The Religious Roots of Western Mathematics

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Introduction

Three questions

Let me begin with three questions.
Q. 1: Is "science and religion" solely a post-retirement pursuit for scientists?

The answer is obviously yes. Most scientists don't dare to talk about issues related to science and religion until they are safely retired. They are well aware of the "shut up, and calculate" view of science among scientists; they are well aware that scientists who engage in philosophy are looked down upon as deviating from the scientist's job of producing innovations for the consumption of state and industry, and they dare not break this unwritten taboo—at least not while it might materially affect their careers. The actions, obviously, present a truer account of their real value system.

What about social scientists then?
Q. 2: Can a social scientist opine on questions of "science and religion" without a working knowledge of science?

The answer again is obviously yes. The discipline of science is one that it takes many years (even decades) to acquire—and social scientists are not usually willing or able to hold back their comments for that long! One might say that in social science it is not always important to know what one is talking about!

The third question again relates to strangely universal matters of practice rather than any stated positions or dogma.
Q. 3: Does the issue of "science and religion" primarily concern the West?

The answer again is obviously yes. For example, in Brooke's celebrated book of 420 pages, all references to Buddhism and Islam fit in one page (and it is not as if that page contains terse comments of great depth).[1] There is no mention at all of Hinduism.

[1] J. H. Brooke, *Science and Religion: Some Historical Perspectives*, Cambridge University Press, Cambridge, 1991.

Stereotypes

Brooke is only being true to the stereotype. That "science and religion" is seen to be exclusively a concern of the West is also made clear by the following stereotypes characterizing the issue of "science and religion" as that of:

- science *versus* religion
- reason *versus* faith
- Western science *versus* Eastern spirituality
- fact *versus* values
- etc.

There are various other subsidiary stereotypes that are associated with this. For example:

- science is universal
- reason is universal
- science originated in the West
- all religion is a matter of faith
- etc.

My position

Be it noted that *I reject all the above stereotypes* as based on ignorance or prejudice.

As for my own case, I would like to point out that I am still not retired. That I continue to be an active scientist—having most recently published[2] the solutions of the new functional differential equations of physics that I proposed some time back,[3] and having pointed out some new empirical consequences of the existence of advanced interactions.[4] On the relation of mathematics to culture I have a number of earlier talks and publications:

- "The Religious Roots of Mathematics", *Theory, Culture & Society* **23**(1–2) Jan-March 2006, pp. 95–97. Spl. Issue on *Problematizing Global Knowledge*, ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillip.

- "Science religion and globalisaton" ICRIC, Dec 2005. Also Bhopal 2005.

[2]C. K. Raju,"The Electrodynamic 2-Body Problem and the Origin of Quantum Mechanics" *Found. Phys.* **34** (2004) pp. 937–62.

[3]C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994. Fundamental Theories of Physics, vol. 65.

[4]C. K. Raju, "Time Travel and the Reality of Spontaneity", *Found. Phys.* July, (2006)

- "Why Deduction is MORE Fallible than Induction", invited talk at International Conference on Methodology and Science, Vishwabharati, Shantiniketan, Dec 2004. Abstract at: http://www.IndianCalculus.info/Santiniketan.pdf

- "Math Wars and the Epistemic Divide in Mathematics", invited talk at the Centre for Research in Mathematics and Science Education, Univ. of San Diego, Oct. 2004, and paper presented at Episteme-1, Goa, Dec 2004.www.hbcse.tifr.res.in/episteme/allabs/raju_abs.pdf and www.hbcse.tifr.res.in/episteme/themes/ckraju_finalpaper.

- "Science:Religion::Reason:Faith?" ICRIC 2003.

- "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the Yuktibhāṣā" Hawai'i, 2000. *Philosophy East and West* **51**(3) 2001 325–62.

- "Mathematics and Culture", in *History, Culture and Truth: Essays Presented to D. P. Chattopadhyaya*, ed. Daya Krishna and K. Satchidananda Murthy, Kalki Prakash, New Delhi, 1999, pp. 179–193. Reprinted in *Philosophy of Mathematics Education* **11** (1999). Available online at http://www.ex.ac.uk/~PErnest/pome11/art18.htm

Some of these thoughts—those related to matheamtics and its history and philosophy—are being consolidated into the forthcoming book *Cultural Foundations of Mathematics*.

As for the term "West", this is explained in the glossary of my book, *The Eleven Pictures of Time*[5] as follows.

> **West.** On the earth, east and west are relative, and Rome was to the West of Constantinople (Istanbul), the shorter way round the earth. The Roman church followed Augustine's theology, creating a division between Western and Eastern Christianity—a division that later broadened into a division between Western Christianity and everyone else. According to Toynbee, every universal state must have a universal church, and Western Christianity is the religion associated with the only surviving universal state. This is the West for which capitalism is a cultural value according to Huntington.

[5]Sage, 2003

On the Alleged Universality of Maths

True to the above stereotypes, mathematics has long been regarded in the West as universal; not merely global, but universal. This thought is captured in the following remark from Huygens[6]

> no matter how inhabitants of other planets might differ from man in other ways, they must agree in music and geometry, since [music and geometry] are everywhere immutably the same, and always will be so.

However, contrary to this long-standing Western belief in the universality of mathematics, in the above cited works, I distinguish Western mathematics, as

- proof oriented,
- spiritual, and
- anti-empirical,

and different from Indian mathematics, as

- calculation oriented,
- practical, and
- empirical.

That is, contrary to the long-standing Western belief in the universality of mathematics, I regard mathematics as not global. Not merely non-universal, but non-global. Apropos Huygens, it is curious that the difference between Indian and Western mathematics parallels the difference between Indian music and Western music (the equal tempered scale), but this would take us too far afield.

A couple of immediate clarifications are in order. First, Western mathematics is what is socially dominant today; this is the kind of mathematics that is today taught in schools and colleges. Secondly, when I speak of "Indian mathematics" this has nothing to do with "Vedic mathematics" (which in turn has nothing whatsoever to do with the Veda, as is pointed out in the very first page of the book on "Vedic mathematics".[7])

The definition of mathematics

Further clarifications are obviously required. By the word "mathematics" do I refer to the same thing that is understood as "mathematics" in the West? This suggests that we should have before us a working definition of mathematics. What then is mathematics?

[6] Christiaan Huygens, *The Celestial Worlds Discover'd: or Conjectures Concerning the Inhabitants, Plants, and Productions of the Worlds in the Planets*, London, 1698, p. 86.

[7] Bharati Krsna Tirtha, *Vedic Mathematics*, Motilal Banarsidass, New Delhi, Rev. ed. 1992, reprint 1997, p. v, and also p. xxxv, "Obviously these formulas are not to be found in the present recensions of the Atharvaveda".

(Definition) **Mathematics.** Something (a) invented in Greece and (b) further developed in Europe.

This definition might seem preposterous, but, in practice, it is even more accurate than the practical characterization of "science and religion" as solely a post-retirement pursuit for scientists.

Thus, tell a mathematician something, and he will ask "Do you have a proof?" (Else it can't be mathematics.) Now, give the mathematician a proof that involves empirical verification—for example try to establish the existence of a chair, by pointing to an instance of a chair. The mathematician will reject this proof. Though he may or may not say it, what he is implicitly asking is this: "Do you have a proof according to *my* understanding of proof." Press him hard enough, and he might even cite the social community of mathematicians in support of his belief that pointing to a chair does not establish the existence of the chair—not mathematically at least. ("So many experts believe this; they can't all be wrong can they?")

The belief is that mathematical proof **ought** not to involve the empirical.

So, although mathematics is regarded as the basis of science and technology, and the basis of (Western) mathematics is proof, the basis of this proof is an ought-type belief: mathematical proof *ought* not to involve the empirical.

What is the basis of this "ought"?

The current understanding of mathematical proof involves

1. imitating the method of demonstration in the *Elements*,
2. as modified and reinterpreted to suit the principles of Christian rational theology, and
3. as subsequently further reinterpreted in a secular way by Hilbert and Russell.

The *Elements* and all that

So the first leg of this ought-type belief is that all mathematics ought to imitate the method of the *Elements*. A few quick historical points about this are in order. Thus, in India, the *Elements* was known for long before it was translated from Persian into Sanskrit in 1723. Why? Because it was regarded as a religious text of no particular practical value. More specifically, it was regarded as part of the Islamic religious eduction, or *tālīm*: Abu'l Fazl studied the *Elements*, as he records in the *Ain-i-Akbari*,[8] non-Muslims did not. Indian mathematics texts *began* with

[8] vol. 2, pp.15–16; vol. 3, p. 24, Jarett's edition

the "Pythagoras" "theorem", but the (first book of the) *Elements ended* with that. Moreover, compared to the long and complex proof of the "Pythagoras" "theorem" in the *Elements*, simple proofs of this result were long known in India, from before Pythagoras—assuming that he really existed. However, the 20th c. mathematician might reject these proofs, on the grounds that they *ought* to agree with *his* "universal" notion of proof which they do not.

The claim that Pythagoras had some special kind of proof—a deductive proof with no empirical component—is pure historical bunkum. Every known manuscript of the *Elements*, without any exception whatsoever, appeals to the empirical in the proofs of its propositions 1.1, 1.4 (SAS theorem) etc., without the aid of which the "Pythagoras" "theorem" could not be proved. If an appeal to the empirical is made at one point in the proof, is there any reason why it should not be made at another point in the proof? It was only in the 20th c. that an attempt was made by Hilbert, Russell, and Birkhoff to altogether bypass the appeal to the empirical by taking 1.4 as an axiom (SAS axiom) rather than a theorem. Therefore, it is misguided to suggest that "Pythagoras" or anyone else after him, until the 20th c. had any sort of "superior" proof which did not involve the empirical.

Incidentally, the claim that Pythagoras had any sort of proof at all is itself suspect for it is attributed to a rumour by Proclus[9] who comes about a thousand years after Pythagoras. We cannot even be very confident about exactly what Proclus said, since we learn of *his* opinions from a manuscript which comes from at least five hundred, and probably another thousand years after him.

The point is that the ought-type belief about imitating the *Elements* requires us to imitate not any actual historical manuscript of the *Elements*, but to imitate a very suspect historical understanding of that work. The allegedly certain and universal truths of Western mathematics are based, like the certainties of religious dogmas, on ought-type beliefs anchored in myth—or deliberately concocted history.

Even the original sources have not been left untouched, so that it is hard even to refer to the original sources: what is typically passed off today as the published original source of the *Elements* is based on a single convenient manuscript in the Vatican which differs from every other manuscript of the *Elements* known till the 19th c. CE—all of which manuscripts were attributed to the theologically incorrect Theon of Alexandria, last librarian of the Great Library of Alexandria and father of Hypatia.[10]

> All our Greek texts of the *Elements* up to a century ago depended upon manuscripts containing Theon's recension of the work; these

[9]Proclus, *A Commentary on the First Book of Euclid's Elements*, trans. Glenn R. Morrow, Princeton University Press, Princeton, 1992, p. 337.

[10]Sir Thomas Heath, *A History of Greek Mathematics*, Dover, New York, 1981, p. 360.

> manuscripts purport, in their titles, to be either "from the edition of Theon"... or "from the lectures of Theon".

Since the faithful had so brutally murdered the daughter, and also probably the father, it is understandable that the church disliked this piece of history and worked hard to change it.

As for "Euclid", the three geometry papyri fragment obtained from Alexandria do not agree with the received text of the *Elements*, suggesting that no definitive version of the *Elements* existed until the time of Theon.[11] This supports Theon's claim to have authored (or to have begun authoring) the *Elements*. The only information we have about "Euclid" comes from an isolated remark[12] in the Monacensis manuscript of Proclus' *Commentary* on the *Elements* (presumably of Theon or Hypatia). This remark is obviously a later-day interpolation, since it speaks of "Archimedes" having referred to the *Elements* in the course of a proof.[13] This reference (to the *Elements*, not "Euclid") has been regarded as spurious[14] since it was not the custom in the time of Archimedes to make such references to texts (since "standard editions" did not then exist). The culture of making such references to texts obviously postdates Christian theology (and printing technology). Since the Monacensis remark (attributed to Proclus) cites a late spurious manuscript of Archimedes, the remark itself must be spurious and probably comes from the 16th c. CE.

It is this spurious remark, attributed to Proclus, which first speaks of "irrefragable demonstration" in the *Elements*. Be it noted, however, that according to the author of this remark, this supposed irrefragable demonstration is based on "causes and signs"[15] and not on deduction!

The name "Euclid" could well have originated in a Toledan howler. Europe first came to know about the *Elements* in the 12th c. CE through translations from Arabic to Latin at Toledo. During the process of translation from Arabic to Latin—by translators who knew no Arabic(!)—it was understandably easy to mistake the word "Euclides" associated with the Arabic *Elements*, and meaning the "key to geometry", for the name of a Greek called "Euclid".

[11] David Fowler, *The Mathematics of Plato's Academy: A New Reconstruction*, Clarendon Press, Oxford, 2nd ed. 1999, p. 216.

[12] T, L. Heath, *The Thirteen Books of Euclid's Elements*, Dover Publications, New York, [1908] 1956, vol. I, p. 1.

[13] *On the Sphere and the Cylinder* I, Proposition 6, in *The Works of Archimedes*, trans. T. L. Heath, *Great Books of the Western World*, vol. 10, Encyclopaedia Britannica, Chicago, 1996, p. 407.

[14] J. Hjelmslev, "Über Archimedes' Grössenlehre", *Kgl. Danske Vid. Selsk. Mat.-fys. Medd.* (Royal Danish Academy of Sciences) **25** (15) 1950.

[15] Proclus, *A Commentary on the First Book of Euclid's Elements*, trans. Glenn R. Morrow, Princeton University Press, Princeton, 1992, p. 57.

As for the modern-day interpretation of the *Elements* it is highly debatable whether at all one can proceed in the manner of Hilbert and Russell. The basic idea of these two worthies, published in their respective tracts on the foundations of geometry, was that proposition 1.4 of the *Elements* could be taken as an axiom to avoid the introduction of the empirical into mathematical proof. However, this required that, for consistency, *all* movement in space was to be avoided. Since measurement, for example, involves moving a ruler in space and putting it next to the object to be measured, all measurement too was to be avoided. In particular, the notion of "equality" as "coincidence" in the original *Elements* was reinterpreted as "congruence". The problem with this "synthetic" geometry, to be done with an unmarked straight edge and a "collapsible" compass (as distinct from ruler and compass), was that it did not fit the *Elements* after 1.35, where "equality" is used in a new sense of the equality of non-congruent areas.[16] Parallelograms on the same base, and between the same parallels are equal—in the sense of equal areas—but they are not congruent. This applies also to the "Pythagorean theorem", where the equality relates to equality of areas.

Now, it is surely not beyond human ingenuity to define area within synthetic geometry. However, it is, in my opinion, an excessively artificial enterprise to try to define area when the definition of length has been disallowed.

On the other hand if one does accept a metric formulation of geometry, as done by Birkhoff, then the entire book of the *Elements* is trivialised, because the "Pythagorean theorem" can again be proved in a single step—as in Indian proofs, without the aid of the preceding 46 propositions. Obviously, the "author of the *Elements*", as s/he is usually called in Greek texts, did not intend to write a trivial text. So, this interpretation, too, does not quite fit the *Elements*.

So, even without addressing the issue of whether axiomatic and deductive proofs are somehow superior to empirical proof, there are enormous problems in accepting any of the the present day interpretations of "Euclid's" *Elements*.

All these problems obviously arise because the real content of the *Elements* is being ignored in favour of an artificially and externally imposed point of view.

What is this real content?

Mathematics and Relgion

The matter was succinctly addressed by Plato. He prescribed the study of geometry for the young men of the Republic on the ground that "geometry will draw

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[16]C. K. Raju, "How Should 'Euclidean' Geometry be Taught", paper presented at the International Workshop on *History of Science, implications for Science Education*, Homi Bhabha Centre, TIFR, Bombay, Feb, 1999. In Nagarjuna G., ed., *History and Philosophy of Science: Implications for Science Education*, Homi Bhabha Centre, Bombay, 2001, pp. 241–260.

the soul towards truth".[17] He was quite explicit that if geometry concerned perishable (empirical) things only, then it was of no value, for "geometry aims [for] knowledge of what eternally exists, and not of anything perishable or transient".[18]

In *Meno*[19] Socrates demonstrates the untutored slave boy's intrinsic knowledge of geometry, and argues that "if he [the slave boy] did not acquire knowledge of geometry in this life, he must have ... learned it at some other time?" That is, so far as Socrates was concerned, the slave boy's knowledge of geometry was a demonstration of the existence of the soul, for "all learning is but recollection [of knowledge the the soul acquired in its past lives].[20]

Proclus explains why Socrates specifically asked questions related to mathematics, and not to some other kind of knowledge—because the soul being eternal is most easily aroused by the eternal truths of mathematics.[21] Hence, he points out the etymological root of mathematics as "mathesiz" which means "learning", for "learning is recollection of the eternal ideas in the soul... [which have been forgotten since birth]", and explains that "the study that especially bring us the recollection of these ideas is called *the science concerned with learning* ([mathematik])", which "through the discovery of the Nous leads us to the blessed life"[22]

He had earlier explained[23]

> For soul is also Nous.... Realizing this, Plato constructs the soul out of all the mathematical forms, divides her according to number, binds her together with proportions, and harmonious ratios, deposits in her the primal principles of figures, the straight line and the circle, and sets the circles in her moving in an intelligent fashion. All mathematicals are thus present in the soul from the first.

Thus, the spiritual and religious content of mathematics was perfectly obvious and explicit to the philosophers from Plato to Proclus. Mathematical truths were valued above empirical truths not on any grounds related to deduction, but on the grounds that mathematical truths were eternal, while empirical things were perishable. The empirical *was* permitted, though only at the beginning of mathematics, because Proclus explains mathematical forms as concerning an intermediate state of being, which led from the perishable to the eternal, so that[24]

[17]Plato, *Republic*, VII, 527; *The Dialogues of Plato*, trans. Benjamin Jowett, *Great Books of the Western World* vol. 6, Encyclopaedia Britannica, Chicago, 1996, p. 394.

[18]*Republic*, p. 394, translation modernised by author.

[19]Plato, *Meno*, in *Dialogues of Plato*, cited above, p. 182.

[20]Socrates in Plato's *Meno*, cited above, p. 180.

[21]Proclus, cited above, p. 37

[22]Proclus, cited above, p. 38; emphasis mine.

[23]Proclus, cited above, p. 14.

[24]Proclus, cited above, p. 29.

> Proofs must... be differentiated according to the kinds of being... since mathematics is a texture of all these strands and adapts its discourse to the whole range of things.

Now it is quite possible that in a long life one might value only some fleeting moments of ecstasy. However, this valuation of the eternal over the transient came naturally to the karma-samskāra-mokṣa view of the world common to India and the Alexandrian "Neoplatonists". (The commonality with Indian beliefs is no great matter of surprise, since religious beliefs obviously did travel from India to Alexandria, as is recorded in the "victory of Dhamma" in the 33 rock edicts of Ashoka, distributed across India, Pakistan, and Afghanistan.[25] Subsequently it is well known that India had a roaring trade with Alexandria, and some 120 ships sailed annually from India to Alexandria, draining $\frac{1}{3}$rd of the surplus of the Roman empire, according to Pliny. The influence of Indian thought is pretty obvious in, for example, the tract on vegetarianism by the influential Neoplatonist Porphyry.

The religious war against philosophers and mathematicians

How did this relation of mathematics to religious beliefs get lost?

We recall that Proclus belonged to the Alexandrian school, earlier headed by Theon and Hypatia. We also recall that this school was attached to the temple of Serapis, an Egypto-Greek God, and that this temple was smashed, and the adjacent Great Library of Alexandria burnt by Christian mobs led by Bishop Theophilus.

We further recall that a few years later, Theophilus' nephew and successor Bishop Cyril assembled another mob, and dragged Hypatia to a church, stripped her, scraped the skin off her body with tiles, dismembered her and scattered the remaining pieces across Alexandria.

"After this", adds Bertrand Russell, "Alexandria was not bothered by Philosophers". However, that is not entirely true. The book burnings continued, and eventually Proclus was declared a heretic, and all philosophical schools were ordered closed by Justinian in 529 CE.

How did mathematics become involved in religious violence. Why were so many prominent mathematicians the target of religious hatred in the Roman empire? What was it about mathematics that so infuriated the Christian bishops

[25] In the Greek version of the Ashokan edict found in Kandahar, "Dhamma" is represented by the Greek word epidoeia, meaning "piety".

of Alexandria? The question seems not to have been asked earlier. But the answer is obvious enough.

Proclus philosophy of mathematics was remarkably similar to Origen's interpretation of early Christianity—also along the lines of karma-samskāra-mokṣa. But this was contrary to the theology of Augustine, preferred by the state church. Since I have written about this at length elsewhere, I will just summarise the key points here. A key point was equity—to emphasize which, most theorems of the *Elements* are about equality. The belief that all people are equal, regardless of their religious beliefs, was contrary to the state church's claim that converting to Christianity offered special benefits in the future life, after resurrection. Another key issue was transcendence versus immanence: the belief in equity was based on the idea that God was equally a part of all persons. Proclus, for example, explicitly argues that mathematics forces the attention of the student inwards (like *haṭha yoga*), thus enabling him to recognize the Nous within. But this belief in an immanent God went against the belief in a powerful transcendent God who would judge all persons on the day of judgement. A third issue was images: the *Elements* is full of images, or figures, and the idea that images help learning was articulated by Proclus as the idea that images stir the soul. Hence, he argues, Socrates draws a diagram in his discourse with the slave boy. Though, after the second Nicene council, the church accepted images, at that time the church characterized all images as idolatry, and offered this as the justification for smashing temples. (The smashing of the temples, of course, continued, as in Goa, on the grounds that the church still rejected the images of strange gods.) The importance of the issue of images is apparent from the fact that Porphyry wrote a whole book *On Images*, and the church specially singled out for burning the books of Porphyry.

Briefly, the religious beliefs underlying Neoplatonic geometry were "spiritual" in the best sense of the term. But these beliefs were unacceptable to the church which regarded them as a hindrance to the exercise of power. Hence, the church singled out people like Porphyry, Hypatia, and Proclus for attack. A few years later, the church also anathemized Origen.[26]

The return of mathematics

If the church so soundly rejected "spiritual" Neoplatonic mathematics, how was mathematics accepted back into the church fold?

The story is as follows. When the mathematicians/philosophers were expelled from the Roman empire, many of them settled in Jundishapur, in neighbouring

[26] C. K. Raju, "The curse on 'cyclic' time", chp. 2 in: *The Eleven Pictures of Time*, Sage, 2003.

Persia, where they set up a hospital (rather than a temple), and imported and translated numerous works from throughout the world (including works like the *Panctantra*) into Persian. When Persia was conquered by Arabs, the diaspora of these philosophers shifted to Baghdad, where they contributed to the House of Wisdom, and came to be known as the *falasifa*. The *Elements* was one of the books translated into Arabic in the 9th c. Baghdad.

At that time there was a conflict regarding the interpretation of the Koran. The Mutazilah or the *aql-ī-kalām* (Islamic rational theology) maintained that the faculty of *aql* had been given to man to be used. Therefore, the difficult passages in the Koran, whose meaning was not transparent, were to be interpreted by applying one's *aql*. Khalifa al Mamun, an ardent Mutazilite, vigorously promoted this view in Islam. In particular, Islamic theology was deeply influenced by "Aristotle's" theology—which texts are today regarded as consisting of the *Enneads* of Plotinus, and Proclus' *Elements of Theology*. The role of the *Elements* in this process is manifest.

Al Ghazali who attacked the *falasifa*, from a traditionalist Asharite perspective was primarily concerned with refuting their doctrine of causes. (This was used by the *falasifa* for their theory of medicine—treatment presupposes an understanding of the causes of the disease.) Al Ghazali's concern was that if everything had a cause, there was no space left for Allah as the cause of the world. To this end, he denied that Allah was tied by cause and effect. he maintained that the world was created afresh each instant, and observations of cause of effect merely illustrated Allah's habits, which he could, of course, break, if he so chose. In the course of this argument, al Ghazali incidentally conceded that Allah was bound by the laws of logic (since that was not the key issue of concern to al Ghazali).

However, this led to a new understanding of logical truths as truths that bound Allah, for these logical truths were true in all possible worlds that Allah could create. In contrast, empirical truths were true only in those worlds Allah *chose* to create, but he could of course create other possible worlds. So, while Allah could create smoke without fire, and fire without smoke, he could not create a world in which $p \wedge \neg p$. (This is similar to the present-day understanding of logical truths as necessary truths in the sense of being true in all possible worlds, while empirical truths are contingent truths, in the sense that they would be true in only some possible worlds.)

This concession that Allah was bound by logic was, of course, seized upon by al Ghazali's opponent, Ibn Rushd, whose abiding influence on European thought until the 16th c. is well known.

It is also well known that the *Elements* first reached Europe via the translations of Arabic books captured at Toledo, beginning ca. 1125 CE. Since this happened at the time of the Crusades, a time when the church was spreading great religious hatred, it was naturally galling for the church to admit to learning from the hated

Islamic enemy. Therefore, the church, which financed the translation, pretended that all this Arabic knowledge had theologically correct Greek origins. It was in this social context that the *Elements* was attributed to an imagined early Greek called Euclid (about whom nothing else was known), rather than to a late Alexandrian opponent of the church like Theon, or Hypatia, or Proclus. It is surprising how historians down to the present-day uncritically cling to this belief, although the most superficial historical examination immediately shows up its falsity.

This was also the time of the Inquisition, when even supposedly early Greek sources too could be (and were) rejected as heretical. Therefore, many of the books became acceptable only after they were appropriately Christianized, by reinterpreting them in line with then-existing Christian theology. It was natural in this social context that the philosophy of the *Elements* was reinterpreted to suit the tenets of state Christianity. Thomas Aquinas and the schoolmen advocated reason not on the Platonic or Neoplatonic grounds that it encouraged virtue by leading one to the soul within, but on the grounds that reason helped to persuade the non-Christian who did not believe in the Christian scripture but nevertheless accepted reason. Hence, also reason was portrayed as universal, and more compelling than empirical proof, since logical truths bound God, unlike empirical facts which did not. (Aquinas, too, naturally agreed with al Ghazali that God was free to create a world of his choice.) This created the grounds for Christian priests to study the *Elements*.

Thus, mathematics was made acceptable by eliminating the relation of mathematics to the soul, and to spirituality. Medieval Christian priests anyway had little to do with calculation, and the early Greeks were obviously not sophisticated as regards calculation as is clear, for example, from their crude numerical notation and equally crude calendar. Thus, the connection of mathematics to calculation remained largely severed. Mathematics became a model of persuasion, just a way of providing convincing arguments—arguments of the sort that even God would listen to respectfully for they bound him! In short, from an instrument of spirituality, mathematics became an instrument of power.

On the purported universality of logic

If mathematics is regarded as primarily concerning proof, as it came to be regarded in Europe, then the universality of mathematics is a statement about the universality of the means of proof. This, as we have observed is not a factual belief, but an ought-type belief which presupposes the universality of logic. But, what, after all, is the basis for the belief in the universality of logic? Apart from the agreement between some theologians, regarding what Allah or God can do or not do, what ultimately is the basis for the belief that logical truths are superior to empirical truths?

The matter seems not to have been examined in Western thought on mathematics. It is true that Christians and Muslims accepted a certain system of logic. It is even true that this "Aristotelian" logic, like the Aristotelian syllogism, is not very different from Nyāya logic—as is to be expected since Nyāya works found their way probably into Jundishapur, if not Alexandria, and certainly into Baghdad, and thence into Arabic books that were all subsequently attributed wholesale to Aristotle at Toledo. However, the agreement between Christians, Muslims, and Hindus does not make a thing universal—it is necessary to consider also Buddhists and Jains, for example.

Buddhists and Jains use different systems of logic, which are different also from "Aristotelian" logic. This created a major problem in Indian debates, where attempted Nyāya refutations of Buddhism tend to be tangential because they did not fully grasp the *pūrva pakṣa* of a different logic underlying *anumāna*.

Briefly, in two valued logic, we have the situation that

$$p \wedge \neg p \ \Rightarrow q$$

A contradiction implies any statement whatsoever. This is the basis of proofs by contradiction, or proofs based on *reductio ad absurdum*. This is not the situation in either Jaina logic or in Buddhist logic. The Jaina logic of *syādavāda* has been interpreted as 3-valued logic by Haldane[27] (although I do not accept this account). On the other hand, Nāgārjuna's tetralemma only follows the Buddha in the Brahmajāla sutta in proposing four alternatives:[28]

- p : "The world is finite"
- not-p: "The world is infinite"
- **both** p and not-p: "The world is both finite and infinite"
- **neither** p nor not-p: "The world is neither finite nor infinite"

The third alternative admits of a simple semantic interpretation, for example, "The world is finite up and down, and infinite across". The fourth alternative also admits of a simple interpretation: for example, a sceptic like Sanjaya who refuses to commit himself to the first three positions, is put into the fourth category to avoid the proliferation of categories, which is described as the "wriggling of

[27] J. B. S. Haldane, "The *Syādavāda* system of Predication", *Sankhya*, Indian Journal of Statistics, **18** (1957) 195–??; reproduced in D. P. Chattopadhyaya, *Formation of the Theoretical Fundamentals of Natural Science*, Appendix IV C, Firma KLM, Calcutta, 1991, pp. 433–440.

[28] Dīgha Nikāya, e.g., trans. Maurice Walshe, Wisdom publications, Boston, pp. 80–81

the eel". The existence of the third alternative, of the middle way, is essential to Nagarjuna's *Mūlamādhyamakakārikā*, for example, which begins by denying both p and not-p in various cases: these eight negations do not correspond to an eight-valued logic, but to four examples where both p and not-p are denied:

अनिरोधम अनुत्पादम अनुच्छेदम अशाश्वतं ।

अनेकार्थम अनानार्थम अनागमम अनिर्गमं ॥

Even in natural languages, such as English, there is a very common semantic interpretation that one can attach to the phrase "neither good nor bad", for one can readily visualise a whole range of possibilities between "good" and "bad", which are negations of each other. Briefly, the middle way requires the rejection of the "law" of the excluded middle, it is the "law" that is wrong, and not the path that is non-virtuous!

Recall that Kant had argued, in the preface to his *Critique of Pure Reason*, that "Logic is perfect since it has not changed since Aristotle". This not only conflated the "Aristotle" of Toledo with the Aristotle of Stagira, but parochially neglected the views about logic in the rest of the world. So, it would seem that the Western belief in the universality of logic is born of mere parochialism and ignorance.

Deduction vs induction

Not only is logic not universal, the belief that deduction is more certain than empirical observation is not universal either. In Indian tradition, *prataykṣa* is the only means of proof that is universally accepted, since Lokayata for instance denies the validity of *anumāna* irrespective of the logic on which it is based.

In any case logic is NOT universal, as we have seen. Now, the theorems of mathematics vary not only with the axioms, but also with the logic used. So we need to enquire: which logic should be used for mathematics? The question has not been asked in Western culture so far, but that need not prevent us from trying to answer it.

If the choice of logic is based on culture, then, since logic is not culturally universal, the truths of (deductive, proof-oriented) mathematics are reduced to mere cultural truths—not universal, and not even global. For, with a different choice of logic (which denies proofs by contradiction, for instance), one would end up with a different set of theorems.

The other possibility is to fix the nature of logic empirically. there are two points here worthy of note. First, if logic itself is to be decided empirically, then there is no way that logical deduction can claim to be superior to empirical observation. Doubtless induction remains fallible, but if the logic underlying deduction is itself based on induction, then this inductive conclusion too is bound to be fallible, so deduction cannot but be MORE fallible than induction (allowing for the possible existence of potentially fallible deductive proofs that are too complex for the human mind to check).

The second point is that the empirical decision need not necessarily be in favour of two-valued logic. Naturally, to make this empirical decision regarding the logic underlying deduction, we would like to use our most sophisticated *physical* theories, such as quantum mechanics. And for this we might need to allow for the possibility that Scrödinger's cat can be both alive *and* dead at a *single* instant of time. Though I no longer believe in formalism, a more formal statement is that the axioms of quantum mechanics are derivable from a quasi truth-functional logic,[29] of which Buddhist logic may be regarded as an example.

Conclusion

A final question: How did mathematical truth which is metaphysical and disconnected from the empirical come to be regarded as necessary truth (or a higher kind of truth) at the present moment of time? How did it come to be regarded as *more* certain than the empirically manifest? This illogical belief can only be understood through a historical understanding of the religious roots of mathematics, as set out above.

[29] C. K. Raju, "Quantum mechanical time", chp. 6b in *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994.