Symposium on Mathematics in Relation to History, Culture and Technology

Background Paper

The key questions

What is the nature of mathematics? Is mathematics universal? Or has it varied with the historical epoch? Is mathematics secular, or is it aligned with a particular brand of theology? Does it vary with cultural context or the state of technology? What sort of mathematics should we be teaching today? What sorts should be supported by the state?

These and similar questions have been around for some time, and have been repeatedly raised in various international forums.[1] But no answers are forthcoming in defence of the stories that mathematics is universal and secular, or that it is perfect and incorporates necessary truth. If no answers are forthcoming for so long, it is because no ready answers are available, so that it is time to contemplate a change in the practice and teaching of mathematics. This Symposium is being organized with this aim.

[1] Ubiratan D'Ambrosio, Ubi D'Ambrosio, *Ethnomathematics*, and *Socio-Cultural Bases for Mathematics Education*, Unicamp, Campinas, 1985; Paul Ernest, *Social Constructivism as a Philosophy of Mathematics*, SUNY, Albany, 1998, C. K. Raju, "Mathematics and Culture", in *History, Culture and Truth: Essays Presented to D. P. Chattopadhyaya*, ed. Daya Krishna and K. Satchidananda Murthy (eds), Kalki Prakash, New Delhi, 1999, pp. 179–193. Reprinted in *Philosophy of Mathematics Education* **11**. Available at `http://www.people.ex.ac.uk/PErnest/pome11/art18.htm`.

The end of formal mathematics

Formal mathematics (mathematics-as-proof) has been dominant in the existing academic system, but its unacceptable assumptions stand exposed, and its end seems only a matter of time. Proofs (or theorems) are the heart of formal mathematics. But, from a practical point of view, given, say, a differential equation, the important thing is to calculate its solution, and not to prove the existence of a solution. A solution may be calculated even in the absence of an existence proof; further, a mere proof of existence without an accompanying method of calculation would make the equation unusable from a practical viewpoint.

However, much of Western philosophy is built on the premise that mathematical proofs (or theorems), being purely deductive, have an intrinsic value because they somehow are the locus of necessary truth. The belief is (as in Popper's philosophy of science) that the logical force of deduction transcends the merely empirical, that deduction is superior to induction. However, the use of another cultural perspective, say a Buddhist or Jain perspective, immediately exposes this as mere metaphysical posturing. The empirically manifest must remain the first means of proof, for, as Buddhist and Jain logic show, there is no agreement even on the logic underlying proof. So an approach which puts metaphysics above physics can never be universal. Since logic is not culturally universal, it too may need to be decided empirically. Granting that the empirical is fallible, logical deduction can only be *more* fallible.[2] In any case, the logic underlying natural language or quantum physics is not 2-valued, so the logic of the real world is unlikely to be two-valued. Mathematical proof, then, is no more than a game that present-day mathematicians play.

Technological perspective

From the present-day technological perspective, playing this game is counter-productive. Children are taught formal arithmetic, which clashes with computer arithmetic, where the associative "law", for example, must fail for floating-point numbers. Why should children be taught to view computer

[2]C. K. Raju, "Why Deduction is MORE Fallible than Induction", invited talk at International Conference on Methodology and Science, Vishwabharati, Shantiniketan, Dec 2004. Abstract at `http://www.IndianCalculus.info/Santiniketan.pdf`.

arithmetic as unnatural and erroneous? Viewed from a pragmatic or realistic perspective (such as *śūnyavāda*), this clash of arithmetics only exposes the inherent errors of idealistic mathematics.[3] This clash is only likely to exacerbate with anticipated future technology such as that of quantum computing with its (quasi truth-functional) logic of parallel worlds.[4]

Historical perspective

Historically, the present-day notion of mathematical proof derives from claims about a certain "Euclid" who perhaps originates in a Toledan howler of the 12th c. CE, for the name might simply mean "the key to geometry". Though conjectures, concoctions, and myths abound, nothing is actually known[5,6,7] about this "Euclid" beyond this name, which is not found in any original manuscripts of the *Elements* attributed to him.[8] The way this name has been used by the proponents of racist history is evident from the current NCERT school texts[9] which pictorially associate this name with a concocted Caucasian stereotype, thus indoctrinating millions of Indian school children with this racist myth.

[3]C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhāṣā", invited plenary talk at the *8th East-West Conference*, University of Hawai'i, Jan, 2000. In *Philosophy East and West*, **51**:3, July 2001, pp. 325–362. Preprint: `http://IndianCalculus.info/Hawaii.pdf`.

[4]In the sense of the structured-time interpretation of quantum mechanics; see C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994, chp. 6b.

[5]C. K. Raju, "How Should 'Euclidean' Geometry be Taught", paper presented at the International Workshop on *History of Science, Implications for Science Education*, Homi Bhabha Centre, TIFR, Bombay, Feb, 1999. In Nagarjuna G., ed., *History and Philosophy of Science: Implications for Science Education*, Homi Bhabha Centre, Bombay, 2001, pp. 241–260.

[6]David Fowler, Historia Matematica discussion list 10 Nov 2002 (in response to a query about the statement that Euclid perhaps did not exist). `http://mathforum.org/kb/thread.jspa?threadID=381990&messageID=1175734`.

[7]For more details, C. K. Raju, "Good-Bye Euclid!" (submitted for publication) preprint available from `http://IndianCalculus.info/Good_bye_Euclid_journal.pdf`.

[8]T. L. Heath, *A History of Greek Mathematics*, Dover, New York, 1981, p. 360.

[9]*Mathematics: Textbook for Class IX*, (J. V. Narlikar, P. Sinclair, et al.), NCERT, New Delhi, 2005.

Cultural perspective

On the other hand, from Egyptian Mysteries through Plato and Proclus to Islamic rational theology (*aql-ī-kalām*) the religious understanding of the *Elements* was roughly constant,[10] though it differed from the post-Toledo reinterpretation of the *Elements* which made it conform to Christian theology, during the Crusades and Inquisition. It is this reinterpretation on which Hilbert's analysis built, and from which the present-day notion of mathematical proof emerged.

Educational perspective

On the principle that phylogeny is ontogeny, a fresh historical understanding of mathematics provides a fresh educational perspective.[11]

The calculus was transmitted from India to Europe, but the attempt to assimilate it into the European understanding of mathematics took over three centuries thorough the formal construction of real numbers using set theory, and until the formalization of set theory itself. A similar epistemological difficulty attended the transmission of Indian arithmetic to Europe via the Arabic "algorismus" texts, first misunderstood by Pope Sylvester II (d. 1003 CE), and abacus methods continued to be used for the "exchequer" (as its name suggests) until they were abolished, with the burning down of the British Parliament building in the 19th c. CE.

The historical difficulty that Europe experienced over a thousand years in learning present-day school mathematics arose because of the following reason. That mathematics (algorismus, calculus) originated in one epistemological context, and the attempt was to assimilated it with a different understanding of mathematics. This involved the kind of contortions (Dedekind's formal real numbers etc.) which have made mathematics hard to understand. Thus, the informal real numbers of the *śulba sūtra* can be under-

[10] C. K. Raju, "The Religious Roots of Mathematics", *Theory, Culture & Society* **23**(1–2) Jan-March 2006, pp. 95–97. Spl. Issue on *Problematizing Global Knowledge*, ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillip. Also, "The Religious Roots of Western Mathematics", invited talk at JNU seminar on "Science and Spirituality", IIC, Feb 2006 (to appear) in Proc.

[11] "Math Wars and the Epistemic Divide in Mathematics", paper presented at Episteme-1, Goa, Dec 2004. At `http://www.hbcse.tifr.res.in/episteme1/allabs/raju_abs.pdf` and `http://www.hbcse.tifr.res.in/episteme1/themes/ckraju_finalpaper`.

stood by a child, but a thorough understanding of formal real numbers (and its associated background) is available today only to a graduate student of mathematics. Hence, also, while the calculus is taught in schools, its formalist understanding requires mathematical analysis which cannot be taught in schools. As the criticism voiced by Descartes, Galileo or Berkeley shows[12] the difficulty with the calculus arose just because idealistic geometry took the straight line as the primary object of geometry, unlike the curved line in Indian tradition, which facilitated the development of the calculus.

The aim of this symposium is to bring together a few experts to confer on the mathematics appropriate to the present place and time. Although formal mathematicians will certainly be represented, they are hardly the sole "experts", for they only *do* what they regard as mathematics, and rarely think about what mathematics has been or ought to be. The hope is to initiate the evolution of an alternative understanding of mathematics, to avoid remaining entrapped in a state of perpetual mimesis.

[12]C. K. Raju, "The Indian Rope Trick: Rope vs Compass Box", submitted for publication. `http://IndianCalcuclus.info//ropetrick_journal.pdf`