Teaching calculus as ganita

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Summary

IITs typically teach calculus from "Thomas' calculus" texts,¹ fat texts of over thousand pages in A4 size,² with small print, used to have a supplemental CD. Despite their prolixity, none of these texts **even properly defines the most basic concepts of calculus**—derivative or integral or function. These texts all have a section on "rigorous definition of limits"; that has all the ritualistic ε 's and δ 's, but leaves out two essential ingredients. (1) The definition of "real" numbers, needed for limits, hence to define derivatives and integrals, or even the exponential function. This is left for

advanced courses on real analysis. Thus, any calculus student can recite the formula $\frac{d}{dx}e^x = e^x$ but they cannot

explain what is meant by e^x or how to do the infinite sum used to define it. This fits nicely with the objective of colonial education to teach how to ape the West **without** proper understanding. (2) Also left out even in advanced courses on real analysis,³ is axiomatic (NOT naive) set theory needed to construct "real" numbers whether as Dedekind cuts or as equivalence classes of Cauchy sequences. Since the desperate desire is only to ape the West, we note that <u>California, the US technology hub, recently canceled calculus teaching in schools</u>. Why? Because the current teaching of calculus is extremely complex, apart from being useless. But teaching data science and statistics without calculus is risky, since it may result in faulty techniques of statistical inference being used, creating bugs in machine learning programs on which tomorrow's world may depend.

This talk will explain that this complexity is not innate to calculus, but is due to its **bad Western understanding**.

The bad Western understanding of calculus arose because the West <u>stole the calculus</u> from India.⁴ Newton and Leibniz <u>claimed its "discovery</u>", on the <u>evil doctrine of Christian discovery</u>, the way Vasco claimed to have "discovered" India (<u>or the sea-route to India</u>⁵). There is ample proof of **theft** of calculus on criminal law: opportunity, motivation, circumstantial and documentary evidence.⁶ The apologia of "independent rediscovery" is ruled out <u>by the epistemic test</u>: knowledge thieves, like students who cheat in an exam, do not **fully** grasp what they steal. Therefore, **the West failed to fully understand the Indian calculus**, particularly how to sum Indian infinite series. Recognizing its failure to understand calculus, even two centuries after Newton, the West eventually invented real numbers and axiomatic set theory (20th c.) needed for that, an excessively complex and practically useless way to understand calculus.

But Indians originally invented calculus in a very simple way, which still works for all practical applications of calculus today. Hence, reverting to calculus as <u>ganita</u> (\neq <u>axiomatic mathematics</u>) makes calculus easy and enables students to solve <u>harder practical problems not covered in Thomas' calculus</u>, as has been demonstrated in teaching experiments with 8 groups in 5 universities in 3 countries.⁷ Instead of teaching calculus as symbolic manipulation of

¹ G. B. Thomas, Maurice D. Weir, Joel Hass, Frank R. Giordano, *Thomas' Calculus*, Dorling Kindersley, 11th ed., 2008. James Stewart, *Calculus: early Transcendentals*, Thomson books, 5th ed, 2007.

² Thomas = 1228 + 34 + 80 + 14 + 6 + 6 + xvi = 1384 pages; size 11 x 8.5 inches, Stewart = 1168 + 134 + xxv pp. = 1327 pages; size 10 x 8.5 inches + CD.

³ W. Rudin, *Principles of Mathematical Analysis* Third ed. New York: McGraw-Hill, 1976.

⁴ C. K. Raju, Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of Calculus from India to Europe in the 16th c, CE (Pearson Longman, 2007).

⁵ Vasco came from Africa to India with the help of an Indian navigator whose instrument, the kamal, he carried back with him. See, "Kamal on Rapalagai" in *Cultural Foundations of Mathematics*, cited above.

⁶ C. K. Raju, 'Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhâsâ'', *Philosophy East and West* 51, no. 3 (2001): 325–62, https://muse.jhu.edu/article/26555/pdf. For an extract see http://ckraju.net/papers/HAWAIIpp26_32.pdf.

C. K. Raju, 'Teaching Mathematics with a Different Philosophy. 1: Formal Mathematics as Biased Metaphysics', *Science and Culture* 77, no. 7–8 (2011): 274–79; C. K. Raju, 'Teaching Mathematics with a Different Philosophy.
2: Calculus without Limits', *Science and Culture* 7, no. 7–8 (2011): 280–85; C. K. Raju, 'Decolonising

integrals and derivatives of elementary functions, this teaches calculus as the numerical solution of differential equations, a technique first invented by the 5th c. Aryabhata (now falsely called "Euler method"), variants of which are used in all current practical applications of calculus, such as sending a rocket to the moon. (Real numbers anyway **cannot** be used in any practical application of calculus or statistics done using computers, which perforce use floating point numbers.) Instead of the "Archimedean" arithmetic of "real" numbers, the ganita method teaches Brahmagupta's (non-Archimedean) arithmetic, of polynomials, which has infinities and infinitesimals, hence no limits. Instead of metaphysical exactitude, ganita uses the opposite philosophy of realistic inexactitude called zeroism⁸ (sunyavada).⁹

However, the colonially indoctrinated are superstitiously **terrified** of anything original or which does not ape the West. Real numbers are NOT needed for calculus, but university mathematicians will persistently and unethically <u>flee from</u> <u>public debate</u> on the validity of what they teach in the calculus classroom. As explained in the <u>prologue</u>, their sole standard of "truth" is approval by Western authority. The West will never approve this ganita thesis because it busts its key myth of Western "superiority" in math, similar to its superstitious myth of racist superiority. Actually, the West was very inferior and backward in math, and at least <u>3000 years behind Indians and Egyptians</u>, and therefore obtained almost all its math, starting from elementary arithmetic, from India,¹⁰ but had difficulties understanding it, <u>such as</u> <u>negative numbers</u>,¹¹ right until the end 19th c. Time to be free of Western falsehoods and hegemony.

A reading list is posted at <u>https://tinyurl.com/decol-list-new</u>. (Note: First read the <u>prologue on colonialism and math</u>.)

About the speaker

C. K. Raju holds an MSc in mathematics, followed by a PhD from the Indian Statistical Institute. (He <u>was selected for</u> <u>IIT JEE</u>, <u>but did not join</u>; <u>joined IIT:Delhi for a PhD in math</u>, but left after attending one class and asking one question which went unanswered.) He taught formal mathematics (real analysis, advanced functional analysis, Schwartz distributions) for several years before abandoning it. He played a key role in building the first C-DAC Param supercomputer, and implemented applications of national importance (space, oil etc.) on it. He later headed India's largest computer science department, and has developed industrial and educational software. He has long been a Professor in various universities India and abroad.

He has authored several radical books such as *Time: Towards a Consistent Theory* (Kluwer Academic, 1994), *The Eleven Pictures of Time* (Sage, 2003), and *Cultural Foundations of Mathematics:* (Pearson 2007). His research work has consistently received high praise across four decades, and he has lectured in leading universities on six continents. He has received various national and international honors and awards. For details, see <u>http://ckraju.net/cv</u>. His current mission is to decolonize mathematics and science.

His interests are wide-ranging and he was part of the initial group of the Project of History of Indian Science Philosophy and Culture, and contributed many articles apart from a book. He was a Fellow and later a Tagore Fellow of the Indian Institute of Advanced Study. He was on the editorial board of the Journal of Indian Council of the Philosophical Research for Indian philosophy of science and ganita.



Mathematics', *AlterNation* 25, no. 2 (2018): 12–43b.

- 8 C. K. Raju, 'Zeroism', in *Encyclopedia of Non-Western Science, Technology and Medicine*, ed. Helaine Selin (Dordrecht: Springer, 2016), http://ckraju.net/papers/Springer/zeroism-springer-f.pdf.
- 9 C. K. Raju, 'California, Indian Calculus and the Technology Race. 2: Don't Cancel the Calculus, Make It Easy!', *Boloji.Com*, 24 December 2021, https://www.boloji.com/articles/52950/california-indian-calculus-and.
- 10 C. K. Raju, 'Precolonial Appropriations of Indian Ganita: Epistemic Issues' (International round table on Indology, IIAS, Shimla, 2020), http://ckraju.net/papers/ckr-indology-abstract.pdf.
- 11 C. K. Raju, 'In Black History Month: A Response to Nature's Editorials on Decolonising Mathematics', *Medium* (blog), 26 February 2023, https://medium.com/@c_k_raju/in-black-history-month-a-response-to-natures-editorials-on-decolonising-mathematics-cadc124fb2c8.