

**Aim :** To study how the time period of a simple pendulum changes when its amplitude is changed.

---

Teacher's Signature

Name: Suvrat Raju  
Class: XIID  
Board Roll No.:

## Table of Contents

<b>Aim</b> .....	<b>1</b>
<b>Apparatus</b> .....	<b>2</b>
<b>Theory</b> .....	<b>3</b>
Introduction .....	3
General Discussion .....	3
The time period for small amplitudes .....	3
The equation of motion for large amplitudes .....	5
The Jacobian elliptic functions .....	9
Time period for large amplitudes .....	12
Details about the method of experimentation .....	15
<b>Procedure</b> .....	<b>16</b>
<b>Observations</b> .....	<b>17</b>
<b>Calculations</b> .....	<b>19</b>
<b>Graph</b> .....	<b>21</b>
<b>Result</b> .....	<b>22</b>
<b>Precautions</b> .....	<b>23</b>
<b>Sources of Error</b> .....	<b>24</b>
<b>Discussion</b> .....	<b>25</b>
<b>Bibliography</b> .....	<b>26</b>

## **Aim**

---

To study how the time period of a simple pendulum changes when its amplitude is changed.

## Apparatus

---

Small heavy bob,

String,

Clamp Stand,

Split Cork,

Glass rod,

Stop watch,

Vernier Calipers,

Scissors,

Match Box,

Candle.

# Theory

---

## Introduction

A simple pendulum ideally consists of a small heavy bob attached to a rigid support by means of a light inextensible string. When we speak of the motion of the simple pendulum, we refer to the oscillations it performs, when the bob is taken to a height (the string remaining taut) and released.

Further, with reference to the above kind of motion, we define amplitude, time-period, and frequency as follows

**Amplitude :-** The amplitude of a simple pendulum is defined as the maximum angular deviation from the mean position of the bob.

**Oscillations :-** If the pendulum moves from one extreme position to the other and back to the first it is said to have performed one oscillation.

**Time Period :-** The time period of the simple pendulum is defined as the time required by the pendulum to complete one oscillation.

**Frequency :-** The frequency of the simple pendulum is defined as the number of oscillations performed per unit time.

## General Discussion

### The time period for small amplitudes

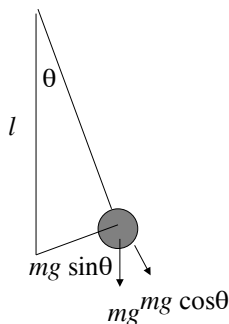


Fig. 1: The geometry of the simple pendulum

For very small amplitudes, the motion of the simple pendulum may be approximated by simple harmonic motion. In this case we have :

$$F = mg \sin \theta, \quad (1)$$

where  $F$  is the restoring force acting on the pendulum,  $m$  is the mass of the bob,  $g$  is the acceleration due to gravity and  $\theta$  is the angular displacement.

Further, for small  $\theta$ ,

$$\sin \theta \approx \theta. \quad (2)$$

Therefore, equation (1) reduces to

$$F \approx mg \theta. \quad (3)$$

By Newton's second law of motion,

$$F = m\ddot{x}. \quad (4)$$

Here,  $x = l \sin \theta = l \theta$ , by virtue of (2), so  $\ddot{x} = l \ddot{\theta}$ , and using (3) and (4), we get

$$\ddot{\theta} = -\frac{g}{l} \theta \quad (5)$$

This is the same as the equation of motion for the simple harmonic motion

$$\dot{\theta} = -p^2 \theta, \quad (6)$$

with

$$\boxed{p^2 = \frac{g}{l}}, \quad (7)$$

which has the solution

$$\theta = A \sin(pt + \phi), \quad (8)$$

representing a simple harmonic motion with amplitude  $A$ , and initial phase  $\phi$ . The time period for this simple harmonic motion is  $\frac{2\pi}{p}$ . Putting back the value of  $p$  from equation (7) we get the well-known expression for the time period of the simple pendulum as:

$$T = 2\pi \sqrt{\frac{l}{g}} \quad (9)$$

### The equation of motion for large amplitudes

**The above expression, however, is not valid for large amplitudes**, since the assumption  $\sin \theta \approx \theta$  no longer provides a good approximation for large  $\theta$ . Therefore, we shall now endeavour to derive an exact equation of motion for the simple pendulum which will include the case of large amplitudes. We will see that the differential equation obtained is not analytically solvable. The analysis is interesting because it shows that *the time period of the simple pendulum is dependent on the amplitude*.

Let  $l$  be the length of the pendulum and let  $P$ ,  $K$ , and  $E$  be respectively the potential energy, kinetic energy and total energy of the system. Then

$$P = -mgl \cos \theta, \quad (10)$$

and

$$K = \frac{1}{2}ml^2\dot{\theta}^2 \quad (11)$$

since the velocity of the center of mass, at any instant, is  $l\dot{\theta}$ . Hence,

$$E = P + K = \frac{1}{2}ml^2\dot{\theta}^2 - mgl \cos \theta \quad (12)$$

At its maximum displacement the pendulum is instantaneously at rest. That is, if  $\alpha$  is the amplitude of the pendulum, then for  $\theta = \alpha$  we have  $\dot{\theta} = 0$ . Substituting in (12) we get

$$E = -mgl \cos \alpha \quad (13)$$

Therefore, equation (12) reduces to

$$-mgl \cos \alpha = \frac{1}{2}ml^2\dot{\theta}^2 - mgl \cos \theta \quad (14)$$

$$\Rightarrow \frac{1}{2}ml^2 \dot{\theta}^2 = mgl \cos \theta - mgl \cos \alpha$$

$$\Rightarrow \dot{\theta}^2 = 2p^2 (\cos \theta - \cos \alpha) = 4p^2 \left( \sin^2 \frac{\alpha}{2} - \sin^2 \frac{\theta}{2} \right), \quad (15)$$

where  $p^2 = \frac{g}{l}$ , as before, and the second equality follows from the well-known identity  $\cos \theta = \cos \left( \frac{\theta}{2} + \frac{\theta}{2} \right) = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = 1 - 2 \sin^2 \frac{\theta}{2}$ .

Let us define  $\varphi$  by

$$\sin \varphi = \frac{\sin \frac{\theta}{2}}{\sin \frac{\alpha}{2}}$$

$$\Rightarrow \sin \frac{\theta}{2} = \sin \frac{\alpha}{2} \sin \varphi. \quad (16)$$

Differentiating (16), we get



$$\frac{1}{2} \cos \frac{\theta}{2} \cdot \dot{\theta} = \sin \frac{\alpha}{2} \cos \varphi \cdot \dot{\varphi} \quad (17)$$

Multiplying (15) by  $\frac{1}{4} \cos^2 \frac{\theta}{2}$ , we get

$$\frac{1}{4} \cos^2 \frac{\theta}{2} \cdot \dot{\theta}^2 = p^2 \cos^2 \frac{\theta}{2} \cdot \left( \sin^2 \frac{\alpha}{2} - \sin^2 \frac{\theta}{2} \right). \quad (18)$$

The l.h.s. of (18) is the square of the l.h.s. of (17), so it may be replaced by the square of the r.h.s. of (17) to obtain

$$\sin^2 \frac{\alpha}{2} \cos^2 \varphi \cdot \dot{\varphi}^2 = p^2 \cos^2 \frac{\theta}{2} \cdot \left( \sin^2 \frac{\alpha}{2} - \sin^2 \frac{\theta}{2} \right). \quad (19)$$

Substituting for  $\sin^2 \frac{\theta}{2}$  in the r.h.s. from (16), we obtain

$$\sin^2 \frac{\alpha}{2} \cos^2 \varphi \cdot \dot{\varphi}^2 = p^2 \cos^2 \frac{\theta}{2} \cdot \left( \sin^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2} \sin^2 \varphi \right) \quad (20)$$

$$\Rightarrow \sin^2 \frac{\alpha}{2} \cos^2 \varphi \cdot \dot{\varphi}^2 = p^2 \cos^2 \frac{\theta}{2} \sin^2 \frac{\alpha}{2} \cos^2 \varphi. \quad (21)$$

$$\Rightarrow \dot{\varphi}^2 = p^2 \left( 1 - \sin^2 \frac{\theta}{2} \right), \quad (22)$$

upon canceling the common factors.

Again using (16),

$$\dot{\varphi}^2 = p^2 \left( 1 - \sin^2 \frac{\alpha}{2} \sin^2 \varphi \right). \quad (23)$$

If we multiply this equation by  $\cos^2 \varphi$ , we get

$$\dot{\varphi}^2 \cos^2 \varphi = p^2 \cos^2 \varphi (1 - \sin^2 \frac{\alpha}{2} \sin^2 \varphi). \quad (24)$$

If we now put,

$$y = \sin \varphi = \frac{\sin \frac{\theta}{2}}{\sin \frac{\alpha}{2}}, \quad (25)$$

we have

$$\dot{y} = \dot{\varphi} \cos \varphi \quad (26)$$

and

$$1 - y^2 = \cos^2 \varphi. \quad (27)$$

Further, putting

$$k = \sin \frac{\alpha}{2} \quad (28)$$

and substituting in (24), we get

$$\dot{y}^2 = p^2 (1 - y^2) (1 - k^2 y^2). \quad (29)$$

To get rid of the constant  $p^2$  let us define a new variable

$$\boxed{x = pt} \quad (30)$$

so that

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\dot{y}}{p} \quad (31)$$

Substituting in (29) we get

$$\boxed{\left(\frac{dy}{dx}\right)^2 = (1 - y^2)(1 - k^2 y^2)} \quad (32)$$

### The Jacobian elliptic functions

The above equation (32) cannot be solved in terms of elementary functions. Nor is the above form suited for numerical computations. Thus, it is obvious from physical considerations that the angular displacement of the pendulum reaches a maximum and then starts diminishing. Hence, from the definition of  $y$  in (25), and the definition of  $\varphi$  in (16), it is clear that  $y$  must reach a maximum and then start decreasing. Hence, the sign of  $\frac{dy}{dx}$  will change as  $y$  crosses its maximum. But, for numerical computation, the sign of the square root must be specified unambiguously. In order to do this, we now need to transform the above equation.

We define the function  $\text{sn}(x)$  to be that solution of (32) which satisfies the conditions

$$\text{sn}(0) = 0, \quad \text{sn}'(0) > 0. \quad (33)$$

Using  $\text{sn}(x)$ , we define two more function  $\text{cn}(x)$  and  $\text{dn}(x)$  as follows

$$\text{cn}^2 x = 1 - \text{sn}^2 x, \quad \text{cn} 0 = 1, \quad (34)$$

$$\text{dn}^2 x = 1 - k^2 \text{sn}^2 x \quad \text{dn} 0 = 1, \quad (35)$$

with the further condition that the function and their derivatives be continuous.

Since  $\text{sn}(x)$  has been defined as a solution of equation (32), it satisfies that equation. That is,

$$\left(\frac{d}{dx}\text{sn} x\right)^2 = (1 - \text{sn}^2 x)(1 - k^2 \text{sn}^2 x). \quad (36)$$

Substituting from (34) and (35) we get

$$\left(\frac{d}{dx}\text{sn} x\right)^2 = \text{cn}^2 x \cdot \text{dn}^2 x. \quad (37)$$

We can now specify the sign of the square root in (32). This specification, due to the mathematician Jacobi, now takes on the elegant form

$$\boxed{\frac{d}{dx}\text{sn} x = \text{cn} x \text{ dn} x.} \quad (38)$$

Differentiating (34), we get

$$2 \text{cn} x \frac{d}{dx} \text{cn} x = -2 \text{sn} x \frac{d}{dx} \text{sn} x = -2 \text{sn} x \text{cn} x \text{dn} x. \quad (39)$$

where the second equality follows from an application of (38). Thus, canceling  $2\text{cn } x$  from both sides, we obtain

$$\boxed{\frac{d}{dx}\text{cn } x = -\text{sn } x \text{ dn } x} \quad (40)$$

Similarly, differentiating (35), we get

$$2\text{dn } x \frac{d}{dx}\text{dn } x = -2k^2\text{sn } x \frac{d}{dx}\text{sn } x = -2k^2\text{sn } x \text{ cn } x \text{ dn } x$$

where the second equality again follows from (38). The above equation simplifies to

$$\boxed{\frac{d}{dx}\text{dn } x = -k^2 \text{sn } x \text{ cn } x} \quad (41)$$

These three simultaneous differential equations (38), (40), and (41) can now be numerically solved to obtain the values of  $\text{sn } x$ ,  $\text{cn } x$ ,  $\text{dn } x$ . This also solves the equation (32), which, by definition of  $\text{sn } x$ , has the solution

$$y = \text{sn } x + \text{constant}. \quad (42)$$

Using the definition of  $y$  in (25), and the definition of  $x$  in (30), this allows us to express the angular displacement  $\theta$  as a function of time  $t$ .

$$\boxed{\theta = 2 \sin^{-1} \left[ \sin \frac{\alpha}{2} \cdot \text{sn}(pt) + \text{constant} \right]} \quad (43)$$

where the constant is fixed by the initial conditions.

The three functions  $\text{sn } x$ ,  $\text{cn } x$ , and  $\text{dn } x$  are known as the Jacobian elliptic functions. As can be seen from the definition (35), for  $k = 0$ , the function  $\text{dn } x \equiv 1$ . For this case of  $k = 0$ , the equations (38) and (34) are satisfied by the usual trigonometric functions, so that  $\text{sn } x$  reduces to  $\sin x$ , and  $\text{cn } x$  reduces to  $\cos x$ . Thus, the Jacobian elliptic functions may be considered generalizations of the trigonometric functions. The graphs of these periodic functions also are very similar to those of the trigonometric functions, though not identical.

### Time period for large amplitudes

The results of numerically computing the time-period of  $\text{sn } x$  are given below. This computation shows that theoretically the time period of the simple pendulum must change with the amplitude. From equation (43) we can see, that the  $\theta$  is periodic with the same period as that of  $\text{sn}$ . Also, according to the standard result,<sup>†</sup>  $\text{sn } x$  has the period  $4K$ , where

$$4K = \int_0^1 \frac{dy}{\sqrt{(1-y^2)(1-k^2y^2)}} \quad (44)$$

Using  $x = pt$ , the time period of oscillation of the simple pendulum is given by

$$T = \frac{4K}{p} = \frac{4}{p} \int_0^1 \frac{dy}{\sqrt{(1-y^2)(1-k^2y^2)}} \quad (45)$$

substituting  $y = \sin \phi$  back into the equation, we get

---

<sup>†</sup> J.L. Synge and B.A. Griffith, *Principles of Mechanics*, McGraw-Hill, New York, 1959, Third Edition, p 334.

$$T = \frac{4}{p} \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{(1 - k^2 \sin^2 \varphi)}} \quad (46)$$

As already observed, this elliptic integral must be evaluated numerically. But an analytical approximation may be used as a quick check of the numerical computation. On binomial expansion of the integrand in (46) we get,

$$T = \frac{4}{p} \int_0^{\frac{\pi}{2}} \left( 1 + \frac{1}{2} k^2 \sin^2 \varphi + \frac{1 \cdot 3}{2 \cdot 4} k^4 \sin^4 \varphi + \dots \right) d\varphi. \quad (47)$$

Now,

$$\int_0^{\frac{\pi}{2}} d\varphi = \frac{\pi}{2}, \quad (48)$$

while,

$$I = \int \sin^2 \varphi d\varphi = \int \sin \varphi \cdot \sin \varphi = \int (-\cos \varphi)' \cdot \sin \varphi d\varphi = -\cos \varphi \cdot \sin \varphi + \int \cos^2 \varphi$$

$$= -\cos \varphi \sin \varphi + \int (1 - \sin^2 \varphi) d\varphi = -\cos \varphi \sin \varphi + \int d\varphi - \int \sin^2 \varphi d\varphi.$$

Hence,

$$2I = -\cos \varphi \sin \varphi + \int d\varphi, \text{ which implies}$$

$$\int \sin^2 \varphi = \frac{1}{2} \int d\varphi - \frac{1}{2} \sin \varphi \cos \varphi \quad (49)$$

so that,

$$\int_0^{\frac{\pi}{2}} \sin^2 \varphi d\varphi = \frac{1}{2} \cdot \frac{\pi}{2}. \quad (50)$$

Similarly,

$$\begin{aligned} I_1 &= \int \sin^4 \varphi d\varphi = \int \sin^3 \varphi \cdot \sin \varphi d\varphi = \int \sin^3 \varphi \cdot (-\cos \varphi)' d\varphi \\ &= -\sin^3 \varphi \cos \varphi + \int \cos \varphi \cdot 3\sin^2 \varphi \cdot \cos \varphi d\varphi \\ &= -\sin^3 \varphi \cos \varphi + \int 3\sin^2 \varphi (1 - \sin^2 \varphi) d\varphi \\ &= -\sin^3 \varphi \cos \varphi + 3 \int \sin^2 \varphi d\varphi - 3I_1, \text{ which implies} \end{aligned}$$

$$\int \sin^4 \varphi = \frac{3}{4} \int \sin^2 \varphi d\varphi - \frac{1}{4} \sin^3 \varphi \cos \varphi \quad (51)$$

so that

$$\int_0^{\frac{\pi}{2}} \sin^4 \varphi d\varphi = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \quad (52)$$

which gives us

$$T = \frac{2\pi}{p} \left( 1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1}{2} \cdot \frac{3}{4}\right)^2 k^4 + \dots \right) \quad (53)$$

Putting back  $p^2 = \frac{g}{l}$  and  $k = \sin \frac{1}{2} \alpha$  and taking only the first term in (53) we get as the first approximation,



$$T = 2\pi \sqrt{\frac{l}{g}} \quad (54)$$

which agrees with equation (9).

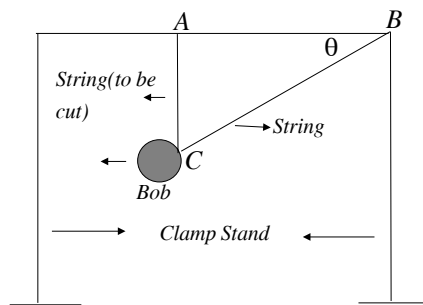
On taking the second term in (53) we get an improved second approximation,

$$T = 2\pi \sqrt{\frac{l}{g}} \left( 1 + \frac{\alpha^2}{16} \right) \quad (55)$$

which clearly shows us that the time period varies rather significantly with the amplitude.

### Details about the method of experimentation

While trying to test the above theory, the primary practical problem that arose was to find a method to measure the amplitude accurately. The second problem was to ensure that the bob oscillated in one plane. The third problem was to ensure, that no force was inadvertently applied while releasing the bob. All three problems were solved by using the apparatus detailed in the diagram.



Mainly, the apparatus consisted of a clamp stand to which a split cork was attached, and through which the string BC (see figure) was passed and tied to the bob. By varying the length of the string, the length of the simple pendulum could be varied. Further, a horizontal glass rod was also attached to the stand. Now another string of known length AC was tied to the bob, and was looped over the glass rod as shown. By measuring AB and using the two known lengths AC and BC, the angle  $\theta$  could be calculated using trigonometry.

**Fig: 2 A schematic depiction of the apparatus**

Now, when the reading was to be taken, the string attached to the glass rod was cut causing the bob to start oscillating. This ensured that no external force was applied to the bob, and this also ensured that it moved in a single plane.

## Procedure

---

- 1) First the radius of the bob was measured using Vernier calipers.
- 2) Now a string was tied to the bob, and a length of 100 cm (including the hook and the radius of the bob) was marked.
- 3) The string was now passed through a split cork attached to a clamp stand. It was lowered, till the length through the split cork exactly equaled 100 cm.
- 4) Now, a horizontal glass rod was also clamped to the stand at one end and another stand, exactly parallel to the original stand, at the other end. The arrangement was such that the top of the rod touched the split cork.
- 5) Now another string was tied to the bob. A known length was marked out on it.
- 6) This string was now tied at the glass rod, at a specific distance from the split cork so that the amplitude equaled a desired angle.
- 7) Now this second string was burnt, using a candle, and the stop watch was started.
- 8) After a specified number of oscillations, the stop watch was stopped. The elapsed time was divided by the number of oscillations to give the time period.
- 9) This procedure was repeated both for different angles and for different lengths of the pendulum.

## Observations

---

Two sets of observations were taken. In one, the length of the simple pendulum was 100 cm and 20 oscillations were timed. In the other, the length of the pendulum was 80 cm and 10 oscillations were timed. These numbers were taken, because for a larger number of oscillations, the amplitude was observed to change considerably due to damping. This was also done, in order to get maximum variety, as factors such as damping would effect the second set less, but random errors would be minimized in the first.

Least count of the stop watch — 0.1 s

Least count of the metre scale — 0.1 cm

radius of the bob — 0.9 cm

Length of the pendulum — 100 cm

No of oscillations timed — 20

S. No	Amplitude ( $^{\circ}$ )	Time required (s)	Observed Time period (s)	Expected Time period (s)	Classical Time period (s)
1	60	41.9	2.095	2.153	2.006
2	55	41.7	2.085	2.128	2.006
3	50	41.5	2.075	2.107	2.006
4	45	41.4	2.070	2.086	2.006
5	40	41.2	2.060	2.069	2.006
6	35	41.0	2.050	2.054	2.006
7	30	40.9	2.045	2.041	2.006
8	25	40.6	2.030	2.030	2.006
9	20	40.4	2.020	2.021	2.006
10	15	40.0	2.000	2.015	2.006

Length of the pendulum — 80 cm

No of oscillations timed — 10

S. No	Amplitude( $^{\circ}$ )	Time required (s)	Observed Time Period (s)	Expected Time Period (s)	Classical Time period (s)
1	60	19.0	1.900	1.926	1.794
2	45	18.3	1.830	1.866	1.794
3	30	18.1	1.810	1.826	1.794
4	15	18.0	1.800	1.802	1.794

## Calculations

---

The values in the “expected time period” column of the observation table were obtained by calculating the time period by simultaneously solving the system of equations (38), (40) and (41) numerically. All computations for numerical solution of the differential equations were carried out on a 486 DX2 processor running DOS 6.22 using a double precision 8th order Runge-Kutta algorithm (the coefficients of Dormand and Prince) in the executable software CALCODE. The software also computes the sections of the solution using a high-order interpolation scheme, and the *regula falsi* method. The time period was calculated by noting the difference between successive zeros of the function  $\text{sn}(x)$ .

Inputs to the executable are the three first-order equations (38), (40), (41), entered symbolically, with the convention that

$$y_1 = \text{sn}, \quad y_2 = \text{cn} \quad y_3 = \text{dn} \quad (56)$$

The input *parameters* for these equations are the values of  $p$ , and  $k$  given respectively by (7), and (28): these values can evidently be calculated from a knowledge of the length of the string,  $l$ , the acceleration due to gravity,  $g$ , and the amplitude  $\alpha$ , and are tabulated below for the actual settings used. These values of  $p$ , and  $k$  suffice to fix the time period of oscillation. A small auxiliary computer program was written to calculate  $p$  and  $k$ . The *initial data* are the same in all cases, and are given by (33), (34), and (35), viz.,  $y_1(0) = 0$ ,  $y_2(0) = 1$ ,  $y_3(0) = 1$ .

The table below also compares the time period calculated numerically by the above sophisticated numerical method with the approximation given by (55).

$p = (g/l)^{1/2}$ ( $l = \text{length}$ )	S. No.	$\alpha$	$k = \sin \frac{\alpha}{2}$ ( $\alpha = \text{amplitude}$ )	Time period from CALCODE	$\frac{2\pi}{p}$	Time period from equation (55)
3.131557 ( $l = 1.00 \text{ m}$ )	1	60	0.5	2.153242	2.006409	2.143926
	2	55	0.461749	2.128457		2.121962
	3	50	0.422618	2.106942		2.101907
	4	45	0.382683	2.086612		2.083763
	5	40	0.342020	2.069291		2.067528
	6	35	0.300706	2.054229		2.053203
	7	30	0.258819	2.041338		2.040789
	8	25	0.216440	2.030548		2.030284
	9	20	0.173648	2.021796		2.021689
	10	15	0.130526	2.015038		2.015004
3.501187 ( $l = 0.8 \text{ m}$ )	1	60	0.5	1.925918	1.794587	1.917580
	2	45	0.382683	1.866322		1.863774
	3	30	0.258819	1.825828		1.825337
	4	15	0.130526	1.802304		1.802274

## Graph

---

Two graphs were plotted, one for each set of readings.

The classical time period, the expected time period, and the observed time period were depicted on the graph together.

## Result

---

The variation in the time period of a simple pendulum, on change in amplitude was studied.

(i) An accurate expression for derived both for the equation of motion and the time period for large amplitudes. This was verified within experimental errors.

(ii) A graph was plotted between the Amplitude and the Time Period.



## Precautions

---

- 1) The length of the simple pendulum must take into account, both the length of the hook and the radius of the bob.
- 2) The string should not be stretched excessively, while measuring the length.
- 3) The stop watch should be started immediately after the second supporting string is burnt.
- 4) The glass rod, should be completely horizontal.
- 5) The glass rod, should touch the cork, and should not have a very large diameter, as it is assumed to be a straight line, in calculations.
- 6) Before the string is burnt, the bob should be absolutely stationary. During motion, care should be taken, that no torque acts on the bob, and the string does not coil up on itself.

## Sources of Error

---

1) The major source of error was damping due to friction at the point of suspension, and also due to air. Due to this friction, the total energy of the bob does not remain exactly constant. Instead, the total energy continuously decreases along with the amplitude. Hence, also, the time period changes from oscillation to oscillation. The experimental arrangement, therefore, does not reflect the theoretical assumptions very accurately. Hence one can observe a systematic departure from the theoretically calculated values.

2) Extraneous sources, such as air currents can also cause random errors.

3) Errors are also caused by the least count of the stop watch and the metre scale, the thickness of the glass rod, the slight extensibility and mass of the string and the slight non spherical shape of the bob

## Discussion

---

The graph of the Time Period vs the Amplitude clearly shows that though the theory provides a very close approximation to the observations initially, a steady deviation is shown as the amplitude increases.

This is partly because of the various sources of error like the extensibility of the string, the slight non-spherical shape of the bob, the least count of the stop watch etc. but also partly because the modeling of the physical situation is not exact.

The project may be improved by using a more sophisticated theoretical model which takes damping into account. This would also require a superior apparatus. A better quality string, a thinner glass rod, and a stop watch with a smaller least count would be useful. A heavier bob would allow more oscillations to be timed, and would improve accuracy.

## Bibliography

---

J.L. Synge and B.A. Griffith, *Principles of Mechanics*, McGraw-Hill, New York, 1959, pp 330–335.

G. F. Simmons, *Differential Equations*, Tata McGraw-Hill, New Delhi, 1972, pp 21–24.

D. Halliday, R. Resnick and K.S. Krane, *Physics*, Wiley, Singapore, 1994, pp 321–324.