

## Grievance against NCERT school texts in mathematics

This grievance concerns the 9<sup>th</sup> standard mathematics textbook produced by the National Council of Educational Research and Training (NCERT). The present grievance is restricted to chapter 5 of that text titled “Euclid’s geometry”, available in electronic form at <http://ncert.nic.in/textbook/textbook.htm?iemh1=5-15>, and related material (Appendix 1) in this and the class 6 mathematics text.

Mathematics is a compulsory subject at the level of class 9, and the text produced by NCERT is followed by most states and all union territories, and by students throughout the country. As such, a responsibility vests with the NCERT to ensure the accuracy of the material in its texts. However, the material in the said chapter is grossly inaccurate, and involves numerous lies.

### FALSEHOOD 1: NO EUCLID.

The chapter 5 of the class 9 math text is titled “Euclid’s geometry”, but there is no evidence that any person such as “Euclid” ever existed.<sup>1,2</sup>

Briefly, NO Greek manuscripts of the *Elements* (the book “Euclid” supposedly wrote) carry the name of Euclid as the author. On the contrary the Greek manuscripts mention someone else, Theon, as the author.<sup>3</sup> In response to my objections, the leading Western historians of Greek mathematics, David Fowler,<sup>4</sup> long ago accepted that there no evidence for Euclid.<sup>5</sup>

NCERT well knows it is unable to supply the evidence about “Euclid”. This came out during my earlier interactions with NCERT, from 2007, with NCERT “experts” including its former math head Hukam Singh,<sup>6</sup> and former Director Pervin Sinclair,<sup>7</sup> an author of the text. Some of these objections to “Euclid” were also raised publicly.<sup>8</sup>

---

1 A comprehensive review of the evidence is in C. K. Raju, *Euclid and Jesus: how and why the church changed mathematics and Christianity across two religious wars*, Multiversity, 2012.

2 An earlier account is in C. K. Raju, “Towards equity in mathematics education. 1: Goodbye Euclid!”, *Bharatiya Samajik Chintan* (New Series) 7 (4) (2009) 255–264. <http://ckraju.net/papers/MathEducation1Euclid.pdf>.

3 *Euclid and Jesus*, cited above, p. 14, citing Thomas Heath, *A History of Greek Mathematics*, Dover, New York, 1981, p. 360. “All our Greek texts of the *Elements* up to a century ago. . . purport in their titles to be either “from the edition of Theon”. . . or “from the lectures of Theon”.”

4 David Fowler, “What is known at present about the person Euclid? Nothing.” <http://mathforum.org/kb/thread.jspa?threadID=381990&messageID=1175734%#1175734>, *Historia Matematica* discussion list, 10 Nov 2002

5 For a fuller discussion, see *Euclid and Jesus*, chp. 2.

6 The interaction with Hukam Singh took place after my phone call to Krishna Kumar, then Director of NCERT, who asked him to attend a seminar in India International Centre, Delhi, on my book *Cultural Foundations of Mathematics*, Pearson Longman 2007. The first chapter of the book is on Euclid and Hilbert. When asked for the evidence for Euclid, Singh remarked “Why do you need evidence, we go by a committee”.

7 The interaction with Pervin Sinclair took place during the presentation of the paper on “Goodbye Euclid!”, cited at 2 above, at the Indian Social Science Congress, Mumbai 2007. Sinclair picked on only the point that the image of Euclid in the text was racist: a Caucasian stereotype. Subsequently, that image was replaced by another, still Caucasian, but no longer a stereotype.

8 “इतिहास के विचलन”, *Jansatta*, 24 Jan 2008. <http://ckraju.net/papers/Jansatta-Euclid.jpg>. Also, “Teaching racist history” *Indian Journal of Secularism* 11(4) (2008) pp. 25–28. <http://ckraju.net/papers/Teaching-racist-history-scanned.pdf>.

Some ten years ago, I publicly announced a “Euclid challenge prize” of Rs 2 lakhs for any serious evidence about “Euclid”, in front of the Malaysian Deputy Education Minister. The caveat of serious evidence means evidence from primary sources.<sup>9</sup> The NCERT and its selected “experts” have been unable to publicly meet this public challenge—obviously because there are no primary sources for “Euclid”. But NCERT has not changed the school text. If the NCERT and its experts cannot PUBLICLY supply the needed evidence, then our school teachers will be unable to answer any questions raised by children about the authenticity of the text, based on public challenges to it.

Thus, the NCERT has tacitly taken the stand that children in India have no right to raise questions or cross-check the Western secondary sources its “experts” blindly trust. This is not acceptable, or if it is NCERT should declare this publicly as its policy so that everyone is aware that children and others have no right to verify what its textbooks say which itself is blindly copied from Western texts. Else the false statements about “Euclid” must be deleted and replaced by the true statement that there is no evidence for any “Euclid” and this is just another widespread Western myth.



## FALSEHOOD 2. “ONLY GREEKS USED REASONING”. NO. INDIANS TOO USED REASONING

The same chapter 5 of the class 9 NCERT text asserts the total falsehood that Greeks did geometry in a way different from and superior to the way it was done all over the world.<sup>10</sup> That is, as we shall see, the false myth of Euclid (for which there is nil evidence and much counter-evidence) is used to promote a bad way of teaching mathematics, starting with geometry.

First, what is the supposed difference? The school text claims that all others across the world (Indians, Maya, Babylonians, Egyptians) did geometry for “practical purposes”, and the Greeks alone used deductive reasoning in mathematics. This text-book claim is 100% false. Certainly, it is well-documented that Indians, for example, did use reasoning. Thus, inference or deduction is accepted by most schools of Indian philosophy as a valid means of proof. Indian mathematicians too used reasoning. Aryabhata DEDUCED that the earth is round from the observation that far off

9 “Decolonising History: Goodbye Euclid!” Special lecture at Universiti Sains Malaysia, 22 July 2011. See, poster at <http://ckraju.net/blog/?p=63>. The prize for RM 10,000, had been announced earlier, but the video of the talk has the proof of its announcement. The video links have changed, and the video of the lecture “Goodbye Euclid” in 3 parts is now posted at Part 1: <https://www.youtube.com/watch?v=sEK1FCrLHjU>, Part 2: <https://www.youtube.com/watch?v=MFf5co3G3R8>, Part 3: <https://www.youtube.com/watch?v=zomZU949Cnw>.

10 “Also, we find that in some civilisations like Babylonia, geometry remained a very practical oriented discipline, as was the case in India and Rome....But in civilisations like Greece, the emphasis was on the reasoning behind why certain constructions work. The Greeks were interested in establishing the truth of the statements they discovered using deductive reasoning” (NCERT, class IX, chp. 5, pp. 78-79).

trees cannot be seen.<sup>11</sup> The Indian proofs of the Pythagorean theorem too used reasoning.<sup>12</sup> Also, there is absolutely nothing wrong in doing geometry for practical purposes: that is exactly how it should be done. Therefore, this false statement must be replaced by the true statement that Indians (and most others) certainly used reasoning.

### **FALSEHOOD 3. “GREEKS GAVE AXIOMATIC (PURE DEDUCTIVE) PROOFS OR USED REASONING IN A SPECIAL WAY AS IN PRESENT-DAY FORMAL MATHEMATICS”**

There are two parts to this claim, (1) that the Greeks did axiomatic mathematics, and (2) that axiomatic mathematics is superior. Both parts are false, but this paragraph looks at the first part.

**Now, the distinguishing feature of formal or axiomatic mathematics, is NOT the use of reasoning but the exclusion of the empirical.**<sup>13</sup>

Historically, this is clear from the fact that Indian (and world) mathematics used reasoning but did not exclude the empirical. Mathematically, it is clear from the definition of a formal mathematical proof<sup>14</sup> that a proof is a sequence of statements in which every statement is either an axiom or is derived from preceding statements by some rule of reasoning. That is, formal mathematics requires that a proof must reason ONLY from axioms, and is not allowed to use any empirical facts at an intermediate stage.

The 10<sup>th</sup> century book *Elements*, and its later versions,<sup>15</sup> do mention “axioms” and “common notions”, and even state proofs. But the similarity is ritualistic: it is false that the proofs in *Elements* are based SOLELY on axioms or postulates and exclude the empirical. That is the proofs in the *Elements* are NOT formal mathematical proofs. Specifically, while such was the false belief widely prevalent in Europe, until the 19<sup>th</sup> c., it is now well known for the last hundred years that this belief is false, and there isn’t a SINGLE formal deductive proof in the *Elements*, the book attributed to “Euclid”.

Specifically, consider Proposition 1, which is “To construct an equilateral triangle on a given line segment”. The proof of Proposition 1 of the *Elements* involves the empirical, and does NOT proceed solely from axioms. Two arcs are constructed with radius equal to the length of the line segment, and centre at each end of the line segment. **The proof involves the intersection of two arcs.** While one can SEE the arcs intersecting, this is an empirical matter, and cannot be proved from any axioms. If the arcs are drawn on a pixelated computer screen, they look continuous, but they may or may not they may or may not have a point in common, as shown in the figures below.

---

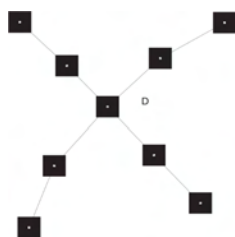
11 Aryabhata, Gola 6-7 said the earth is round like a Kadamba flower. Lalla, (*Sishyadhivrdhida*, chp. 20) explained that this is because far off trees cannot be seen. *Aryabhatiya*, ed. and trans. Shukla and K. V. Sarma, Indian National Science Academy, Delhi, 1976. *Sishyadhivrdhida*, ed. and trans. Bina Chatterjee, Indian National Science Academy, Delhi, 1981.

12 One proof may be found in C. K. Raju, “Computers, mathematics education, and the alternative epistemology of the calculus in the YuktiBhāā”, *Philosophy East and West*, 51:3 (2001) pp. 325–362. <http://ckraju.net/papers/Hawaii.pdf>. This is different from the proof (*upapatti*) demanded in Bhaskara’s *Beejaganita*. in the context of the solution of quadratic equations (verse 13 chapter on quadratic equations, ed. M. M. Pandit, Sudhakara Dwivedi, Varanasi Sanskrit series, p. 69. ed. D. P. Devchandra Jha, Chowkhamba, Varanasi, p. 246).

13 See abstract of Hawaii paper (note 11 above) at <http://ckraju.net/papers/Hawaii-abstract.pdf>. Also, C. K. Raju, “Decolonising mathematics”, *AlterNation* 25(2) 2018, pp. 12-43b. <https://doi.org/10.29086/2519-5476/2018/v25n2a2>.

14 E.g., E. Mendelson, *Introduction to mathematical logic*, van Nostrand, New York, 1964, p. 29.

15 T. Heath, *The thirteen books of Euclid’s Elements*, Dover, New York, 1956, vol. 1.



This is also true of Proposition 4 (the side-angle-side theorem). The proposition is that if two triangles have adjacent sides and included angle equal, then the two triangles are equal. This is proved empirically by moving one triangle in space, rotating it, and placing it on top of another to SEE that they are identical. This is an empirical proof. That 4<sup>th</sup> proposition is used in the proof of the 47<sup>th</sup> proposition (the “Pythagorean theorem”).

Therefore, there is solid counter-evidence that the proofs in the book *Elements* DO use the empirical and are NOT, therefore, different in principle from the Indian proof of the Pythagorean theorem.

However, the proofs in the *Elements* are extremely prolix: if empirical proofs are permitted at one stage, then, since a chain is only as strong as its weakest link, they may as well be permitted everywhere. Then the “Pythagorean theorem” can be proved in one step, as was done in India. So, if empirical proofs are permitted, there is no virtue to the order of the theorems in the *Elements*, and 46 intermediate propositions are NOT needed.

However, until the end of the 19<sup>th</sup> c., Cambridge University foolishly required<sup>16</sup> that, for its examinations, the exact order of the theorems in the *Elements* be rigidly and ritualistically followed. This was even after it “liberalised” its math syllabus to allow that the proofs in the book *Elements* need not be imitated. However, to stick to the order of propositions in the *Elements* it was necessary to include propositions 1 and 4 which have empirical proofs. Once empirical proofs are admitted at one step in the chain, they may as well be admitted everywhere. So, the “Pythagorean theorem” (proposition 47) can be proved in one step as in the Indian proof. But the foolish Cambridge system of teaching geometry was blindly copied in India since Indian education was colonial education which taught blind imitation, minus common sense.

At the beginning of the 20<sup>th</sup> c. it was acknowledged in the West that there are no actual deductive proofs (proofs solely from axioms) in the *Elements*, and Bertrand Russell declared the proofs in the *Elements* to be full of errors.<sup>17</sup> He added, “Thus Euclid fails entirely to prove his point in the very first proposition.... it is a scandal that he should still be taught to boys in England”.<sup>18</sup> However, in the spirit of blind imitation of the West and total lack of accountability, the NCERT is still pursuing the bigger scandal of teaching “Euclid” in independent India.

Instead, the statement that the Greeks did something different in mathematics in the current school text should be replaced by the statement that all evidence shows that “Greeks” did NOT reason in any different way, from Indians, Egyptians etc., though that was a widespread Western myth

16 H. M. Taylor, *Euclid's Elements of geometry*, Cambridge university press, 1893. It begins with the related note of the Cambridge examination board which is posted online at <http://ckraju.net/geometry/cambridge-note.html>.

17 B. Russell, “The Teaching of Euclid”, *The Mathematical Gazette* 2 (33) (1902), pp. 165-167. Posted online at <http://ckraju.net/geometry/Bertrand%20Russell%20on%20Euclid.htm>.

18 B. Russell, “Mathematics and the metaphysician”, in *Mysticism, logic, and other essays*, George Allen and Unwin, 1918, London, chp. 5. Available online at [https://en.wikisource.org/wiki/Mysticism\\_and\\_Logic\\_and\\_Other\\_Essays/Chapter\\_05](https://en.wikisource.org/wiki/Mysticism_and_Logic_and_Other_Essays/Chapter_05).

contrary to all known facts. Children should be taught instead to beware of believing in such myths, still extensively available online.

## **FALSEHOOD 4. “FORMAL (AXIOMATIC) MATHEMATICAL PROOFS ARE SUPERIOR: DEDUCTION IS SUPERIOR TO INDUCTION, DEDUCTIVE PROOFS ARE SUPERIOR TO EMPIRICAL PROOFS”**

This is the second part of the false claim of superiority that axiomatic or formal mathematics is superior (irrespective of whether Greeks actually did formal mathematics). The same NCERT class 9 text has an appendix (Appendix 1) on mathematical proofs, which seeks to explain that theorems of formal mathematics are deduced from axioms. The idea that formal mathematics is superior is **the core superstitious belief** that the NCERT seeks to promote by using the false “Euclid” myth.

That is, formal mathematics is based on the Western superstition that deduction is infallible or at any rate less fallible than induction. This superstition is contrary to elementary common sense. There are many reasons why deduction is fallible.<sup>19</sup>

As the Indian Lokayata (“people’s philosophers”) pointed out, there is no guarantee that deductive proof leads to valid knowledge. One simple reason is that the axioms or postulates may be invalid.

As the NCERT class 9 text (p. 296) correctly states, the term “axiom” (which once meant uncontested truth) is today used interchangeably with the term “postulate”. Both have come to mean anything that one ASSUMES to be true, without proof (pp. 297-98). As Bertrand Russell put it, “We then take any hypothesis that seems amusing, and deduce its consequences”.<sup>20</sup> But if the assumption is false the deduced conclusion or mathematical theorem will also be false or invalid knowledge. Therefore, formal mathematical theorems are NOT valid knowledge. Students must be warned about this.

To illustrate this, the favourite Lokayata example was that of a wolf’s footprints. A man makes a wolf’s footprints at night. The next morning villagers incorrectly deduce that a wolf was around.

An even simpler example is that of the horned rabbit.<sup>21</sup> Consider the propositions (a) all animals have two horns, (b) a rabbit is an animal, therefore (c) all rabbits have two horns. The conclusion is validly deduced, but it is patently absurd, and shows how deduction can easily lead to faulty conclusions. Now, the conclusion is faulty because the assumption (a) is faulty.

Formal mathematicians accept that the theorems of formal mathematics are at best relative truths, relative to the assumptions or postulates. As Russell (cited earlier) put it, “[Formal] mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of *anything*, then such and such another proposition is true of that thing.”

However, the NCERT text deliberately hides this weakness of formal mathematics from students.

---

19 C. K. Raju, “Decolonising mathematics”, *AlterNation* 25(2) 2018, pp. 12-43b. <https://doi.org/10.29086/2519-5476/2018/v25n2a2>.

20 Bertrand Russell, “Mathematics and the metaphysicians”, cited above, 2<sup>nd</sup> paragraph (labeled 75).

21 C. K. Raju, “Mathematics and censorship”, Kafila online, <https://kafila.online/2017/06/25/mathematics-and-censorship-c-k-rajju/>

Is there some way in which the theorems of formal mathematics may be valid knowledge? This would require that we check whether the assumptions or postulates are valid. But how does one check the validity of the assumptions or postulates of formal mathematics? The school text is confused on this point and misleads students about the fact that THERE IS NO EMPIRICAL WAY TO VERIFY THE POSTULATES OF FORMAL MATHEMATICS.

Thus, formal mathematics historically began in the 20<sup>th</sup> c. with the rejection of empirical proofs in "Euclid's" Elements, by Hilbert, Russell et al., particularly proposition 1 (involving intersecting arcs) and proposition 4 (SAS). The attempts to "repair" the proof of proposition 1 involved Dedekind cuts, or formal real numbers. This needs a METAPHYSICS OF INFINITY. Even a single real number such as  $\sqrt{2}$  cannot be written down EXACTLY, for putting down all the infinite decimal places will take an eternity of time. This can only be done in the imagination, that is to say non-physically. But an uncountable infinity of real numbers is asserted to exist. This requires a vaster metaphysics of infinity in the form of axiomatic set theory<sup>22</sup> (since Cantorian set theory was not good enough). Any such beliefs about infinity are not physically testable; they are irrefutable, unverifiable, hence metaphysics in the Popperian sense. Such metaphysics can never be universal.

As a simple example, consider Hilbert's first axiom for his (synthetic) geometry<sup>23</sup>: "two distinct points A and B always completely determine a straight-line a." If we try to verify this with two dots A and B marked on a piece of paper we find that this is FALSE. The two dots always have some finite extent and can therefore be connected in different ways by straight lines which are NOT identical. That is two dots do NOT completely determine a straight-line. Hilbert's first axiom is empirically false.



To hide this empirical falsehood, the NCERT text tries to nudge the imagination of the student to believe in the axiom. The process of indoctrination begins in class 6 where the NCERT text tries to make students believe that a "true" geometric point must be INVISIBLE.<sup>24</sup> Obviously, statements about something invisible cannot be readily empirically verified. (A geometric point also does not have any indirect consequences from which its existence may be inferred like that of an invisible electron.) So, the student is left with no option but to believe the authority of the school text. Obviously also, and this is deliberately never explained to students, imagined or metaphysical entities may be imagined in multiple ways: one may imagine instead that two dots only approximately determine a straight line, and this persists even when the dots are infinitesimally small. But such alternative ways of imagining things is prohibited. It fetches zero marks in the exam.

---

22 Many people, including mathematics teachers seem confused about axiomatic set theory. For axiomatic set theory, see, e.g., Mendelson, cited earlier.

23 D. Hilbert, *The Foundations of Geometry*, trans. E. J. Townsend, Open Court, La Salle, reprint 1950, <http://ckraju.net/geometry/Hilbert-Foundations-of-Geometry.pdf>.

24 NCERT Class 6 mathematics text, 69: "By a sharp tip of the pencil, mark a dot on the paper. Sharper the tip, thinner will be the dot. This almost invisible tiny dot will give you an idea of a point." The idea that a point is invisible, hence not accessible to the senses is reiterated for emphasis on p. 70: "Of course, the dots have to be invisibly thin." How do students know the properties of an invisible point?

That is, formal mathematics is FAITH-based mathematics. One must have faith in its axioms or postulates, which are laid down by Western authorities. This fact is never explained to students that they are required to have blind faith in some Western authorities who have laid down the axioms of geometry or set theory. It is therefore complete bunkum to claim, as NCERT does, that this faith-based formal mathematics is superior to normal mathematics (or FACT-based mathematics, like that of Aryabhata).

Further, irrespective of the validity of the axioms or postulates at the beginning of formal mathematics, THE PROCESS OF DEDUCTION IS STILL A HIGHLY FALLIBLE AND ERROR-PRONE PROCESS. Though contrary to Western superstitious faith in deduction, this is easily verified. When it comes to a complex mathematical proof, even the foremost mathematicians often make mistakes, erroneously imagining that they have proved some theorem. A recent example is the claim by Michael Atiyah that he had proved the Riemann hypothesis.<sup>25</sup>

These are not isolated mistakes. There is a systematic reason why deduction is more fallible than induction. Errors in empirical proofs involve errors of the senses; errors in deductive proofs involved errors of the mind; and the mind (especially the indoctrinated mind) is far more easily deceived than the senses. The game of chess provides a common example of the errors that arise in a complex chain of deduction. EVERY human being ALWAYS errs in chess, and therefore every human being always loses to a machine.

The fact that errors frequently arise in complex deductive proofs has two consequences. First, since a deductive proof is fallible it must be repeatedly rechecked (as anyone who has performed a complex calculation or done a formal proof knows). Checking it a dozen times does not, however, eliminate the philosophical doubt about its validity. That is **a deductive proof must be inductively verified, so deduction is no less fallible than induction.**

Second, the fact is that the present (colonial) system of education followed by NCERT makes most people mathematically illiterate. For example, our school texts teach about real numbers and set theory, but no student I have examined could clearly and correctly state even the definition of a real number or a set.<sup>26</sup> No mathematician I know has checked the 378 page proof of  $1+1=2$  given by Russell. The proof of  $1+1 = 2$  in formal real numbers (from first principles) is far more complex and may run into a thousand pages. It has never been written out fully to my knowledge.

Therefore, for even the proofs of the simplest mathematical theorems like  $1+1 = 2$ , the vast majority of educated people have no choice but to trust and have faith in mathematical authority. That is, **the belief in the validity of deductive proofs is almost entirely a matter of faith** (in Western authority) both as regards assumptions and postulates AND as regards the theorems derived from them. But highly authoritative mathematicians may be wrong or plain dishonest.<sup>27</sup> Faith in authority is always more fallible than facts and empirical observations; hence **deduction is MORE fallible than induction.**

Therefore, the NCERT text must be corrected to point out that common sense shows that formal (deductive) mathematical proofs are always inferior to inductive and empirical proofs though Western philosophers, driven by their indoctrination into church superstitions, have long erroneously believed to the contrary.

---

25 <https://www.newscientist.com/article/2180504-riemann-hypothesis-likely-remains-unsolved-despite-claimed-proof/>.

26 See the pre-test question paper posted at <http://ckraju.net/sgt/3-question-paper-pre-test-sgt.pdf>

27 <http://ckraju.net/atiyah/atiyahcase.html>.

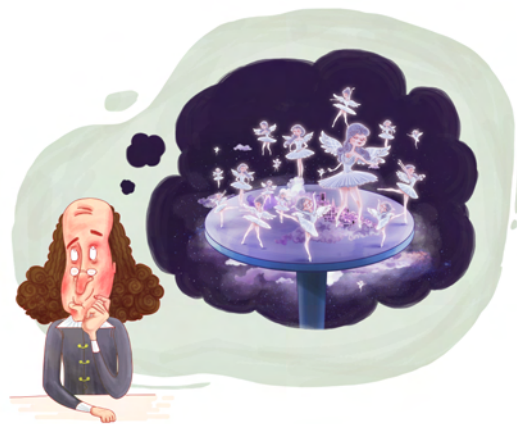
## FALSEHOOD 5. HIDING THE CHURCH CONNECTION

One solid fact about “Euclid’s” *Elements* is that it was used as a church text for centuries. Europe first came to know about the book through Crusading spies like Adelard of Bath and the Toledo nass translations of Arabic texts ca. 1125 CE.<sup>28</sup> It remained an essential text for priests trained in church institutions such as Cambridge University etc. until the 20<sup>th</sup> c. As we saw, until 1888 students of Cambridge were required to blindly copy proofs in the book; later they were told it was necessary only to stick to the order of proofs in the book. The NCERT text never mentions that the book *Elements* attributed to “Euclid” was used as a church text, to train priests, for centuries. The moment we acknowledge this fact, it needs an explanation: what benefit did the church derive from this?

The fact is that Christianity and the church had nothing to do with reason until the Crusades, when the church adopted the theology of reason, adapting it from Muslims (*aql-i-kalam*). It did so because it could find no other way of persuading Muslims to convert. (The Crusades were fought to convert Muslims by force, the way pagan Europe had been earlier converted by force. Muslim Europe then was the richest part of Europe; however, the combined armies of Christian Europe proved to be militarily too weak, forcing the church to resort to persuasion as the preferred means of conversion, to enlarge its kitty.)

Now, it is simple common sense that facts are fatal to church dogmas about virgin birth etc. Accordingly, when the church adopted the Christian theology of reason, it glorified faith-based reasoning and declared fact-based reasoning (as in science) to be inferior. For example, Aquinas reasoned about the number of angels that can fit on a pin, he obviously had no facts about angels, but just postulated whatever he liked.<sup>29</sup> (Aquinas is called a “saint” on the superstitious belief that he performed at least two miracles AFTER his death.)

That is, faith-based reasoning suited the church, therefore, the church glorified it as superior to fact-based reasoning. But it does not suit Indian children who learn mathematics for its practical value for science and engineering, not with a view to propagate church beliefs. Indeed, a formal mathematical proof of  $1+1=2$  adds nothing to the practical value of  $1+1=2$  for commerce or science or engineering, which existed from long before formal mathematics. However, the complexity of the proof makes it impossible for most students to understand. Therefore, it forces everyone to trust authority, which is localised in the West.



*Aquinas' “superior” axiomatic proof about the number of angels on a pin! He explored the reasons why more than one angel can fit on a pinhead.*

The church connection also explains the myth of Euclid. First, the (post-Nicene) church (e.g. Eusebius) regarded the EARLY Greeks as its sole friends, and declared all others to be its enemies to be killed en masse or enslaved, as happened across centuries with pagans, Muslims, native

<sup>28</sup> More details and references in *Euclid and Jesus* cited earlier.

<sup>29</sup> Thomas Aquinas, *Summa Theologica*, First Part, Q. 52, article 3.  
<http://www.newadvent.org/summa/1052.htm#article3>.



Americans, Maya, Inca, Australian aborigines, and Egyptians and other blacks in Africa.<sup>30</sup> Therefore, church historians always chauvinistically attributed all “pre-renaissance” science to the Greeks, real or imaginary. Like all the multitudinous false claims made by the church, there is no primary evidence for any of these chauvinistic claims about Greek achievements. But there are piles of secondary literature, and Wikipedia promotes these belief by insisting on Western secondary sources, so that control of authorised knowledge always stays in Western hands.

Second, colonial education came as church education (there was no secular education in Britain before the British Elementary Education Act of 1870; even after 1870 it was still Christian education but non-denominational). The aim of church education was to do church propaganda. The key aspect of church propaganda is to teach faith. Because church ideology keeps changing as convenient, the faith must be faith in church authority. The effect of this indoctrination is clearly visible in NCERT and its “experts”, all of whom implicitly believe in the authority of church institutions such as Cambridge.

The stock aspect of church propaganda was to say “:we are superior, you are inferior, imitate us”. It started with telling pagans that the Christian God was bigger and more powerful than the pagan gods, a distinction made clear in the grammatical structure of the English language. Thus, the first things mission schools taught was that everything about Indian culture was inferior: the language we speak (Hindi etc.), the way we eat (sitting on the floor, using hands), what we eat (vegetarian food), the clothes we wear (slippers, pyjamas etc.), and that it is a mark of superiority to speak English, eat at a table, eat with spoon and fork, eat non-vegetarian food, wear suits-boots etc. in hot weather. Along the same lines we mindlessly rejected our superior Indian calendar and adopted the inferior Christian calendar, which forces us to recite a variety of Christian superstitions, such as AD, BC, with every date we utter.

The NCERT school text follows the same rant in the teaching of geometry: whatever the rest of the world did was inferior, what the early Greeks (sole friends of Christians) did was superior, and we should imitate it. Actually, under the guise of imitating the Greeks, students are being taught to imitate the church, for the book *Elements* does NOT reject facts the way the church did. Church propaganda is supported by myths. Hence the myth of Euclid, since, the later Greeks, such as Theon etc., and the real author of the *Elements*, his daughter Hypatia, were against the church (A Christian mob burnt the library of Alexandria of which Theon was the last librarian, Hypatia was lynched by another Christian mob and raped and killed most brutally on the altar of a church.) Therefore, a theologically correct but unknown “Euclid” (probably a subtitle Ucli + des meaning key to geometry), was concocted since that made it easier to reinterpret the book to suit the church theology of “reasoning minus facts”. And any intention whatsoever could be attributed to an unknown person, especially with the help of a few suitable forgeries.

Another key consequence of the church’s constant claims of Christian superiority, and consequent papal bulls, was the genocide on three continents (North America, South America, and Australia) and slavery on a fourth (Africa). To promote Western/White mental dominance, and this idea of Western/White superiority, colonial education glorified early Greeks and whites. Instead of examining the evidence, the NCERT “experts” are hell-bent on copying this practice. In a completely racist way, the NCERT school text depicts all Greek mathematicians as white-skinned even though they are all supposedly from Alexandria in Africa, where the default skin color is black.

---

30 For details of the relevant papal bulls see, e.g., C. K. Raju, “The meaning of Christian ‘discovery’”, *Frontier Weekly*, <http://www.frontierweekly.com/archive/vol-number/vol/vol-47-2014-15/47-29/47-29-The%20Meaning%20of%20Christian%20Discovery.html>.

Thus, it is elementary common sense that if nothing is known about the names such as Euclid, Pythagoras, Thales etc., then nothing is known about the color of their skins, and the default skin color (black) should be used (and some Muslim scholars did describe Archimedes as a short black man). However, when the use of a racist stereotype was brought to the attention of an author of that school text and later NCERT Director, Pervin Sinclair, she slyly changed the image to depict a white person though not a stereotype. Clearly her aim is to indoctrinate children to be in awe of not only the West but also of white-skinned people. That is, the NCERT authors and “experts” are intent on doing both racist and church propaganda. This way of trying to poison the minds of children is a criminal depravity, in my opinion. Apart from punishing the authors, children must be protected from the ill effects.

All the images of “white” “Greek”: mathematicians must be deleted from the NCERT school text, and it must be instead explained to students that the images in Wikipedia etc. are false. It must be explained to students that there is no evidence not only for “Euclid” but for any of the claims of Greek origins of mathematics and science, and plenty of counter-evidence. The church and racist interest in falsely promoting early Greeks must also be made clear to children.

## A HOTCH-POTCH OF INCOMPATIBLE GEOMETRIES

These multitudinous falsehoods in the NCERT text have a very bad effect also on the *content* of what is taught.

Because the Western myth of “Euclid” was false, it had to be repeatedly “adjusted”, to “save the story”. Because of NCERT’s policy of blind imitation, the NCERT school text ends up teaching the original myth PLUS all its “adjustments” simultaneously! This results in the comic teaching of a hotch-potch of incompatible geometries.

Thus, the West tried to “save the story” of “Euclid and his deductive proofs” by modifying the proofs given in the *Elements*.

### Dedekind cuts

Robert Dedekind first noticed that the proof of Proposition 1 in the *Elements* is “faulty” in the sense that it did not follow from any axiom. He tried to “save the story” of axiomatic proofs in “Euclid” by inventing “Dedekind cuts” which ensure that two arcs which are seen to intersect will always intersect. However, he was unable to formulate his axioms about cuts correctly. That needed Cantor’s set theory which was itself problematic and incoherent. These problems were supposedly settled by the formal set theory of the 1930’s. However, axiomatic set theory is far too complicated to teach to 9<sup>th</sup> standard students. In fact, today most people including professional mathematicians and heads of IIT mathematics departments cannot correctly define a set.

What 9<sup>th</sup> standard students are told (indirectly) is that it is incorrect to define a set as a “collection of objects” (as other NCERT text books do define) for that would require us to define “collection” and “object”. As the text puts it (pp. 80-81) “to define one thing you need to define many other things”, precipitating an infinite regress (“you may get a chain of definitions without end”). What the text avoids explaining is that the objects such as point, line, etc. that the geometry book talks about are unreal, and purely imaginary, so the infinite regress can NEVER be terminated by pointing empirically towards a real object.

Second, among others, Bertrand Russell noticed that the proof of Proposition 4 in the *Elements* is faulty in the sense that it is an empirical proof. However, he argued that the myth (of special proofs in the *Elements*) could be preserved by rejecting the actual proof of the 4<sup>th</sup> proposition (SAS) of the *Elements*, and replacing the SAS theorem by the SAS postulate (as taught in our NCERT texts today). However, the piecemeal efforts of Dedekind or Russell are not enough for a full axiomatic treatment of “Euclid’s” geometry.

## Hilbert’s synthetic geometry

The first fully axiomatic treatment was given at the turn of the 20<sup>th</sup> c. by David Hilbert. To try to explain the seeming extreme proximity of the *Elements*, Hilbert proposed his synthetic geometry.<sup>31</sup>

“Synthetic” means that distances are NOT allowed to be measured, since measuring distances by using a ruler is an empirical process, which involves picking and carrying the ruler, and placing it above the line segment to be measured. The original proof of Proposition 4 (SAS) is disallowed on exactly this ground since it uses such an empirical process. To copy Hilbert, the NCERT school text (p. 113) declares that SAS is an axiom.

Hence, Hilbert’s synthetic geometry is incompatible with the use of the “compass-box” or “geometry box” to teach geometry. If it is possible to measure lengths, then SAS is a theorem not a postulate.

But this incompatibility is never clearly explained to students. The NCERT text makes one single cryptic remark “ideally a ruler has no markings on it” (Class 6, p. 274).. If so, NCERT should insist that only unmarked rulers are supplied in the compass box. Else, if the aim is to teach synthetic geometry it should replace the compass box merely by straight edge and “collapsible” compasses which cannot be used to pick and carry distances. Else it should explain that Hilbert’s synthetic geometry has no practical use, and should reject it. In this case, SAS reverts to a theorem instead of a postulate. However, the NCERT aim is still that of colonial education: to teach blind imitation, and trust in Western authority and to teach students to distrust elementary common sense. Hence, it teaches both that it is possible to measure lengths, but that “ideally” this should be avoided!

There are other difficulties. Thus Hilbert replaced the term “equality” in the original *Elements* by “congruence” (the term used in NCERT texts). However, this does not work beyond Proposition 34 of the *Elements* which asserts that two parallelograms between the same parallels are equal. This asserts the equality of incongruent areas. Hence, Hilbert’s synthetic geometry which does not define distance or length measurement, does define area (to be able to prove the “Pythagorean theorem”). This peculiarity (of not defining length but defining area) is never explained to students.

## Birkhoff’s axiomatic metric geometry

Dissatisfied with Hilbert’s synthetic approach, G. D. Birkhoff proposed a metric set of axioms “based on ruler and compass”.<sup>32</sup> After the 1957 Sputnik crisis, the US comprehensively restructured its math teaching, and the School Mathematics Study Group<sup>33</sup> recommended that Birkhoff’s metric

---

31 D. Hilbert, *The Foundations of Geometry*, trans. E. J. Townsend, Open Court, La Salle, reprint 1950, <http://ckraju.net/geometry/Hilbert-Foundations-of-Geometry.pdf>.

32 G. D. Birkhoff, 1932. A set of postulates for plane geometry, based on scale and protractor. *Annals of Mathematics* 33, pp. 329-345.

33 School Mathematics Study Group, *Geometry*. Yale University Press, New Haven, 1961.

geometry be taught. This, however, goes against the book *Elements* and makes the proof of the “Pythagorean theorem” very simple. Of course, due to its policy of blind imitation, NCERT changed its teaching of geometry in the 70’s (earlier it copied Cambridge texts on “Euclid”). But it does not explain the difference between axiomatic metric geometry (Birkhoff’s) and empirical metric geometry (“compass-box geometry”). With empirical metric geometry it is still possible to prove the “Pythagorean theorem” in one step. So, what exactly is the advantage of axiomatics? Obviously that it forces faith in Western authority. But this is never explained to students.

## Religious geometry

To summarise, to save the story, it was claimed that providing deductive proofs was the mythical *intention* of the mythical Euclid, and that “Euclid” erred in executing that intention. This is yet another total falsehood, contrary to the available evidence, since Greek literature from Plato<sup>34</sup> to Proclus<sup>35</sup> speaks of a completely different RELIGIOUS intention for geometry.<sup>36</sup> The very word mathematics derives from Plato’s belief in mathesis: that mathematics arouses the soul to make it remember its innate knowledge acquired in previous lives. Students are never informed about this reality. The Wikipedia falsely claims that mathematics derives from mathema, not mathesis, but this false claim is based solely on the authority of Heath, not on any actual Greek sources like Proclus. It is contrary also to what Plato states in the *Republic*, that mathematics ought NOT to be taught for its practical value, but with a view to make the young men of the Republic virtuous, because it helps to arouse the soul, like music.

To avoid misleading and confusing students in this manner, NCERT should not conflate and teach a hotch-potch of five mutually-conflicting systems of geometry as if they are one and the same: (1) early Greek religious geometry (focus on soul and virtue), (2) church religious geometry (focus on persuasion and metaphysical proof), (3) Hilbert’s synthetic geometry, (4) Birkhoff’s axiomatic metric geometry, and (5) compass-box geometry which is empirical and metric. This confused hotch-potch arises from blindly following various European attempts to save the false story of “Euclid”. Instead NCERT should teach a single system of geometry of practical importance.

## THE ALTERNATIVE

If people decide the NCERT must be told to NOT teach silly church myths and superstitions, then it also needs to be considered which INSTRUMENTS are the most appropriate: the compass box or the traditional string, used in India since the days of the sulba sutra-s and in Egypt as well. The compass box has ritualistic Elements such as set squares and the divider. No instrument in it can be used to measure a curved line, the way a string can. The case in favour of the string is clear.<sup>37</sup>

34 Plato, *Meno*, trans. B. Jowett, <http://classics.mit.edu/Plato/meno.html>.

35 Proclus, *Commentary*, [*A Commentary on the First Book of Euclid’s Elements*], trans. Glenn R. Morrow, Princeton University Press, Princeton, New Jersey, 1970, Prologue, p. 38.

36 The religious intent of early Greek geometry, borrowed from Egyptian mystery geometry, is explained for the layperson in detail in *Euclid and Jesus: how and why the church changed mathematics and geometry across two religious wars*, Multiversity, 2012. Also, see, C. K. Raju, “The religious roots of mathematics”, *Theory, Culture & Society* 23 Jan-March 2006, Special Issue ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillips, pp. 95–97, and “Teaching Mathematics with a Different Philosophy. 1: Formal mathematics as biased metaphysics”. *Science and Culture* 77 (2011) pp. 275–80. . arxiv:1312.2099, and in “Eternity and Infinity: the Western misunderstanding of Indian mathematics and its consequences for science today.” *American Philosophical Association Newsletter on Asian and Asian American Philosophers and Philosophies* 14(2) (2015) pp. 27–33. <http://ckraju.net/papers/Eternity-and-infinity-Pages-from-APA.pdf>.

37 C. K. Raju, “Towards Equity in Math Education 2. The Indian Rope Trick” *Bharatiya Samajik Chintan* (New Series) 7 (4) (2009) pp. 265–269. Also, “Decolonised mathematics”, cited earlier.

It is also a more practical instrument whether it is used to measure the area of an agricultural field with curved boundaries or to draw a D in a hockey or football field.

This issue (of an alternative) however can be taken up only AFTER the numerous errors in the NCERT text are removed. The NCERT is not willing to do that, and in typical church fashion it resorted to censorship to protect its irrational position: it refused to allow me to present this<sup>38</sup> in a recent conference. The related email exchange with NCERT is posted online.<sup>39</sup>

C. K. Raju

---

38 The papers were earlier and later discussed in two workshops. The second workshop was in the math department of IIT-BHU, <http://ckraju.net/blog/?p=163>. The papers are posted at <http://www.ckraju.net/IIT-BHU/Alternative-math-1-Rajju-ganit-with-references.pdf>, and <http://www.ckraju.net/IIT-BHU/Alternative-math-2-calculus-without-limits-with-references.pdf>. The context is set by the “Institute lecture” on the preceding day, an extended abstract of which is at <http://www.ckraju.net/IIT-BHU/Alternative-math-2-calculus-without-limits-with-references.pdf>.

39 <http://www.ckraju.net/geometry/correspondence-with-NCERT-NCME.pdf>, and <http://ckraju.net/geometry/NCERT-correspondence-2.pdf>.