mathematics Education at the School of Education in Exeter, Dr Zadorzhnyy, Department of Classics, University of Liverpool and another person for plagiarism of his innovative research.

According to Dr Raju, he had placed an advertisement

ment for Research Associate on his project, Madhava and the origin of calculus on the internet in 1996, following which, Almeida had contacted him showing his interest in history of mathematics.

Almeida came to India to meet Raju and after his return to England raised some funding from the university for the project. In process, he reportedly gained access to Raju's unpublished works by saying that this was needed to raise more funds.

The association between Raju and consequential educational implications" in a journal in 2001.

However, Almeida then allegedly co-opted Dr

George Joseph, Reader in Economics Department of the University of Manchester, on the grounds that his British citizenship will help in raising more funding. The terms subsequently became unacceptable for Dr Raju because it appeared to him that Almeida wanted to take over Dr Raju's ideas but giving him only a side role.

The association thus ended but to Raju's astonishment, he found that his thesis was presented by Dr Joseph in a conference in December 2000 and then Almeida, John and Zadorzhnyy wrote a paper, "Kovalev mathematics: its possible transmission to Europe and consequential educational implications" in a journal in 2001.

Dr C.K. Raju, who played

a leading role in the development of India's first supercomputer, Parus, is an author of several books. He has questioned scientists like Einstein and Isaac Newton and challenged the long-existing beliefs that calculus was invented in Europe.

He has written several books including the famous, 'The Eleven Pictures of Time'.

The eminent Bhopal mathematician was astounded when he found that his unpublished work was freely used without credit to him and it appeared on a website recently. The complaint to the VC, University of Exeter was made and now the reply from the university and the findings of enquiry have at last proved the Indian

mathematician correct.

PEARSON LONGMAN

History of Science, Philosophy and Culture
in Indian Civilization

General Editor D. P. Chattopadhyaya

Volume X Part 4

Cultural Foundations of Mathematics
The Nature of Mathematical Proof and
the Transmission of the Calculus
from India to Europe in the 16th c. CE

C. K. RAJU

PHISPC

Centre for Studies in Civilizations

ANALYSE THIS

Kerala scholars cracked math code before Newton

Vijay Dutt
London, August 13

A LITTLE-KNOWN school of scholars in Kerala discovered one of the founding principles of modern mathematics much before Sir Isaac Newton, to whom the finding is currently attributed, a new research here says.

Dr George Gheverghese Joseph, an

Honorary Reader of the University of Manchester, says the 'Kerala School of Mathematics and Astronomy' identified the 'Infinite Series' — one of the basic components of calculus — in about 1350.

"The 'Infinite Series' was identified by these little-known scholars in Kerala all of whom were from within 500 km of Cochin," Dr Joseph, hailing from Kottayam, told HT. The scholars

of the school also discovered what amounted to the Pi series and used it to calculate Pi correct to 17 decimals.

The research, carried out by teams led by Dr Joseph and Dennis Almeida of the University of Exeter, found evidence that the Indians passed on their discoveries to Jesuit missionaries who visited India during the 15th century.

vdutt@aol.com



JAYANTO



HT CORRECTION

The claim made by two British researchers that they were the ones who unearthed the fact that Kerala mathematicians invented the calculus long before Sir Isaac Newton (Hindustan Times, August 14, 2007) was incorrect. The Kerala infinite series has been known to British scholars since 1832. Recent work on transmission of the calculus was first done by C.K. Raju, Editorial Fellow of the Project of History of Indian Science, Philosophy and Culture and is published in his book, Cultural Foundations of Mathematics: the Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th century. One of the British researchers, Dennis Almeida, was even warned in 2004 by Exeter University against plagiarising Raju's work." The error is regretted.

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- ▶ Why did we forget our traditional knowledge?

Question 1

- ▶ Why did we forget our traditional knowledge?
- ▶ Because there were no knowledgeable people left.

Question 1

- ▶ Why did we forget our traditional knowledge?
- ▶ Because there were no knowledgeable people left.
- ▶ Why?

- ▶ Many reasons: Loss of patronage after 17th c., Colonial epistemicide, etc.

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- ▶ Many reasons: Loss of patronage after 17th c., Colonial epistemicide, etc.
- ▶ But: *anti-vidvān policy post-independence.*

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- ▶ Many reasons: Loss of patronage after 17th c., Colonial epistemicide, etc.
- ▶ But: *anti-vidvān policy post-independence*.
 - ▶ We no longer even know *who* is a *vidvān*:

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 - ▶ commonly mistake science babus for “eminent scientists”.

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- ▶ 100% reservation for ignorant loyalists.
 - ▶ Pañcatantra story: “Loyal monkey cuts off your nose”,
 - ▶ repeatedly makes a joke of tradition:
 - ▶ in 10 minutes discredits 10 years work of the knowledgeable.



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News / India /



Don't miss: 5 howlers from the Indian Science Congress

Inaugurating the conference, Health Minister Dr Harsh Vardhan on Saturday said India had given the world algebra and the Pythagoras's Theorem, which is used to calculate the length of the hypotenuse of a right-angle triangle.

IndiaToday.in

New Delhi, January 5, 2015 | UPDATED 14:26 IST

A + A -



Picture for representational purpose

Nothing Vedic in 'Vedic Maths'

Advocating 'Vedic mathematics' as a replacement for traditional Indian arithmetic is hardly an act of nationalism; it only shows ignorance of the history of mathematics

C.K. Raju

Cujarat has made it compulsory for school students to read the texts of (pseudo) books authored by Prime Minister Narendra Modi. According to news reports, Mr. Modi has now proposed a non-governmental education commission which will Indianise education through, for instance, Vedic mathematics. The Minister for Education has also mentioned Vedic mathematics as part of her agenda.

Ignorant of tradition

One appreciates the desire of these people to work for Indian traditions, but where in the Vedas is "Vedic mathematics" to be found? Nowhere. Vedic mathematics has no relation whatsoever to the Vedas. It actually originates from a book misleadingly titled *Vedic Mathematics* by Bharati Krishna Tirtha. The book admits on its first page that its title is misleading and that the (elementary arithmetic) algorithms expounded in the book have nothing to do with the Vedas. This is repeated on p. xxvii: "Obviously these formulas are not to be found in the present recensions of Atharvaveda." I have been pointing this out since 1998. Regrettably, the advocates of "Vedic mathematics," though they claim to champion Indian tradition, are ignorant of the actual tradition in the Vedas. Second, they do not even know what is stated in the final sentence — the real source of "Vedic mathematics." Third, they are unaware of scholarly writing on the subject. When education policy is dictated by such ignorant people, they only end up making a laughing stock of themselves and the Vedas, and thus do a great disservice to the very tradition which they claim to champion.

Everyone learns how to add, subtract, multiply and divide in school. Why should we replace these algorithms with "Vedic mathematics"? Will that Indianise education? No. The standard arithmetic algorithms actually originated in India, where they were known by various names such as *paṭhana* (class arithmetic). However, the word "algorithm" comes from "algorism"; the Latinised name of al-Khwarizmi of the 9th century (base of Whiston in England). He wrote an explanatory book on Indian arithmetic called *Hisab al Hind* (Gerbert of Aurillac called Pope Sylvester II), the leading European mathematician of the 10th century



techniques from the Umayyad Khalifa of Córdoba. He did so because the primitive Greek and Roman system of arithmetic (tied to the abacus), then prevailing in Europe, was no match for Indian arithmetic. However, accustomed to the abacus (on which he wrote a book), Gerbert was perplexed by algorithms based on the place-value system, and foolishly got a special abacus (apex) constructed for these "Arabic numerals" in 976 CE. Hence the name "Arabic numerals" — because a learned pope amusingly thought there was some magic in the shape of the numerals which made arithmetic efficient.

Later, Florentine merchants realised that efficient Indian arithmetic algorithms conferred a competitive advantage in commerce. Fibonacci, who traded across Islamic Africa, translated al-Khwarizmi's work, as did many others, which is why they came to be known as algorithms. Eventually, after 600 years, Indian algorithms displaced the European abacus and were introduced in the Jewish syllabus as "practical mathematics" circa 1574 by Christoph Clavius. These algorithms are found in many early Indian texts, such as the *Paṭhana* of Śrīdhara or the *Ganita Sara Sangraha* of

Mahāvīra, or the *Līlāvati* of Bhāskara II. So, advocating "Vedic mathematics" as a replacement for traditional Indian arithmetic is hardly an act of nationalism. On the contrary, it only shows ignorance of the history of mathematics. Spreading this ignorance among future generations will weaken the nation, not strengthen it.

The techniques of "Vedic mathematics" are designed for mental arithmetic; traditionally used by lower caste artisans such as carpenters or by people like Shakuntala Devi. There are many other such systems of mental arithmetic today. If that is what we intend to promote, we should first do a systematic comparison. We should also be honest and refrain from using the misleading label "Vedic" which is the main selling point of Bharati Krishna Tirtha's system, and which attracts gullible people who infer value just from the wrapper.

Suppressing real Mathematics

Promoting the wrongly labelled "Vedic mathematics" suppresses the mathe-

matics that really does exist in the Vedas. For example, Yajurveda 17.2 elaborates on the decimal place value system (the basis of Indian algorithms) and some of those names for numbers are still in use, though terms such as *arab* (inclusion) have changed meaning. That passage shows that the place value system extends back to Vedic times, and it was a late acquisition only in mathematically backward Europe.

Likewise, the theory of permutations and combinations is built into the Vedic metre (and Indian music in general), as explained in various texts from Pingala's *Chandaśāstra* to Bhāskara's *Līlāvati*. The *śloka śūtra* of the *gāyā* gives a beautiful account of the game of dice, which is the foundation of the theory of probability. The romantic story of Nala and Damayanti in the *Mahābhārata* further relates dice to summing theory (to count the number of fruits in a tree).

More details are in my article on "Probability in Ancient India" available online and published in the *Eisenstein Handouts of the Philosophy of Statistics*. However, all these scholarly efforts are jeopardised, for they too are viewed with suspicion.

We need to change the Western and colonial education system, especially with regard to mathematics. Traditional Indian *ganita* has much to offer in this process, but "Vedic mathematics" is definitely not the right way.

Wrong solutions like "Vedic mathematics" persist because an insecure political dispensation values the politically loyal over the learned who are loyal to the truth. ("Merit" apparently is important only in the context of reservations.) Such political processes are historically known to damage real tradition.

As I wrote over a decade ago in my book *The Eleven Pictures of Time*, those who attain or retain state power through religion are the worst enemies of the religion, whatever be the religion they claim to represent: Christianity, Islam, or Hinduism.

C.K. Raju is author of *Cultural Foundations of Indian Civilisation*. He was professor of mathematics, and Editorial Fellow of the Project of History of Indian Science, Philosophy and Culture.

Wrong solutions persist because an insecure political dispensation values the politically loyal over the learned

वैदिक गणित में वैदिक कुछ नहीं

गुजरात के सूक्तों में दीनानाथ बजा की किताबें पढ़ना अनिवार्य किया गया है। प्रधानमंत्री नरेंद्र मोदी ने इन किताबों का समर्थन किया है। खखों के अनुवाद बजा ने एक गैर-सरकारी शिक्षा आयोग बनाया है, जो शिक्षा के भारतीयकरण और वैदिक गणित आदि के बारे में सुझाव देगा। भारत के शिक्षामंत्री ने भी अपने एजेंडे में वैदिक गणित का उल्लेख किया है।

भारतीय परंपराओं के लिए काम करने की इच्छा सराहनीय है। लेकिन 'वैदिक गणित' वेदों में है कहां? कहां नहीं? वैदिक गणित का किसी वेद से कोई संबंध नहीं है। 'वैदिक गणित' नाम निकाल खला इसी शीर्षक की एक पुस्तक से, जो भारतीय कृष्णतीर्थ ने लिखी। यह पुस्तक भी अपने पहले पन्ने पर यह मानती है कि 'वैदिक गणित' का वेद से कोई संबंध नहीं। गीता बता आगे चल कर भी दोहराई गई है: 'इन सूत्रों का जिक्र अथर्ववेद में नहीं पाया जाता है।' यह बात > **विवाद**

मैं 1998 से कह रहा हूँ। खख है कि जो लोग अपने को भारतीय परंपराओं का चैपियन मानते हैं, वे खुद उन परंपराओं और वेदों से अनभिज्ञ हैं। न वे वह जानते हैं कि जो पुस्तक 'वैदिक गणित' का असली खेत है, उसमें क्या लिखा है। तीसरे, वे इस विषय पर लिखे शोध-पत्रों से भी अनभिज्ञ हैं। जब ऐसे अज्ञानी लोग शिक्षा नीतियां बनाते हैं तो वे खुद को और वेद को एक उपहास का पात्र बना देते हैं।

साधारण गणित के गुणा-भाग के तरीके सभी ने स्कूल में सीखे। उन्हें छोड़ 'वैदिक गणित' क्यों करना चाहिए? क्या इससे शिक्षा का भारतीयकरण होगा? हमें जिन नहीं!

'एल्गोरिथ्म' (जोड़-घटाना, गुणा-भाग की पद्धति) बाल्य में भारत से विकसित हुई। इसके भारत में पाठ्यगणित जैसे कई नाम थे। 'एल्गोरिथ्म' शब्द अल ख्वारिज्मी के लातीनी नाम से आता है। नौवीं सदी में अल ख्वारिज्मी ने भारतीय गणित पर 'हिसाब अल हिंद' नामक एक मशहूर वर्णनात्मक पुस्तक लिखी। नवें (जो बाद में पोप मिलेव्स्टेर द्वितीय बना) दसवीं सदी का प्रमुख यूरोपीय गणितज्ञ था। उसने काठिया के उमय्यद

खिलफत से इस गणित का आयात किया। उस समय यूरोप में अबैकस (अंकगणक) से बंधी गणित की आदिम ग्रीक और रोमन प्रणाली प्रचलित थी, जो भारतीय गणित की तुलना में बहुत पिछड़ी थी। गर्बट अंबेकस का जानकार था। उसने उस पर एक किताब भी लिखी थी। इसलिए वह 'स्थान मूल्य प्रणाली' पर आधारित एल्गोरिथ्म को समझ न पाया, और अज्ञानतावश इन 'अरबी अंकों' के लिए 976 में एक विशेष अबैकस बनवाया। पोप को लगा कि अंकों के आकार में ही कुछ जादू था, इसलिए उसने इसे 'अरबी अंक' कहा। !

बाद में फ्लोरेंस के व्यापारियों ने समझा कि भारतीय गणित से लेन-देन में प्रतिस्पर्धीत्वक लाभ है। फिबोनाची ने अल ख्वारिज्मी की किताब का अनुवाद किया, छह सौ साल बाद यूरोपियों ने अबैकस छोड़ भारतीय एल्गोरिथ्म अपनाया। 'व्यावहारिक

गणित' के नाम से जेसुइट पादरुक्रम में इसे क्रिस्टोफ क्लाविडस ने 1570 के आसपास शुरू किया। गणित के ये तरीके श्रीधर की 'पाटीगणित' या महावीर के 'गणित सार संग्रह' जैसे कई प्राचीन भारतीय ग्रंथों में पाए जाते हैं। इसलिए इस पारंपरिक भारतीय गणित को हटा कर 'वैदिक गणित' पढ़ाना कोई रादुत्व नहीं। यह केवल गणित के इतिहास के अज्ञान का प्रदर्शन है। आने वाली पीढ़ियों में ऐसा अज्ञान फैलाने से रादुत मजबूत नहीं, बल्कि कमजोर होगा।

'वैदिक गणित' के यह एल्गोरिथ्म मन ही मन में गणित करने के लिए उपयुक्त हैं। पारंपरिक रूप से या तो निम्न जाति के कार्यगण या फिर जकुंतला देवी जैसे लोगों ने इसे इस्तेमाल किया। मानसिक गणित की कई अन्य ऐसी प्रणालियां आज उपलब्ध हैं, और अगर हमारा

इरादा इनको बढ़ावा देना है तो हमें पहले एक ज्वरस्थित तुलना करना चाहिए। और इमानदारी से 'वैदिक' जैसे भ्रामक लेबल को हटा देना चाहिए, जो भारतीय कृष्ण तीर्थ की प्रणाली का मुख्य चिह्न बंधु है और उनको आकर्षित

चंद्रकांत राजू

करता है, जो बाहरी आवरण से किसी चीज के मूल्य का अंदाजा लगाता है।

'वैदिक गणित' को बढ़ावा देने से वेद में जो असली गणित है वह हटा जाता है। उदाहरण के लिए, यजुर्वेद 17.2 से पाठ चलता है के दयालव्य स्थान मूल्य प्रणाली बहुत प्राचीन है। अंकों के उन नामों में कुछ नाम आज भी प्रचलित हैं (हालांकि अरब या अर्बुदम जैसे कुछ का अर्थ बल्ल गण है)। केवल गणितीय रूप से पिछड़े यूरोपियों ने हजारों साल के विलंब के बाद उसे अपनाया।

इसी तरह क्रमपरिवर्तन और संयोजन के सिद्धांत वैदिक छंद (और भारतीय संगीत) में बुने हुए हैं, और इसका वर्णन पिंगल के 'छंदःसूत्र' से भास्कर की 'लोलखलती' में मिलता है। ऋग्वेद के अक्ष मुक्त में पांसे के खेल का सूत्र वर्णन है। यह खेल संभावना के सिद्धांत का आधार है, और महाभारत में नल और दाम्यंती की रोमांटिक कहानी इसे नमूना सिद्धांत से जोड़ती है (एक पेड़ पर पत्तों की संख्या गिनने के लिए)। परंपरा के स्वयंपू चैपियन जब वैदिक गणित को महज भ्रामक फिल्मों के आधार पर प्रोत्साहन देते हैं, तो ऐसे सभी गंभीर प्रवास खतरों में पड़ जाते हैं, क्योंकि उन्हें भी कोई गंभीरता से नहीं लेता।

परिचयों और औपनिवेशिक शिक्षा-ज्वररुका को बदलना जरूरी है, विशेष रूप से गणित के संबंध में। भारतीय परंपराओं का इस प्रक्रिया में कामी योगदान है। लेकिन भ्रामक लेबल वाले 'वैदिक गणित' को फेलाना निश्चित रूप से सही तरीका नहीं है।

'वैदिक गणित' जैसे गलत समाधान इसलिए बने रहते हैं, क्योंकि असुस्थित राजनीति राजनीतिक व्यवहारी चाहते हैं और वचनार्थ के प्रति वक्तावर कितानी को दरकिनार करते हैं। ('योग्यता' केवल आरक्षण के संदर्भ में महत्वपूर्ण है!) इतिहास बताता है कि ऐसी राजनीतिक प्रक्रियाओं से परंपराओं को नुकसान होता है।

जैसे मैंने अपनी किताब 'इनेवेन रिक्कर्स ऑफ टाइटम' में लिखा था कि जो लोग धर्म का इस्तेमाल कर राज सत्ता पाते हैं या बनाए रखते हैं, वे उस धर्म के सबसे बड़े दुश्मन होते हैं, चाहे वह जिन भी धर्म के प्रतिनिधित्व का दावा करें, ईसाई, इस्लाम या हिंदू। □

परिधमी और अपनिवेशिक शिक्षा-व्यवस्था का बदलना जरूरी है, विशेष रूप से गणित के संबंध में। भारतीय परंपराओं का इस प्रक्रिया में काफी योगदान है। लेकिन भ्रामक लेबल वाले 'वैदिक गणित' को फेलाना निश्चित रूप से सही तरीका नहीं है।

Invest in knowledge

- ▶ Baghdad invested in knowledge from 8th c. and flourished.

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contribution to
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 - ▶ Fell in 12th c. just because it imprisoned Nasir-ud-din Tusi who then helped Hulegu conquer Baghdad.

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- ▶ Then US, and Russia, and today China invested in knowledge and flourished.
- ▶ Let us invest in knowledge, and create some space for truly knowledgeable truth-seekers.

Gaṇita vs math

Question 2

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- ▶ The two are NOT the same.
- ▶ Whether or not we had it earlier, we had it different.

Gaṇita vs math

Key differences

- ▶ *Gaṇita* \neq math

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- ▶ *Gaṇita* \neq math
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 - ▶ Formal math is **anti-empirical** (pure metaphysics)
- ▶ *Gaṇita* (normal math) accepts approximate calculation in **real world**
 - ▶ formal math claims to be **exact** knowledge (but in a fantasy world inaccessible to the senses; e.g. a geometric point is declared invisible).

E.g. 1. Geometry: “Pythagorean theorem”

- ▶ We had it earlier than Pythagoras: *śulba sūtra*-s state the “Pythagorean” proposition, **BUT**

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- ▶ Difference: Manava *śulba sūtra* uses **square roots**.

Manava śulba sūtra

▶ Manava śulba sūtra 10.10

manava-sulba-sutra-10-10.png

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▶ Manava śulba sūtra 10.10

manava-sulba-sutra-10-10.png

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Rajju Ganit

Part 2



C. K. Raju



ರಾಜ್ಜು ಗಣಿತ ಕಾರ್ಯಾಗಾರ

Aide et Action Widadha Conservation Trust

WORKSHOP ON RAJJU GANIT
18TH TO 24TH, MAY, 2017

Venue : BRC, ChamaraJanagora

SUPPORTED BY
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E.g. 2: Different concept of zero

Zeroism

- ▶ Bhaskar (*Līlāvātī* 47) allows division by zero (*kha-hara*) disallowed in formal arithmetic.

खेनोद्धृता दश च, कः खगुणो निजार्ध-
युक्तस्त्रिभिश्च गुणितः खहृतस्त्रिषष्टिः ॥ ४७ ॥

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$$\frac{(x \times 0 + \frac{x}{2} \times 0) \times 3}{0} = 63$$
- ▶ $x = 14.$

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- ▶
$$\frac{(x \times 0 + \frac{x}{2} \times 0) \times 3}{0} = 63$$
- ▶ $x = 14$.
- ▶ Division by zero (a) NOT a mistake, (b) NOT *pāṭīgaṇita*, (c) NOT to be understood using limits.

Calculus without limits

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0 as **infinitesimal** completely changes how calculus is done
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- ▶ This involves doing calculus and summing infinite series with so-called non-Archimedean arithmetic (*avyakt* fractions).
- ▶ Something very recently understood in the West.
- ▶ Not understood by Newton etc. or colonised mathematicians.

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- ▶ Repeat: Newton not only did not invent calculus, he did not fully understand it.

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- ▶ Correcting that error also corrects his theory of gravitation.

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- ▶ Newton's physics **hence** failed
- ▶ (he made a conceptual error about time, because he misunderstood Indian calculus).
- ▶ Correcting that error also corrects his theory of gravitation.
- ▶ Here are some recent papers on my new theory of gravitation.

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Retarded gravitation theory

C. K. Raju

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ckr@ckraju.net*

Abstract. We propose a Lorentz-covariant theory of gravity, and explain its theoretical origins in the problem of time in Newtonian physics. In this retarded gravitation theory (RGT), the gravitational force depends upon both retarded position and velocity, and the equations of motion are retarded functional differential equations. We explicitly solve these equations, under simplifying assumptions, for various NASA spacecraft. This shows that the differences from Newtonian gravity, though tiny, are just appropriate to explain the flyby anomaly as a $\dot{\tau}$ effect due to earth's rotation. We also show that, in the case of a spiral galaxy, the combined velocity drag from a large number of co-rotating stars enormously speeds up a test particle. RGT can also be tested in the laboratory. It necessitates a reappraisal of current laboratory methods of determining the Newtonian gravitational constant G .

Keywords: Retarded gravity, galactic rotation curves, flyby anomaly, functional differential equations, experimental tests of gravity

PACS: 04.80.Cc, 04.50.Kd, 98.62.Dm, 45.20.D-, 02.30.Ks

INTRODUCTION

The notion of time is fundamental to this reformulation of gravity. Though published long ago[1], my analysis of the notion of time is perhaps not so widely known, and this is briefly summarized below.

The problem of equal intervals of time in Newtonian physics

It is well known that Newtonian physics had a difficulty in defining equal intervals of time. One cannot bring back one hour from the past and put it side by side with one hour in the future and compare the two in the present to say by inspection that the two time intervals are equal. The equality of two time intervals is a matter of definition. Such a definition is needed to make sense of Newton's first law: "uniform" motion means a particle covers equal distances in equal times. This presupposes a theoretical clock: what is uniform motion according to a pendulum would not be uniform motion according to one's heart beats, or an atomic clock. Newtonian physics does not explicitly specify which clock to use. For applications to planetary motion or ballistics, there are many clocks which one could use in practice. However, the lack of a theoretical definition of equal intervals of time became prominent when Newtonian physics had to be reconciled with Maxwellian electrodynamics. If we define equal intervals of time so as to make Newtonian physics valid, that makes Maxwellian electrodynamics invalid.

Functional differential equations.

4: Retarded gravitation

C. K. Raju

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(Submitted 27-06-2015)

Abstract

Are functional differential equations (FDEs) only about electrodynamics? No. They apply also to gravitation. We explain a recent reformulation of gravitation, called retarded gravitation theory (RGT), which is Lorentz covariant, and uses functional differential equations. RGT modifies the Newtonian "inverse square law" gravitational force: the RGT force depends upon (a) retarded distance, and (b) includes a velocity-dependent term. RGT, since Lorentz covariant, theoretically improves on Newtonian gravitation. At the same time, RGT has the practical advantage over general relativity theory (GRT) that a solution of the many-body problem is feasible in RGT. Hence, RGT can and ought to be applied to the galaxy where Newtonian physics apparently fails but GRT cannot be applied. The tiny velocity dependence of the RGT force is amplified across a hundred billion co-rotating stars in the galaxy, so that non-Newtonian velocities of stars in spiral galaxies are to be expected on RGT, even without dark matter. Possible experimental tests of RGT include the flyby anomaly observed for NASA spacecraft which depends systematically on velocity-effects due to the rotation of the earth. We further clarify that Laplace's objection to pre-relativistic naive theories of retarded gravitation (NRG) does not apply to RGT. We solve the 2-body FDEs of RGT for the sun-Jupiter case: the system is stable despite tiny differences from Newtonian gravitation. Thus, FDEs are a general feature of post-relativity physics.

1 Recap

In three earlier articles[1, 2, 3] in this series, we saw that functional differential equations (FDEs) are fundamentally different from ordinary differential equations (ODEs). Hence, doing physics with FDEs leads to a paradigm shift in physics. Further, FDEs arise naturally in classical electrodynamics: *without* any new physical hypotheses but just by doing the math right. The right way to solve for the classical hydro-

gen atom, even without radiation damping, is to use FDEs and that changes the qualitative features of the solution.

What happens if we have radiation damping? The problem of the motion of even a single charged particle, in classical electrodynamics, with radiation damping has remained mathematically unsolved for a century because of runaways. These runaways can be controlled by modifying Maxwell's equations at the microphysical level, so that the equations of motion

A proposed experiment to test theories of the flyby anomaly

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G. D. Parikh Centre, J. P. Naik Bhavan
Mumbai University Kalina Campus
Vidyanagari, Santacruz (E)
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June 25, 2017

Abstract

We use Lorentz covariant retarded gravitation theory (RGT), without simplifications, to validate the earlier calculations for the flyby anomaly as a gravitational effect of Earth's rotation at the special relativistic ($\frac{v}{c}$) level. Small differences persist between the theoretical predictions of RGT and the data reported by Anderson et al. That reported data, however, is not direct observational data but consists of un-modeled residues. To settle doubts, we propose a 3-way experimental test to discriminate between RGT, Newtonian gravitation (no flyby anomaly), and Anderson et al.'s formula. This involves two satellites orbiting Earth in opposite directions in the equatorial plane in eccentric orbits. For these orbits, Earth's rotation should not affect velocity on (1) Newtonian gravitation and (2) the formula of Anderson et al. However, (3) on RGT, one satellite gains and the other loses velocity, by typically a few cm/s/day, which is easily measurable by satellite laser ranging.

- ▶ So, switching from math to *gaṇita*

- ▶ So, switching from math to *gaṇita*
 - ▶ makes math easy

- ▶ So, switching from math to *gaṇita*
 - ▶ makes math easy
 - ▶ makes science better.

- ▶ Trick: Western (formal) math allows church metaphysics to be smuggled into science

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- ▶ and makes church superstitions credible.

- ▶ Trick: Western (formal) math allows church metaphysics to be smuggled into science
- ▶ and makes church superstitions credible.
- ▶ As in the creationism of Stephen Hawking.



TV Sunday

THE mag

Monday, January 16, 2011

The Christian propaganda in Hawking's work

What do the Pope and Stephen Hawking have in common? Both propagate a Christian view of how the universe came into being. While the Pope is direct, seeing the hand of God in the Big Bang, Hawking does it more subtly. His popular books provide a scientific veneer to Christian theology, while projecting non-Christian views of creation as unscientific, reveals Professor CK Raju, who is currently with the School of Mathematical Sciences, Universiti Sains Malaysia



Stephen Hawking's The Universe in a Nutshell. Photo: Amazon.com

Stephen Hawking's new book, The Universe in a Nutshell, has caused a stir in the Christian community. In it, he claims that the Big Bang is evidence of God's hand in creation. This is a bold statement for a scientist who is known for his atheistic views. The book is a mix of science and religion, and it is clear that Hawking is trying to convince his readers that his faith is based on science.

The introduction contains a list of 100 names of scientists who have contributed to the field of cosmology. This is a list of names that are well-known in the scientific community, and it is clear that Hawking is trying to establish his credibility as a scientist.

Christianity and the Bible. The Bible is a book that has been read by billions of people around the world. It is a book that has shaped the lives of many people, and it is a book that has been the source of much controversy. The Bible is a book that is full of stories and teachings, and it is a book that has been the source of much inspiration.

The introduction of Hawking's book is a good example of how a scientist can use religion to support his work. Hawking is a scientist who is known for his work in cosmology, and he is a scientist who is known for his work in physics. He is a scientist who is known for his work in science, and he is a scientist who is known for his work in research.



Prof CK Raju

In a nutshell

Politics of creation. The politics of creation is a complex issue that has been the subject of much debate. It is a topic that has been discussed by many people, and it is a topic that has been the source of much controversy.

Half-hearted science. Science is a field of study that has been the source of much progress. It is a field of study that has been the source of much discovery, and it is a field of study that has been the source of much knowledge.

God's hand in Big Bang. The Big Bang is a theory that has been the source of much controversy. It is a theory that has been the source of much debate, and it is a theory that has been the source of much discussion.

Creationism. Creationism is a belief that the universe was created by a divine being. It is a belief that has been the source of much controversy, and it is a belief that has been the source of much debate.

Science and religion. Science and religion are two fields of study that have been the source of much controversy. They are two fields of study that have been the source of much debate, and they are two fields of study that have been the source of much discussion.

Creation and evolution. Creation and evolution are two theories that have been the source of much controversy. They are two theories that have been the source of much debate, and they are two theories that have been the source of much discussion.

Science and faith. Science and faith are two concepts that have been the source of much controversy. They are two concepts that have been the source of much debate, and they are two concepts that have been the source of much discussion.

Prof CK Raju, who is currently with the School of Mathematical Sciences, Universiti Sains Malaysia, says that Hawking's book is a mix of science and religion. He says that Hawking is trying to convince his readers that his faith is based on science, and he says that Hawking is trying to convince his readers that his faith is based on science.

Hawking's book is a good example of how a scientist can use religion to support his work. Hawking is a scientist who is known for his work in cosmology, and he is a scientist who is known for his work in physics. He is a scientist who is known for his work in science, and he is a scientist who is known for his work in research.

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The Eleven Pictures of Time

*The Physics, Philosophy, and
Politics of Time Beliefs*



C. K. Raju

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- ▶ It also damages traditional beliefs.

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- ▶ E.g. teaching formal math teaches that all systems of Indian philosophy are wrong, for they all accept empirical proof (*pratyakṣa pramāṇa*).

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- ▶ E.g. teaching formal math teaches that all systems of Indian philosophy are wrong, for they all accept empirical proof (*pratyakṣa pramāṇa*).
- ▶ But there is more. . . .

Half-Hindus

- ▶ It rules out the core of Hinduism as anti-scientific.

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- ▶ Hinduism is about *dharma*, *artha*, *kāma*, *mokṣa*

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- ▶ But only half-Hindus today, believe only in *artha*, and *kāma*!

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- ▶ Avoid all talk of *dharma* (ethics) related to *mokṣa*!

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- ▶ It rules out the core of Hinduism as anti-scientific.
- ▶ Hinduism is about *dharma*, *artha*, *kāma*, *mokṣa*
- ▶ But only half-Hindus today, believe only in *artha*, and *kāma*!
- ▶ Avoid all talk of *dharma* (ethics) related to *mokṣa*!
- ▶ Why? Because Western math-in-science has made core of Hinduism disreputable.

Ātman and mokṣa

- ▶ The Upanishads teach that

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- ▶ The Upanishads teach that
- ▶ that there is repeated birth.

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- ▶ The ultimate aim of life is deliverance from rebirth, or *mokṣa*.
- ▶ Is that science?

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Quasi-cyclic time

► Yes!

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- ▶ On the Upanishadic notion not only are individuals reborn, **the entire cosmos approximately repeats.**

Quasi-cyclic time

- ▶ Yes!
- ▶ On the Upanishadic notion not only are individuals reborn, **the entire cosmos approximately repeats.**
- ▶ **Time is quasi-cyclic.**

Cosmic recurrence

- ▶ This cosmic recurrence takes a long time (day and night of Brahmā = 8.64 billion years, according to Bhagvad Gitā (8.17–20), and *Viṣṇu Purāṇa*).

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- ▶ On Newtonian physics, cosmic **must** recur if it is closed. (Poincaré recurrence theorem.)
- ▶ So, **rebirth** across cosmic cycles **is scientifically possible**
- ▶ but not *mokṣa*!

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- ▶ On Newtonian physics, cosmic **must** recur if it is closed. (Poincaré recurrence theorem.)
- ▶ So, **rebirth** across cosmic cycles **is scientifically possible**
- ▶ but not *mokṣa*!
- ▶ (Newtonian physics mechanistic: allows only eternal recurrence, or repeated rebirth, with no escape [as in Nietzsche].)

- ▶ This conflict arises due to a bad way of doing calculus.

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- ▶ and not quasi-cyclic.
- ▶ **Correcting Western misunderstanding of calculus essential for a non-mechanistic physics.**

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 - ▶ May restore core ethical values.